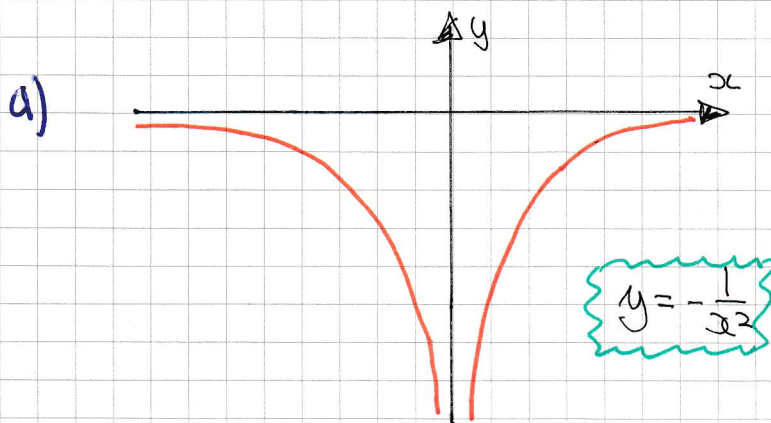
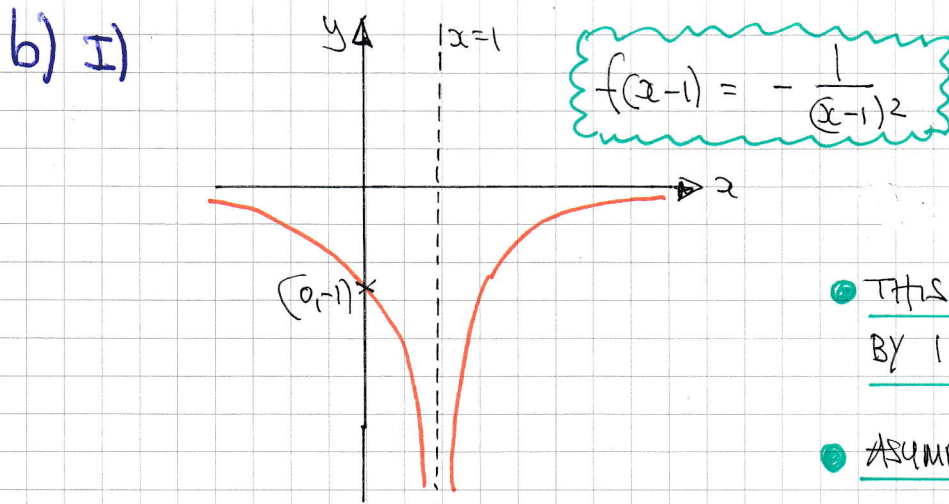


1YGB - MPI PAPER 1 - QUESTION 1



• THIS IS THE STANDARD
 $y = \frac{1}{x^2}$ "UPWARDS UPSIDE
DOWN"

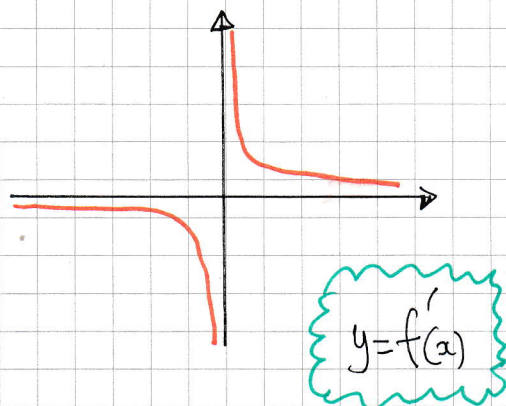
• ASYMPTOTES
 $x=0$ (y AXIS)
 $y=0$ (x AXIS)



• THIS IS A TRANSLATION
BY 1 UNIT TO THE "RIGHT"

• ASYMPTOTES
 $x=1$
 $y=0$ (x AXIS)

II) $f'(x) = \frac{d}{dx} \left(-\frac{1}{x^2} \right) = \frac{2}{x^3}$



• THIS IN THE ABSENCE OF
SCALE LOOKS LIKE $\frac{a}{x}$

• ASYMPTOTES
 $x=0$ (y AXIS)
 $y=0$ (x AXIS)

