

LYGB - MP2 PAPER I - QUESTION 1

BY THE QUOTIENT RULE

$$y = \frac{x^2}{\ln x} \Rightarrow \frac{dy}{dx} = \frac{\ln x \times 2x - x^2 \times \frac{1}{x}}{(\ln x)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x \ln x - x}{(\ln x)^2}$$

$$\Rightarrow \left. \frac{dy}{dx} \right|_{x=\sqrt{e}} = \frac{2\sqrt{e} \ln \sqrt{e} - \sqrt{e}}{(\ln \sqrt{e})^2}$$

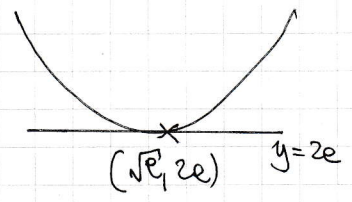
$$\Rightarrow \left. \frac{dy}{dx} \right|_{x=\sqrt{e}} = \frac{\sqrt{e} - \sqrt{e}}{\frac{1}{4}} = 0$$

IT STATIONARY POINT

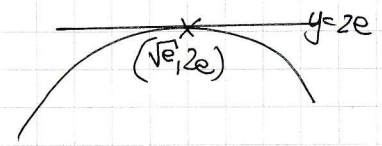
FIND THE y CO-ORDINATE

$$y = \frac{(\sqrt{e})^2}{\ln \sqrt{e}} = \frac{e}{\frac{1}{2}} = 2e \quad \text{at } (\sqrt{e}, 2e)$$

HENCE THE EQUATION OF THE TANGENT IS $y = 2e$



OR



1YGB - MP2 PAPER I - QUESTION 2

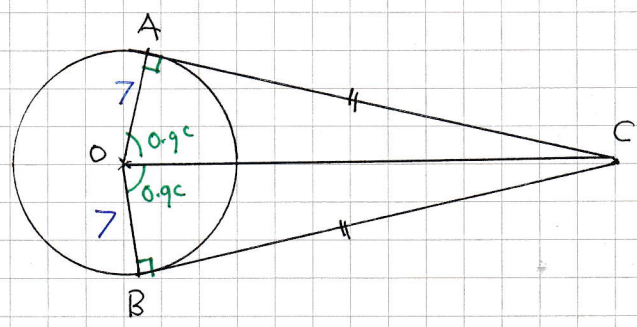
LOOKING AT THE DIAGRAM, ON THE RIGHT ANGLED TRIANGLE $\triangle AOC$

$$\tan(0.9) = \frac{|AC|}{|AO|}$$

$$\tan(0.9) = \frac{|AC|}{7}$$

$$|AC| = 7 \tan(0.9)$$

$$|AC| = 8.821107... \text{ cm}$$



NOW THE AREA OF $\triangle AOC$

$$= \frac{1}{2} |AO| |AC| = \frac{1}{2} \times 7 \times 8.821... = 30.87387... \text{ cm}^2$$

AND THE AREA OF THE KITE $\diamond AOBC$ IS EVIDENTLY TWICE THE AREA $\triangle AOC$

$$\text{"KITE } \diamond AOBC \text{"} = 2 \times 30.87387... = 61.74775... \text{ cm}^2$$

NEXT THE AREA OF THE SECTOR

$$\text{"} \frac{1}{2} r^2 \theta \text{"} = \frac{1}{2} \times 7^2 \times 1.8 = 44.1 \text{ cm}^2$$

FINALLY THE REQUIRED SHADDED AREA IS GIVEN BY

$$61.74775... - 44.1 = \underline{\underline{17.64 \text{ cm}^2}}$$

1YGB-MP2 PAPER I - QUESTION 3

a) OBTAIN THE LIMITS FIRST

• AT A, $x=0$

$$t^2=0$$

$$t=0$$

• AT B, $y=0$

$$1 + \cos t = 0$$

$$\cos t = -1$$

$$t = \pi$$

← ONLY SOLUTION IN $0 \leq t \leq 2\pi$

SETTING UP AN INTEGRAL

$$\text{AREA} = \int_{x_1}^{x_2} y(x) dx = \int_{t_1}^{t_2} y(t) \frac{dx}{dt} dt = \int_0^{\pi} (1 + \cos t)(2t) dt$$

