Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number

Candidate Number

A level Mathematics

Practice Paper
Pure Mathematics - Trigonometry (part 3)

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 96.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. (i) (*a*) Show that $2 \tan x - \cot x = 5 \operatorname{cosec} x$ may be written in the form

$$a \cos^2 x + b \cos x + c = 0$$

stating the values of the constants a, b and c.

(4)

(*b*) Hence solve, for $0 \leq x < 2\pi$, the equation

$$2 \tan x - \cot x = 5 \operatorname{cosec} x$$

giving your answers to 3 significant figures.

(4)

(ii) Show that

$$\tan \theta + \cot \theta \equiv \lambda \operatorname{cosec} 2\theta, \qquad \theta = \frac{n\pi}{2}, \qquad n \in \mathbb{Z}$$

stating the value of the constant λ.

(4)

(Total 12 marks)

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2. (*a*) Express $4 \operatorname{cosec}^2 2\theta - \operatorname{cosec}^2 \theta$ in terms of $\sin \theta$ and $\cos \theta$.

(2)

(*b*) Hence show that

$$4 \operatorname{cosec}^2 2\theta - \operatorname{cosec}^2 \theta = \sec^2 \theta .$$

(4)

(*c*) Hence or otherwise solve, for $0 < \theta < \pi$,

$$4 \operatorname{cosec}^2 2\theta - \operatorname{cosec}^2 \theta = 4$$

giving your answers in terms of π.

(3)

(Total 9 marks)

Turn over

3. (*a*) Prove that

$$\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} = \tan\theta, \qquad \theta \neq 90n°, \quad n \in \mathbb{Z}.$$

(4)

(*b*) Hence, or otherwise,

(i) show that $\tan 15° = 2 - \sqrt{3}$,

(3)

(ii) solve, for $0 < x < 360°$,

$$\text{cosec}\, 4x - \cot 4x = 1.$$

(5)

(Total 12 marks)

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4. (*a*) Show that

$$\text{cosec}\, 2x + \cot 2x = \cot x, \qquad x \neq 90n°, \qquad n \in \mathbb{Z}$$

(5)

(*b*) Hence, or otherwise, solve, for $0 \leq \theta < 180°$,

$$\text{cosec}\,(4\theta + 10°) + \cot\,(4\theta + 10°) = \sqrt{3}$$

You must show your working.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(5)

(Total 10 marks)

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5. (*a*) Prove that

$$2 \cot 2x + \tan x \equiv \cot x, \qquad x \neq \frac{n\pi}{2}, \qquad n \in \mathbb{Z}$$

(4)

(*b*) Hence, or otherwise, solve, for $-\pi \leq x < \pi$,

$$6 \cot 2x + 3 \tan x = \operatorname{cosec}^2 x - 2.$$

Give your answers to 3 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

(Total 10 marks)

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6. (*a*) Prove that

$$\sec 2A + \tan 2A \equiv \frac{\cos A + \sin A}{\cos A - \sin A}, \qquad A \neq \frac{(2n+1)\pi}{4}, \qquad n \in \mathbb{Z}.$$

(5)

(*b*) Hence solve, for $0 \leq \theta < 2\pi$,

$$\sec 2\theta + \tan 2\theta = \frac{1}{2}.$$

Give your answers to 3 decimal places.

(4)

(Total 9 marks)

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7. $$f(x) = 7 \cos 2x - 24 \sin 2x.$$

Given that $f(x) = R \cos (2x + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$,

(*a*) find the value of R and the value of α.

(3)

(*b*) Hence solve the equation

$$7 \cos 2x - 24 \sin 2x = 12.5$$

for $0 \leq x < 180°$, giving your answers to 1 decimal place.

(5)

(*c*) Express $14 \cos^2 x - 48 \sin x \cos x$ in the form $a \cos 2x + b \sin 2x + c$, where a, b, and c are constants to be found.

(2)

(*d*) Hence, using your answers to parts (*a*) and (*c*), deduce the maximum value of

$$14 \cos^2 x - 48 \sin x \cos x.$$

(2)

(Total 12 marks)

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8. (*a*) Starting from the formulae for $\sin(A + B)$ and $\cos(A + B)$, prove that

$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}.$$

(4)

(*b*) Deduce that

$$\tan\left(\theta + \frac{\pi}{6}\right) = \frac{1 + \sqrt{3} \tan \theta}{\sqrt{3} - \tan \theta}.$$

(3)

(*c*) Hence, or otherwise, solve, for $0 \leq \theta \leq \pi$,

$$1 + \sqrt{3} \tan \theta = (\sqrt{3} - \tan \theta) \tan (\pi - \theta).$$

Give your answers as multiples of π.

(6)

(Total 13 marks)

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9. (*a*) Prove that

$$\sin 2x - \tan x \equiv \tan x \cos 2x, \qquad x \neq (2n+1)90°, \quad n \in \mathbb{Z}$$

(4)

(*b*) Given that $x \neq 90°$ and $x \neq 270°$, solve, for $0 \leqslant x < 360°$,

$$\sin 2x - \tan x = 3 \tan x \sin x$$

Give your answers in degrees to one decimal place where appropriate.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(5)

(Total 9 marks)

TOTAL FOR PAPER: 96 MARKS