1YGB - MPI PAPER 0 - QUESTION 2

FORMING THE EQUATION & TRY TO A 3-TERM QUADRATIC

$$\Rightarrow px^2 + 4x(p+3) + 5p = -19$$

$$\Rightarrow \underbrace{px^2}_a + \underbrace{4(p+3)x}_b + \underbrace{(5p+19)}_c = 0$$

TWO DISTINCT ROOTS IMPLY $b^2 - 4ac > 0$

$$\Rightarrow [4(p+3)]^2 - 4 \times p \times (5p+19) > 0$$

$$\Rightarrow 16(p+3)^2 - 4p(5p+19) > 0$$

$$\Rightarrow 4(p+3)^2 - p(5p+19) > 0$$

$$\Rightarrow 4(p^2 + 6p + 9) - 5p^2 - 19p > 0$$

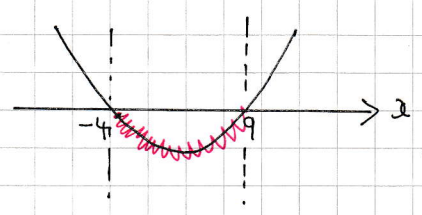
$$\Rightarrow 4p^2 + 24p + 36 - 5p^2 - 19p > 0$$

$$\Rightarrow -p^2 + 5p + 36 > 0$$

$$\Rightarrow p^2 - 5p - 36 < 0$$

$$\Rightarrow (p-9)(p+4) < 0$$

CRITICAL VALUES ARE 9 & -4



$$\text{SO } -4 < p < 9$$

BUT $p \neq 0$, OTHERWISE NO QUADRATIC

$$\underline{-4 < p < 0 \cup 0 < p < 9}$$

- 1 -

IYGB - MPI PAPER U - QUESTION 3

GETTING RID OF THE DENOMINATORS

$$\Rightarrow \frac{2 + \cos 2x}{3 + \sin^2 2x} = \frac{2}{5}$$

$$\Rightarrow 10 + 5\cos 2x = 6 + 2\sin^2 2x$$

USING $\cos^2 2x + \sin^2 2x \equiv 1$

$$\Rightarrow 10 + 5\cos 2x = 6 + 2(1 - \cos^2 2x)$$

$$\Rightarrow 10 + 5\cos 2x = 6 + 2 - 2\cos^2 2x$$

$$\Rightarrow 2\cos^2 2x + 5\cos 2x + 2 = 0$$

$$\Rightarrow (2\cos 2x + 1)(\cos 2x + 2) = 0$$

$$\Rightarrow \cos 2x = \begin{cases} -\frac{1}{2} \\ -2 \end{cases} \quad -1 \leq \cos 2x \leq 1$$

PROCEED WITH SOLUTION

$$\arccos\left(-\frac{1}{2}\right) = 120^\circ$$

$$\begin{cases} 2x = 120 \pm 360n \\ 2x = 240 \pm 360n \end{cases} \quad n=0,1,2,3,\dots$$