$$\text{AREA} = \int_0^{\pi} 2t(1 + \cos t) dt$$

AS REQUIRED

b) INTEGRATION BY PARTS (IGNORING LIMITS)

$$\int 2t(1 + \cos t) dt$$

$$= 2t(t + \sin t) - 2 \int (t + \sin t) dt$$

$$= 2t^2 - 2t \sin t - 2 \left(\frac{1}{2} t^2 - \cos t \right) + C$$

$$= t^2 - 2t \sin t + 2 \cos t + C$$

$2t$	2
$t + \sin t$	$1 + \cos t$

FINALLY THE AREA

$$\text{AREA} = \left[t^2 - 2t \sin t + 2 \cos t \right]_0^{\pi} = (\pi^2 - 0 - 2) - (0 - 0 + 2)$$

$$= \pi^2 - 4$$

IXGB - MP2 PAPER I - QUESTION 4.

SUPPOSE THAT THERE EXIST INTEGERS a & b SO THAT

$$a^2 - 8b = 7$$

THEN WE HAVE

$$a^2 = 8b + 7$$

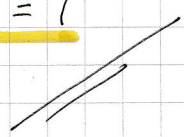
AS THE R.H.S IS ODD (MULTIPLE OF $8 + 7$), IMPLIES THAT a^2 IS ALSO ODD, AND THEREFORE a MUST ALSO BE ODD

LET $a = 2n + 1$ WHICH IS ODD FOR n BEING AN INTEGER

$$\begin{aligned} \Rightarrow (2n+1)^2 &= 8b + 7 \\ \Rightarrow 4n^2 + 4n + 1 &= 8b + 7 \\ \Rightarrow 4n^2 + 4n - 8b &= 6 \\ \Rightarrow 2n^2 + 2n - 4b &= 3 \\ \Rightarrow 2(n^2 + n - 2b) &= 3 \\ \Rightarrow n^2 + n - 2b &= \frac{3}{2} \end{aligned}$$

