Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number | Candidate Number

A level Mathematics

Practice Paper
Pure Mathematics - Differentiation (part 2)

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 11 questions in this question paper. The total mark for this paper is 100.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. The curve C has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{4x+1}{x-2}, \qquad x > 2$$

(a) Show that

$$\mathrm{f}'(x) = \frac{-9}{(x-2)^2}$$

(3)

Given that P is a point on C such that $\mathrm{f}'(x) = -1$,

(b) find the coordinates of P.

(3)

(Total 6 marks)

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2. The curve C has equation

$$y = (2x - 3)^5$$

The point P lies on C and has coordinates $(w, -32)$.

Find

(*a*) the value of w,

(2)

(*b*) the equation of the tangent to C at the point P in the form $y = mx + c$, where m and c are constants.

(5)

(Total 7 marks)

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3. The curve C has equation $x = 8y \tan 2y$.

The point P has coordinates $\left(\pi, \frac{\pi}{8}\right)$.

(a) Verify that P lies on C.

(1)

(b) Find the equation of the tangent to C at P in the form $ay = x + b$, where the constants a and b are to be found in terms of π.

(7)

(Total 8 marks)

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4. (i) Given that $x = \sec^2 2y, \quad 0 < y < \frac{\pi}{4}$

show that $\frac{dy}{dx} = \frac{1}{4x\sqrt{(x-1)}}$

(4)

(ii) Given that $y = \left(x^2 + x^3\right) \ln 2x$

find the exact value of $\frac{dy}{dx}$ at $x = \frac{e}{2}$, giving your answer in its simplest form.

(5)

(iii) Given that $f(x) = \frac{3\cos x}{(x+1)^{\frac{1}{3}}}, \qquad x \neq -1$

show that $f'(x) = \frac{g(x)}{(x+1)^{\frac{4}{3}}}, \qquad x \neq -1$

where $g(x)$ is an expression to be found.

(3)

(Total 12 marks)

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5. The point P lies on the curve with equation

$$x = (4y - \sin 2y)^2.$$

Given that P has (x, y) coordinates $\left(p, \frac{\pi}{2}\right)$, where p is a constant,

(a) find the exact value of p.

(1)

The tangent to the curve at P cuts the y-axis at the point A.

(b) Use calculus to find the coordinates of A.

(6)

(Total 7 marks)

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6. (i) Find, using calculus, the x coordinate of the turning point of the curve with equation

$$y = e^{3x} \cos 4x, \quad \frac{\pi}{4} \leq x < \frac{\pi}{2}.$$

Give your answer to 4 decimal places.

(5)

(ii) Given $x = \sin^2 2y$, $0 < y < \frac{\pi}{4}$, find $\frac{dy}{dx}$ as a function of y.

Write your answer in the form

$$\frac{dy}{dx} = p \operatorname{cosec}(qy), \quad 0 < y < \frac{\pi}{4},$$

where p and q are constants to be determined.

(5)

(Total 10 marks)

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7. (i) Given $y = 2x(x^2 - 1)^5$, show that

(a) $\dfrac{dy}{dx} = g(x)(x^2 - 1)^4$ where g(x) is a function to be determined.

(4)

(b) Hence find the set of values of x for which $\dfrac{dy}{dx} \geqslant 0$

(2)

(ii) Given

$$x = \ln(\sec 2y), \qquad 0 < y < \frac{\pi}{4}$$

find $\dfrac{dy}{dx}$ as a function of x in its simplest form.

(4)

(Total 10 marks)

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8. $f(x) = \dfrac{4x - 5}{(2x + 1)(x - 3)} - \dfrac{2x}{x^2 - 9}, \qquad x \neq \pm 3,\ x \neq -\dfrac{1}{2}.$

(*a*) Show that

$$f(x) = \frac{5}{(2x + 1)(x + 3)}.$$

(5)

The curve C has equation $y = f(x)$. The point $P\left(-1, -\dfrac{5}{2}\right)$ lies on C.

(*b*) Find an equation of the normal to C at P.

(8)

(Total 13 marks)

Turn over

9.

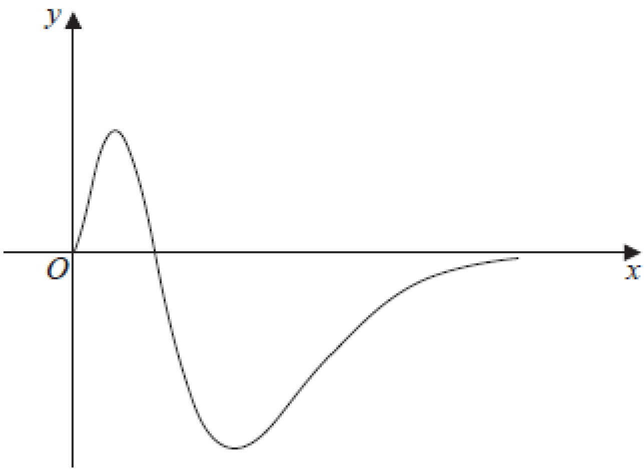

Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$g(x) = x^2(1 - x)e^{-2x}, \qquad x \geq 0.$$

(a) Show that $g'(x) = f(x)e^{-2x}$, where $f(x)$ is a cubic function to be found.

(3)

(b) Hence find the range of g.

(6)

(c) State a reason why the function $g^{-1}(x)$ does not exist.

(1)

(Total 10 marks)

10. Given that k is a **negative** constant and that the function f(x) is defined by

$$\mathrm{f}(x) = 2 - \frac{(x-5k)(x-k)}{x^2 - 3kx + 2k^2}, \qquad x \geq 0,$$

(a) show that $\mathrm{f}(x) = \dfrac{x+k}{x-2k}$.

(3)

(b) Hence find $\mathrm{f}'(x)$, giving your answer in its simplest form.

(3)

(c) State, with a reason, whether f (x) is an increasing or a decreasing function.

Justify your answer.

(2)

(Total 8 marks)

11.

$$\mathrm{f}(x) = \frac{x^4 + x^3 - 3x^2 + 7x - 6}{x^2 + x - 6}, \qquad x > 2, \qquad x \in \mathbb{R}.$$

(a) Given that

$$\frac{x^4 + x^3 - 3x^2 + 7x - 6}{x^2 + x - 6} \equiv x^2 + A + \frac{B}{x-2},$$

find the values of the constants A and B.

(4)

(b) Hence or otherwise, using calculus, find an equation of the normal to the curve with equation $y = \mathrm{f}(x)$ at the point where $x = 3$.

(5)

(Total 9 marks)

TOTAL FOR PAPER: 100 MARKS