$$\begin{cases} x = 60 \pm 180n \\ x = 120 \pm 180n \end{cases}$$

$x = 60^\circ, 240^\circ, 120^\circ, 300^\circ$

LYGB - MPI PAPER U - QUESTION 4

a) $x-2$ IS A FACTOR OF $f(x)$; $2x+1$ IS A FACTOR OF $f(x)$

$$f(2) = 0$$

$$2x^3 - 9x^2 + px + q = 0$$

$$16 - 36 + 2p + q = 0$$

$$2p + q = 20$$

$$q = 20 - 2p$$

$$f(-\frac{1}{2}) = 0$$

$$2(-\frac{1}{2})^3 - 9(-\frac{1}{2})^2 + p(-\frac{1}{2}) + q = 0$$

$$-\frac{1}{4} - \frac{9}{4} - \frac{1}{2}p + q = 0$$

$$-\frac{5}{2} - \frac{1}{2}p + q = 0$$

$$q = \frac{1}{2}p + \frac{5}{2}$$

$$20 - 2p = \frac{1}{2}p + \frac{5}{2}$$

$$40 - 4p = p + 5$$

$$35 = 5p$$

$$p = 7$$

$$q = 6$$

b) $2\sqrt{y} + \frac{7}{\sqrt{y}} = 9 - \frac{6}{y}$

Let $x = \sqrt{y}$

$$\Rightarrow 2x + \frac{7}{x} = 9 - \frac{6}{x^2}$$

$$\Rightarrow 2x^3 + 7x = 9x^2 - 6$$

$$\Rightarrow 2x^3 - 9x^2 + 7x + 6 = 0$$

THIS IS THE CUBIC OF PART (a) - FACTORIZE BY INSPECTION

$$\Rightarrow (2x+1)(x-2)(x-3) = 0$$

$$\Rightarrow x = \begin{cases} 3 \\ 2 \\ -\frac{1}{2} \end{cases}$$

LYGB - MPI PAPER 1 - QUESTION 4

$$\Rightarrow \sqrt{y} = \begin{cases} 2 \\ 3 \\ \cancel{\frac{1}{2}} \end{cases}$$

$$\Rightarrow \sqrt{y} = \begin{cases} 4 \\ 9 \end{cases}$$



1YGB - MPI PAPER U - QUESTION 5

METHOD A - BY CO-ORDINATE GEOMETRY

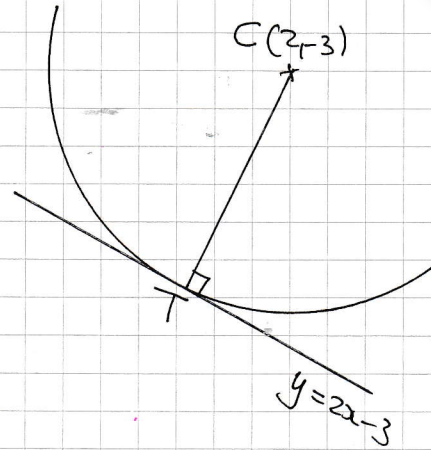
- GRADIENT OF THE TANGENT IS 2
- GRADIENT CT MUST BE $-\frac{1}{2}$
- EQUATION OF LINE THROUGH C & T

$$y - (-3) = -\frac{1}{2}(x - 2)$$

$$y + 3 = -\frac{1}{2}(x - 2)$$

$$2y + 6 = -x + 2$$

$$2y + x + 4 = 0$$



- SOLVING SIMULTANEOUSLY WITH $y = 2x - 3$

$$2(2x - 3) + x + 4 = 0$$

$$5x - 2 = 0$$

$$x = \frac{2}{5}$$

AND $y = 2\left(\frac{2}{5}\right) - 3 = \frac{4}{5} - 3 = -\frac{11}{5} \quad \therefore \left(\frac{2}{5}, -\frac{11}{5}\right)$

- DISTANCE CT FINALLY, $C(2, -3)$ & $T\left(\frac{2}{5}, -\frac{11}{5}\right)$

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

$$|CT| = \sqrt{\left(-3 + \frac{11}{5}\right)^2 + \left(\frac{2}{5} - 2\right)^2}$$

$$r = \sqrt{\left(-\frac{4}{5}\right)^2 + \left(-\frac{8}{5}\right)^2} = \sqrt{\frac{16}{25} + \frac{64}{25}} = \sqrt{\frac{80}{25}} = \frac{4}{5}\sqrt{5}$$

METHOD B - USING DISCRIMINANTS

- LET THE CIRCLE HAVE EQUATION

$$(x - 2)^2 + (y + 3)^2 = r^2$$

1YGB - MPI PAGE 11 - QUESTION 5

● SOLVING SIMULTANEOUSLY WITH $y = 2x - 3$ TO "FIND" T

$$\Rightarrow (x-2)^2 + (2x-3+3)^2 = r^2$$

$$\Rightarrow (x-2)^2 + (2x)^2 = r^2$$

$$\Rightarrow x^2 - 4x + 4 + 4x^2 = r^2$$

$$\Rightarrow 5x^2 - 4x + (4 - r^2) = 0$$

● THIS EQUATION MUST PRODUCE REPEATED ROOTS AS THE POINT T IS A POINT OF TANGENCY

$$b^2 - 4ac = 0 \Rightarrow (-4)^2 - 4 \times 5 \times (4 - r^2) = 0$$

$$\Rightarrow 16 - 20(4 - r^2) = 0$$

$$\Rightarrow 16 - 80 + 20r^2 = 0$$

$$\Rightarrow 20r^2 = 64$$

$$\Rightarrow r^2 = \frac{64}{20} = \frac{64}{100} \times 5$$

$$\Rightarrow r = \frac{4}{5} \sqrt{5}$$

AS BEFORE

METHOD C - BY MINIMIZATION (COMPLETING THE SQUARE)