BUT THE LHS HAS TO BE AN INTEGER, WHILE THE R.H.S IS NOT

\therefore THIS IS A CONTRADICTION TO THE ASSERTION THAT THERE EXIST INTEGERS a & b WHICH SATISFIES $a^2 - 8b = 7$



1YGB - MP2 PAPER I - QUESTIONS

a) USING STANDARD METHODOLOGY

$$y = 4 - \frac{1}{x-1}$$

$$\frac{1}{x-1} = 4-y$$

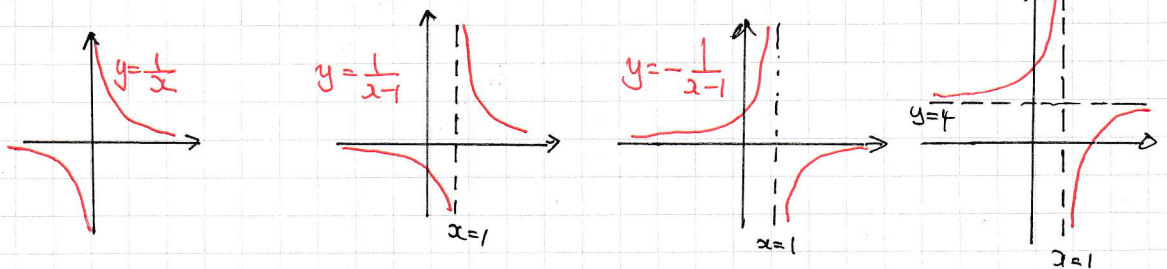
$$x-1 = \frac{1}{4-y}$$

$$x = 1 + \frac{1}{4-y}$$

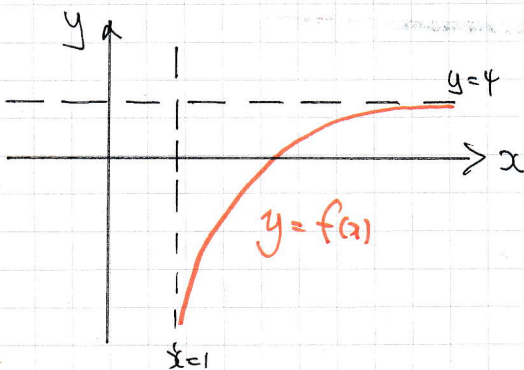
$$y = 1 + \frac{1}{4-x}$$

$$\therefore f^{-1}(x) = 1 - \frac{1}{x-4}$$

b) SKETCHING $f(x)$ FIRST - STARTING WITH $y = 4 - \frac{1}{x-1}$



Now $f(x)$ with $x > 1$



i.e. RANGE of $f(x)$ is
 $f(x) < 4$

	f	f^{-1}
D	$x > 1$	$x < 4$
R	$f(x) < 4$	$f(x) > 1$

∴ DOMAIN $x < 4$
RANGE $f(x) > 1$

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YCB - MP2 PAPER I - QUESTION 6

a) FROM THE DEFINITION OF $g(x)$ & $f(x)$ AND USING $\sin(A \pm B)$

$$\Rightarrow g(x) = f\left(x + \frac{\pi}{4}\right) - f\left(x - \frac{\pi}{4}\right)$$

$$\Rightarrow g(x) = \sin\left[2\left(x + \frac{\pi}{4}\right)\right] - \sin\left[2\left(x - \frac{\pi}{4}\right)\right]$$

$$\Rightarrow g(x) = \sin\left[2x + \frac{\pi}{2}\right] - \sin\left[2x - \frac{\pi}{2}\right]$$

$$\Rightarrow g(x) = \cancel{\sin 2x} \cos \frac{\pi}{2} + \cos 2x \sin \frac{\pi}{2} - \left[\cancel{\sin 2x} \cos \frac{\pi}{2} - \cos 2x \sin \frac{\pi}{2} \right]$$

$$\Rightarrow g(x) = 2 \cos 2x \sin \frac{\pi}{2}$$

$\sin \frac{\pi}{2} = 1$

$$\Rightarrow \underline{g(x) = 2 \cos 2x}$$

// AS REQUIRED

b) DIFFERENTIATING $g(x)$

$$g(x) = 2 \cos 2x$$

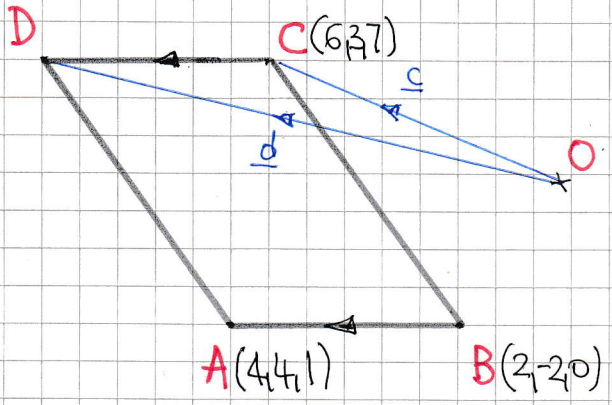
$$g'(x) = -4 \sin 2x$$

$$\underline{g'(x) = -4 f(x)}$$

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1YGB-MP2 PAPER I - QUESTION 7

LOOKING AT THE DIAGRAM BELOW



$$\vec{OD} = \vec{OC} + \vec{CD}$$

$$\vec{OB} = \vec{OC} + \vec{BA}$$

$$\underline{d} = \underline{c} + (\underline{a} - \underline{b})$$

$$\underline{d} = \begin{pmatrix} 6 \\ 3 \\ 7 \end{pmatrix} + \begin{pmatrix} 4 \\ 4 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}$$

$$\underline{d} = \begin{pmatrix} 8 \\ 9 \\ 8 \end{pmatrix}$$

D(8,9,8)

NEXT CALCULATE SOME LENGTHS

$$|\vec{OC}| = |\underline{c}| = |6,3,7| = \sqrt{36+9+49} = \sqrt{94}$$

$$|\vec{OD}| = |\underline{d}| = |8,9,8| = \sqrt{64+81+64} = \sqrt{209}$$

$$|\vec{CD}| = |\underline{d} - \underline{c}| = |(8,9,8) - (6,3,7)| = |2,6,1| = \sqrt{4+36+1} = \sqrt{41}$$