● CONSIDER A POINT ON THE LINE $y = 2x - 3$, i.e. $(x, 2x - 3)$

● THE DISTANCE FROM $(x, 2x - 3)$ TO THE CENTRE $(2, -3)$ IS GIVEN BY

$$\Rightarrow d = \sqrt{(x-2)^2 + [2x-3-(-3)]^2}$$

$$\Rightarrow d = \sqrt{(x-2)^2 + 4x^2}$$

$$\Rightarrow d^2 = x^2 - 4x + 4 + 4x^2$$

1YGB - MAI PAPER U - QUESTION 5

$$\Rightarrow d^2 = 5x^2 - 4x + 4$$

$$\Rightarrow d^2 = 5 \left[x^2 - \frac{4}{5}x + \frac{4}{5} \right]$$

$$\Rightarrow d^2 = 5 \left[\left(x - \frac{2}{5} \right)^2 - \frac{4}{25} + \frac{4}{5} \right]$$

$$\Rightarrow d^2 = 5 \left(x - \frac{2}{5} \right)^2 - \frac{4}{5} + 4$$

$$\Rightarrow d^2 = 5 \left(x - \frac{2}{5} \right)^2 + \frac{16}{5}$$

∴ MINIMUM VALUE OF d^2 IS $\frac{16}{5}$ (OCCURS AT $x = \frac{2}{5}$)

$$\therefore d_{\text{MIN}} = r = \sqrt{\frac{16}{5}} = \sqrt{\frac{16 \times 5}{25}} = \frac{4}{5} \sqrt{5}$$

- 1 -

IYGB - MPI PAPER 1 - QUESTION 6

C: $y = ax^{\frac{3}{2}} + bx^{\frac{1}{2}}, x > 0$ • L: $y = 8x - 32$

• "L IS A TANGENT TO THE CURVE AT $x=4$ "

• $x=4$

$y = 8 \times 4 - 32$

$y = 0$

• GRADIENT OF L IS 8

• $y = ax^{\frac{3}{2}} - bx^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{3}{2}ax^{\frac{1}{2}} - \frac{1}{2}bx^{-\frac{1}{2}}$

• with $x=4, \frac{dy}{dx}=8$

$\Rightarrow 8 = \frac{3}{2}a \times 4^{\frac{1}{2}} - \frac{1}{2}b \times 4^{-\frac{1}{2}}$

$\Rightarrow 8 = 3a - \frac{1}{4}b$

$32 = 12a - b$

• with $x=0, y=0$

$\Rightarrow 0 = a \times 4^{\frac{3}{2}} - b \times 4^{\frac{1}{2}}$

$\Rightarrow 0 = 8a - 2b$

$\Rightarrow 2b = 8a$

$\Rightarrow b = 4a$

• SOLVING THE EQUATIONS YIELDS

$\Rightarrow 32 = 12a - (4a)$

$\Rightarrow 32 = 8a$

$\Rightarrow a = 4$

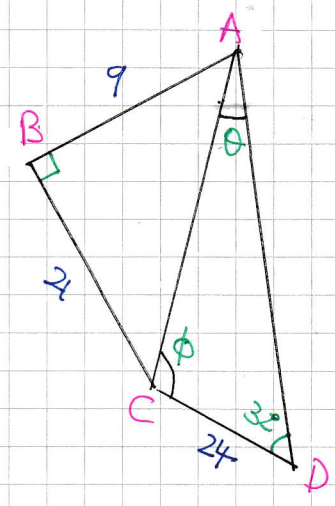
and

$b = 16$

IVGB - MPI PAPER U - QUESTION 7

LOOKING AT THE DIAGRAM

$$\begin{aligned} \Rightarrow |AB|^2 + |BC|^2 &= |AC|^2 \\ \Rightarrow 9^2 + 21^2 &= |AC|^2 \\ \Rightarrow |AC|^2 &= 522 \\ \Rightarrow |AC| &= \sqrt{522} \end{aligned}$$



BY THE SINE RULE ON $\triangle ACD$

$$\begin{aligned} \frac{\sin \theta}{24} &= \frac{\sin 32^\circ}{\sqrt{522}} \Rightarrow \sin \theta = \frac{24 \sin 32^\circ}{\sqrt{522}} \\ \Rightarrow \sin \theta &= 0.55665... \\ \Rightarrow \theta &= 33.8247...^\circ \end{aligned}$$

← NOT OBTUSE AS $\sqrt{522} \approx 22.8$ WHICH IS COMPAREBLY WITH 24

FINALLY FIND ϕ & THE AREA OF $\triangle ACD$

$$\begin{aligned} \phi &= 180 - (32 + 33.8247...) \\ \phi &= 114.1753...^\circ \end{aligned}$$

$$\begin{aligned} \text{AREA OF } \triangle ACD &= \frac{1}{2} |AC| |CD| \sin \phi \\ &= \frac{1}{2} \sqrt{522} \times 24 \times \sin(114.1753...^\circ) \\ &= \underline{250.122... \text{ cm}^2} \end{aligned}$$

NEXT THE AREA OF $\triangle ABC$

$$\text{AREA} = \frac{1}{2} |AB| |BC| = \frac{1}{2} \times 9 \times 21 = \underline{94.5 \text{ cm}^2}$$

$$\begin{aligned} \therefore \text{REQUIRED AREA} &= 250.122... + 94.5 \\ &= 344.622... \\ &\approx \underline{345 \text{ cm}^2} \end{aligned}$$

3 s.f.

1XB-MPI PAGE U-QUESTION 8

PROCEED AS FOLLOWS

$$\Rightarrow \frac{\log_4 x^2}{5 + \log_4 x^2} + (\log_4 x)^2 = 0$$

$$\Rightarrow \frac{2\log_4 x}{5 + 2\log_4 x} + (\log_4 x)^2 = 0$$

$$\Rightarrow \frac{2y}{5 + 2y} + y^2 = 0$$

LET $y = \log_4 x$

$$\Rightarrow 2y + y^2(5 + 2y) = 0 \quad (5 + 2y)$$

$$\Rightarrow 2y + 5y^2 + 2y^3 = 0$$

$$\Rightarrow 2y^3 + 5y^2 + 2y = 0$$

$$\Rightarrow y(2y^2 + 5y + 2) = 0$$

$$\Rightarrow y(2y + 1)(y + 2) = 0$$

$$\Rightarrow y = \begin{cases} 0 \\ -\frac{1}{2} \\ -2 \end{cases} \quad \text{i.e.} \quad \log_4 x = \begin{cases} 0 \\ -\frac{1}{2} \\ -2 \end{cases}$$

NUMBERING BY INSPECTION

$$x = \begin{cases} 4^0 \\ 4^{-\frac{1}{2}} \\ 4^{-2} \end{cases}$$

$$\text{ie } x = \begin{cases} 1 \\ \frac{1}{2} \\ \frac{1}{16} \end{cases}$$

YGB - MPI PAPER 0 - QUESTION 9

$A(-1,4) \bullet B(2,3) \bullet C(8,1)$

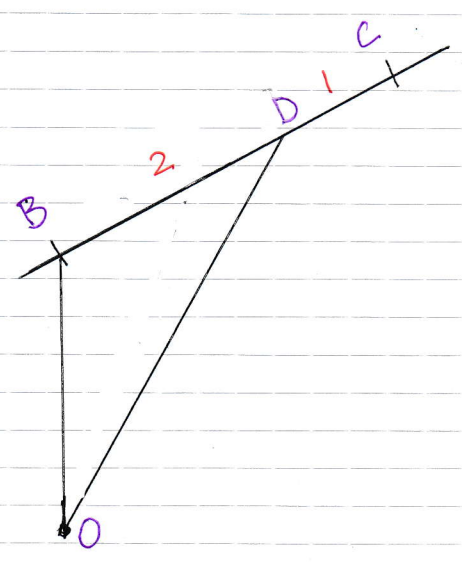
a) FIND THE VECTORS \vec{AB} & \vec{BC}

$\vec{AB} = \underline{b} - \underline{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