BY THE COSINE RULE ON \widehat{OCD} - THE LARGEST ANGLE IS OPPOSITE THE LONGEST LENGTH

$$\Rightarrow |\vec{OD}|^2 = |\vec{CD}|^2 + |\vec{OC}|^2 - 2|\vec{CD}||\vec{OC}|\cos\theta, \text{ WHERE } \theta = \widehat{OCD}$$

$$\Rightarrow 209 = 41 + 94 - 2\sqrt{41}\sqrt{94}\cos\theta$$

$$\Rightarrow 2\sqrt{41}\sqrt{94}\cos\theta = -74$$

$$\Rightarrow \cos\theta = -\frac{37}{\sqrt{3854}}$$

$$\Rightarrow \theta \approx 126.6^\circ$$

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1YGB - MP2 PAPER I - QUESTION 8

a) EXPANDING BINOMIALLY UP TO x^3

$$\Rightarrow \sqrt{1-x} = (1-x)^{\frac{1}{2}} = 1 + \frac{\frac{1}{2}}{1}(-x) + \frac{\frac{1}{2}(\frac{1}{2}-1)}{1 \times 2}(-x)^2 + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{1 \times 2 \times 3}(-x)^3 + o(x^4)$$

$$\Rightarrow \sqrt{1-x} = 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3 + o(x^4)$$

b) PROCEED AS FOLLOWS

$$8 \times \sqrt{1 - \frac{1}{64}} = 8 \sqrt{\frac{63}{64}} = \frac{8\sqrt{63}}{\sqrt{64}} = \frac{8\sqrt{9}\sqrt{7}}{\sqrt{64}}$$

$$= \frac{8 \times 3\sqrt{7}}{8} = 3\sqrt{7}$$

* AS REQUIRED

c) COMBINING RESULTS

$$\sqrt{1-x} \approx 1 - \frac{1}{2}x \quad (\text{FOR SMALL } x, \text{ FIRST 2 TERMS})$$

LET $x = \frac{1}{64}$

$$\Rightarrow \sqrt{1 - \frac{1}{64}} \approx 1 - \frac{1}{2} \times \frac{1}{64}$$

$$\Rightarrow 8\sqrt{1 - \frac{1}{64}} \approx 8 \left[1 - \frac{1}{128} \right]$$

$$\Rightarrow 3\sqrt{7} \approx 8 - \frac{1}{16}$$

$$\Rightarrow 3\sqrt{7} \approx \frac{127}{16}$$

$$\Rightarrow \sqrt{7} \approx \frac{127}{48}$$

* AS REQUIRED

1YGB - MP2 PAPER 1 - QUESTION 9

START BY OBTAINING THE GRADIENT FUNCTION

$$\Rightarrow ax^3 - 3xy + by^2 = 224$$

$$\Rightarrow \frac{d}{dx}(ax^3 - 3xy + by^2) = \frac{d}{dx}(224)$$

$$\Rightarrow 3ax^2 - 3y - 3x \frac{dy}{dx} + 2by \frac{dy}{dx} = 0$$

OBTAIN THE GRADIENT AT (-2, 6)

$$\Rightarrow 3a(-2)^2 - 3 \times 6 - 3(-2) \frac{dy}{dx} + 2b(6) \frac{dy}{dx} = 0$$

$$\Rightarrow 12a - 18 + 6 \frac{dy}{dx} + 12b \frac{dy}{dx} = 0$$

$$\Rightarrow (6 + 12b) \frac{dy}{dx} = 18 - 12a$$

$$\Rightarrow \frac{dy}{dx} = \frac{18 - 12a}{6 + 12b}$$

$$\Rightarrow \frac{dy}{dx} = \frac{3 - 2a}{1 + 2b} \quad \leftarrow \text{GRADIENT OF THE TANGENT}$$