$\vec{BC} = \underline{c} - \underline{b} = \begin{pmatrix} 8 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \end{pmatrix} = 2 \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

AS \vec{AB} & \vec{BC} ARE IN THE SAME DIRECTION & SHARE THE POINT B, A, B & C WILL BE COLLINEAR.

b)



WORKING AT THE DIAGRAM

$\Rightarrow \vec{OD} = \vec{OB} + \vec{BD}$

$\Rightarrow \vec{OD} = \vec{OB} + \frac{2}{3} \vec{BC}$

$\Rightarrow \underline{d} = \underline{b} + \frac{2}{3}(\underline{c} - \underline{b})$

$\Rightarrow 3\underline{d} = 3\underline{b} + 2\underline{c} - 2\underline{b}$

$\Rightarrow 3\underline{d} = \underline{b} + 2\underline{c}$

$\Rightarrow 3\underline{d} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + 2 \begin{pmatrix} 8 \\ 1 \end{pmatrix}$

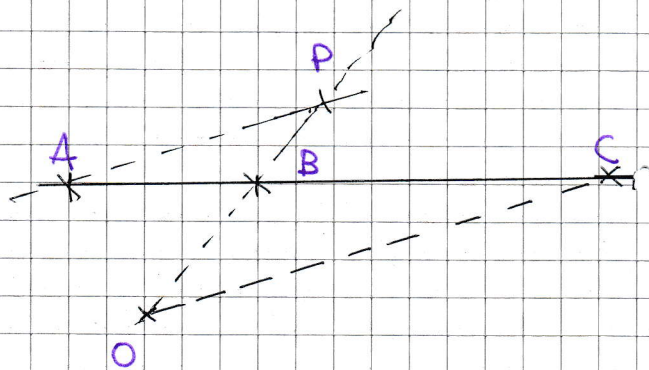
$\Rightarrow 3\underline{d} = \begin{pmatrix} 18 \\ 5 \end{pmatrix}$

$\Rightarrow \underline{d} = \begin{pmatrix} 6 \\ \frac{5}{3} \end{pmatrix}$

$\therefore \underline{D} \left(6, \frac{5}{3} \right)$

1YGB - MPI PAPER 1 - QUESTION 9

b) LOOKING AT THE DIAGRAM BELOW



$$\Rightarrow \vec{OP} = \vec{OA} + \vec{AP}$$

$$\Rightarrow \lambda \vec{OB} = \vec{OA} + \mu \vec{OC}$$

$$\Rightarrow \lambda \underline{b} = \underline{a} + \mu \underline{c}$$

$$\Rightarrow \lambda \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 8 \\ 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2\lambda \\ 3\lambda \end{pmatrix} = \begin{pmatrix} -1 + 8\mu \\ 4 + \mu \end{pmatrix}$$

$$\Rightarrow \frac{2\lambda}{-24\lambda} = \frac{-1 + 8\mu}{-32 - 8\mu}$$

$$\Rightarrow -22\lambda = -33$$

$$\Rightarrow \lambda = \frac{3}{2}$$

HENCE AS $\vec{OP} = \lambda \vec{OB}$

$$\vec{OP} = \frac{3}{2} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 9/2 \end{pmatrix}$$

$$\therefore \underline{P(3, \frac{9}{2})}$$

1 YGB - MPI PAPER U - QUESTION 10

a) EXPANDING BY THE STANDARD BINOMIAL FORMULA

$$\begin{aligned}
 (6x-3)^8 &= \binom{8}{0}(6x)^8(-3)^0 + \binom{8}{1}(6x)^7(-3)^1 + \binom{8}{2}(6x)^6(-3)^2 + \binom{8}{3}(6x)^5(-3)^3 + \dots \\
 &= (1 \times 1 \times 6561) + (8 \times 6x \times (-2187)) + (28 \times 36x^2 \times 729) \\
 &\quad + (56 \times 216x^3 \times (-243)) + \dots \\
 &= \underline{6561 - 104976x + 734832x^2 - 2939328x^3 + \dots}
 \end{aligned}$$

b) PROCEED AS FOLLOWS

$$\frac{y+9}{3} = 6x+3$$

$$y+9 = 18x+9$$

$$18x = y$$

$$x = \frac{1}{18}y$$

USING PART (a)

$$\left[6\left(\frac{1}{18}y\right)-3\right]^8 = 6561 - \dots - 2939328\left(\frac{1}{18}y\right)^3 + \dots$$

$$\left[\frac{y+9}{3}\right]^8 = 6561 - \dots - 504y^3 + \dots$$

↳ 504

c) WORK AS FOLLOWS

$$\begin{aligned}
 (\sqrt{2}z-1)^8 (\sqrt{2}z+1)^8 &= \left[(\sqrt{2}z-1)(\sqrt{2}z+1)\right]^8 \\
 &= (2z^2-1)^8 \\
 &= \frac{1}{3^8} \times 3^8 \times (2z^2-1)^8
 \end{aligned}$$

1YGB - MPI PAPER 10 - QUESTION 10

$$= \frac{1}{3^8} \times [3(2z^2-1)]^8$$

$$= \frac{1}{6561} [6z^2-3]^8$$

$$= \frac{1}{6561} [6561 - 104976(z^2) + 734832(z^2)^2 - 2939328(z^2)^3 + \dots]$$

$$= \frac{1}{6561} [\dots - 2939328z^6 + \dots]$$

$$= \dots - 448z^6 + \dots$$

1.E - 448

- 1 -

IYGB - MPI PAPER U - QUESTION 11

a)

● START WITH A DIAGRAM (NOT TO SCALE)

$$\Rightarrow 3x + y = 12$$

$$\Rightarrow y = -3x + 12$$

● GRADIENT OF \overline{ABC} WILL BE $+\frac{1}{3}$

● EQUATION OF LINE \overline{ABC}

$$y - 1 = \frac{1}{3}(x + 2)$$

● SOLVING SIMULTANEOUSLY WITH $3x + y = 12$

$$\left. \begin{array}{l} y - 1 = \frac{1}{3}(x + 2) \\ y = 12 - 3x \end{array} \right\} \Rightarrow (12 - 3x) - 1 = \frac{1}{3}(x + 2)$$

$$\Rightarrow 11 - 3x = \frac{1}{3}(x + 2)$$

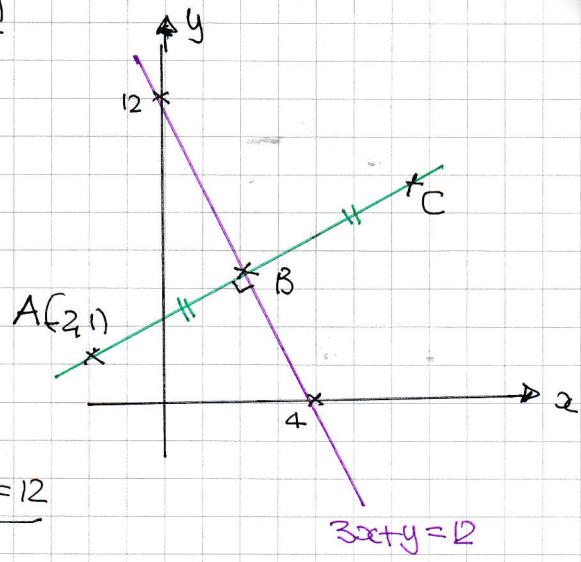
$$\Rightarrow 33 - 9x = x + 2$$

$$\Rightarrow 31 = 10x$$

$$\Rightarrow x = \underline{3.1}$$

$$\& y = 12 - 3(3.1) = 12 - 9.3 = \underline{2.7}$$

THUS SO FAR $B(3.1, 2.7)$



● NOW THE REQUIRED POINT C MUST BE SUCH SO THAT B IS THE MIDPOINT OF AC

$$\begin{array}{ccc} \left(\begin{array}{c} -2 \\ 1 \end{array} \right) & \xrightarrow{\begin{array}{c} +5.1 \\ +1.7 \end{array}} & \left(\begin{array}{c} 3.1 \\ 2.7 \end{array} \right) & \xrightarrow{\begin{array}{c} +5.1 \\ +1.7 \end{array}} & \left(\begin{array}{c} 8.2 \\ 4.4 \end{array} \right) \\ \uparrow & & \uparrow & & \uparrow \\ \mathbf{A} & & \mathbf{B} & & \mathbf{C} \end{array}$$