\therefore GRADIENT OF THE NORMAL IS $\frac{2b+1}{2a-3}$

NEXT WE REARRANGE THE NORMAL TO "READ" ITS GRADIENT

$$\Rightarrow 15x - 13y + 108 = 0$$

$$\Rightarrow 15x + 108 = 13y$$

$$\Rightarrow y = \frac{15}{13}x + \frac{108}{13}$$

$$\uparrow$$

$$\frac{2b+1}{2a-3}$$

$$\Rightarrow \frac{2b+1}{2a-3} = \frac{15}{13}$$

17GB - MP2 - PAPER I - QUESTION 9

$$\Rightarrow 13(2b+1) = 15(2a-3)$$

$$\Rightarrow 26b + 13 = 30a - 45$$

$$\Rightarrow 26b - 30a = -58$$

$$\Rightarrow 30a - 26b = 58$$

$$\Rightarrow \underline{15a - 13b = 29}$$

THE POINT (-2,6) MUST ALSO SATISFY THE EQUATION OF THE CURVE

$$\Rightarrow ax^3 - 3xy + by^2 = 224$$

$$\Rightarrow -8a - 3(-2) \times 6 + b \times 36 = 224$$

$$\Rightarrow -8a + 36 + 36b = 224$$

$$\Rightarrow -8a + 36b = 188$$

$$\Rightarrow \underline{-2a + 9b = 47}$$

FINALLY SOLVING THE EQUATIONS SIMULTANEOUSLY

$$\begin{array}{l}
 15a - 13b = 29 \\
 -2a + 9b = 47
 \end{array}
 \left. \begin{array}{l}
 \Rightarrow \\
 \Rightarrow
 \end{array} \right\} \begin{array}{l}
 \times 2 \\
 \times 15
 \end{array}
 \Rightarrow \left. \begin{array}{l}
 30a - 26b = 58 \\
 -30a + 135b = 705
 \end{array} \right\} \Rightarrow$$

$$\Rightarrow 109b = 763$$

$$\Rightarrow \underline{b = 7}$$

$$\& \ -2a + 9b = 47$$

$$\ -2a + 63 = 47$$

$$\ -2a = -16$$

$$\underline{a = 8}$$

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1YGB - MP2 PAPER I - QUESTION 10

a) LOOKING AT THE PATTERN

WEEK:	1	2	3	...	52
COMM.:	15	15 15 <u>30</u>	15 15 <u>15</u> <u>45</u>		?

$a = 15$
 $d = 15$
 $n = 52$

$\Rightarrow U_n = a + (n-1)d$

$\Rightarrow U_{52} = 15 + 51 \times 15$

$\Rightarrow U_{52} = 780$

i.e. £780

b) SUMMING THE COMMISSIONS USING PART (a)

$S_n = \frac{n}{2} [a + L]$

$S_{52} = \frac{52}{2} [15 + 780] = 26 \times 795 = 20670$

i.e. £20670

c) CONTINUING THE PATTERN & LINKING WITH THE FIRST YEAR

WEEK	COMMISSION	
51	£765	} FIRST YEAR
52	£780	

1	$(52 \times 10) + 20$	} SECOND YEAR
2	$(52 \times 10) + 20 + 20$	
3	$(52 \times 10) + 20 + 20 + 20$	
⋮	⋮	
52	$(52 \times 10) + 20 \times 52$	

$a = 540$

$L = 1560$

SUMMING USING $S_n = \frac{n}{2} [a + L]$

$S_{52} = \frac{52}{2} [540 + 1560] = 54600$

i.e. £54600

YGB - MP2 PART 2 I - QUESTION 11

USING THE SUBSTITUTION GIVEN

$$\int_0^{\frac{\pi}{4}} \frac{1 - \tan^2 x}{\sec^2 x + 2 \tan x} dx = \int_0^1 \frac{1 - \tan^2 x}{\sec^2 x + 2 \tan x} \left(\frac{du}{2 \cos 2x} \right)$$

$$= \frac{1}{2} \int_0^1 \frac{1 - \tan^2 x}{(\sec^2 x + 2 \tan x) \cos 2x} du$$

$u = \sin 2x$
 $\frac{du}{dx} = 2 \cos 2x$
 $dx = \frac{du}{\cos 2x}$

 $x = 0 \rightarrow u = 0$
 $x = \frac{\pi}{4} \rightarrow u = 1$

SWITCH EVERYTHING INTO SINES & COSINES

$$= \frac{1}{2} \int_0^1 \frac{1 - \frac{\sin^2 x}{\cos^2 x}}{\left(\frac{1}{\cos^2 x} + \frac{2 \sin x}{\cos x} \right) (\cos^2 x - \sin^2 x)} du$$