$\therefore \underline{C(8.2, 4.4)}$

YGB - MPI PAPER 1 - QUESTION 11

b)

● STARTING WITH A DIAGRAM - AGAIN

● GRADIENT AP = $\frac{2-1}{4+2} = \frac{1}{6}$

● THUS GRADIENT AQ = -6

● $|AP| = \sqrt{(2-1)^2 + (4+2)^2}$

$|AP| = \sqrt{1+36}$

$|AP| = \sqrt{37}$

● THE EQUATION OF THE LINE THROUGH A & Q IS

$\Rightarrow y-1 = -6(x+2)$

● THIS MUST BE SATISFIED BY Q(a,b)

$\Rightarrow b-1 = -6(a+2)$

● ALSO THE DISTANCE $|AQ| = \sqrt{37}$

$\Rightarrow \sqrt{(a+2)^2 + (b-1)^2} = \sqrt{37}$

$\Rightarrow (a+2)^2 + (b-1)^2 = 37$

$\Rightarrow (a+2)^2 + [-6(a+2)]^2 = 37$

$\Rightarrow (a+2)^2 + 36(a+2)^2 = 37$

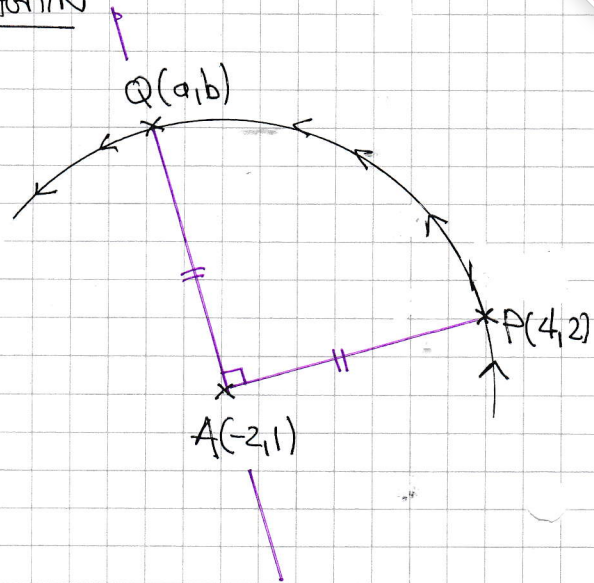
$\Rightarrow 37(-2+a)^2 = 37$

$\Rightarrow (a+2)^2 = 1$

$\Rightarrow a+2 = \begin{cases} 1 \\ -1 \end{cases}$

$\Rightarrow a = \begin{cases} -1 \\ -3 \end{cases} \quad b = \begin{cases} -5 \\ 7 \end{cases} \quad \leftarrow \begin{matrix} (-1, -5) \\ (-3, 7) \end{matrix}$

$\therefore \underline{Q(-3, 7)}$



YGB - NPI PAPER 15 - QUESTION 12

LOOKING AT THE FIRST EQUATION

$$\int_1^2 kx^2 + a \, dx = 11 \Rightarrow \left[\frac{1}{3}kx^3 + ax \right]_1^2 = 11$$
$$\Rightarrow \left(\frac{8}{3}k + 2a \right) - \left(\frac{1}{3}k + a \right) = 11$$
$$\Rightarrow \frac{7}{3}k + a = 11$$

LOOKING AT THE SECOND EQUATION

$$\int_1^k \frac{6}{x^2} \, dx = a \Rightarrow \left[-\frac{6}{x} \right]_1^k = a$$
$$\Rightarrow -\frac{6}{k} + 6 = a$$

SOLVING SIMULTANEOUSLY BY SUBSTITUTION

$$\Rightarrow \frac{7}{3}k + \left(-\frac{6}{k} + 6 \right) = 11$$
$$\Rightarrow \frac{7}{3}k - \frac{6}{k} + 6 = 11 \quad \left. \begin{array}{l} \nearrow \\ \searrow \end{array} \right\} \times 3k$$
$$\Rightarrow 7k^2 - 18 + 18k = 33k$$
$$\Rightarrow 7k^2 - 15k - 18 = 0$$
$$\Rightarrow (7k + 6)(k - 3) = 0$$