MULTIPLY "TOP & BOTTOM" OF THE DOUBLE FRACTION BY $\cos^2 x$

$$= \frac{1}{2} \int_0^1 \frac{(\cancel{\cos^2 x} - \sin^2 x)}{(1 + 2 \sin x \cos x) (\cancel{\cos^2 x} - \sin^2 x)} du = \frac{1}{2} \int_0^1 \frac{1}{1 + 2 \sin x \cos x} du$$

$$= \frac{1}{2} \int_0^1 \frac{1}{1 + \sin 2x} du = \frac{1}{2} \int_0^1 \frac{1}{1 + u} du$$

$$= \frac{1}{2} \left[\ln |1 + u| \right]_0^1 = \frac{1}{2} \left[\ln 2 - \ln 1 \right] = \frac{1}{2} \ln 2$$



IYGB - MP2 PAPER I - QUESTION 6

a) FORMING AN O.D.E.

$$\Rightarrow \frac{dv}{dt} = -kV$$

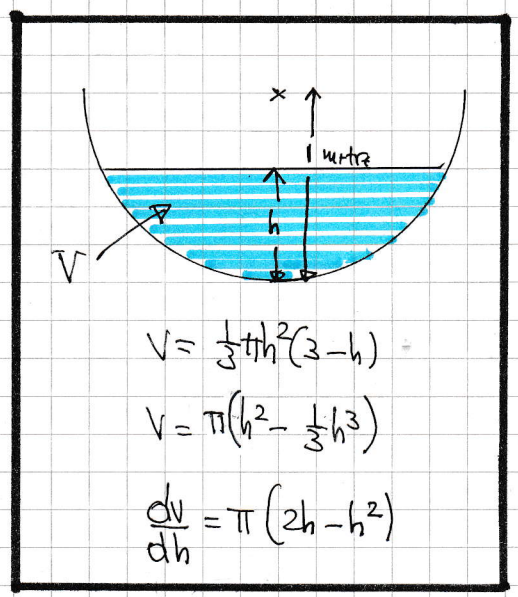
$$\Rightarrow \frac{dv}{dh} \times \frac{dh}{dt} = -kV$$

$$\Rightarrow \pi(2h-h^2) \frac{dh}{dt} = -k \times \frac{1}{3} \pi h^2(3-h)$$

$$\Rightarrow h(2-h) \frac{dh}{dt} = -\frac{1}{3} kh^2(3-h)$$

$$\Rightarrow (2-h) \frac{dh}{dt} = -\frac{1}{3} kh(3-h)$$

$$\Rightarrow \frac{dh}{dt} = -\frac{kh(3-h)}{3(2-h)} \quad \text{As required}$$



b) SEPARATING VARIABLES

$$\Rightarrow \frac{3(2-h)}{h(3-h)} dh = -k dt$$

$$\Rightarrow \int \frac{3(2-h)}{h(3-h)} dh = \int -k dt$$

OBTAIN THE PARTIAL FRACTIONS

$$\frac{3(2-h)}{h(3-h)} \equiv \frac{A}{h} + \frac{B}{3-h}$$

$$3(2-h) \equiv A(3-h) + Bh$$

- If $h=3$ • If $h=0$
- $-3 = 3B$ $6 = 3A$
- $B = -1$ $A = 2$

$$\frac{3(2-h)}{h(3-h)} = \frac{2}{h} - \frac{1}{3-h}$$

IYGB - MP2 PAPER 1 - QUESTION 12

CARRYING OUT THE INTEGRATIONS

$$\Rightarrow \int \frac{2}{h} - \frac{1}{3-h} dh = -kt + C$$

$$\Rightarrow 2\ln|h| + \ln|3-h| = -kt + C$$

$$\Rightarrow \ln h^2 + \ln|3-h| = -kt + C$$

$$\Rightarrow \ln|h^2(3-h)| = -kt + C$$

$$\Rightarrow h^2(3-h) = e^{-kt+C}$$

$$\Rightarrow 3h^2 - h^3 = e^{-kt} \times e^C$$

$$\Rightarrow 3h^2 - h^3 = Ae^{-kt}$$

APPLY THE INITIAL CONDITION $t=0, h=1$ (INITIALLY ROW)

$$\Rightarrow 3-1 = Ae^0$$

$$\Rightarrow A=2$$

$$\therefore \underline{3h^2 - h^3 = 2e^{-kt}}$$

~~As required~~