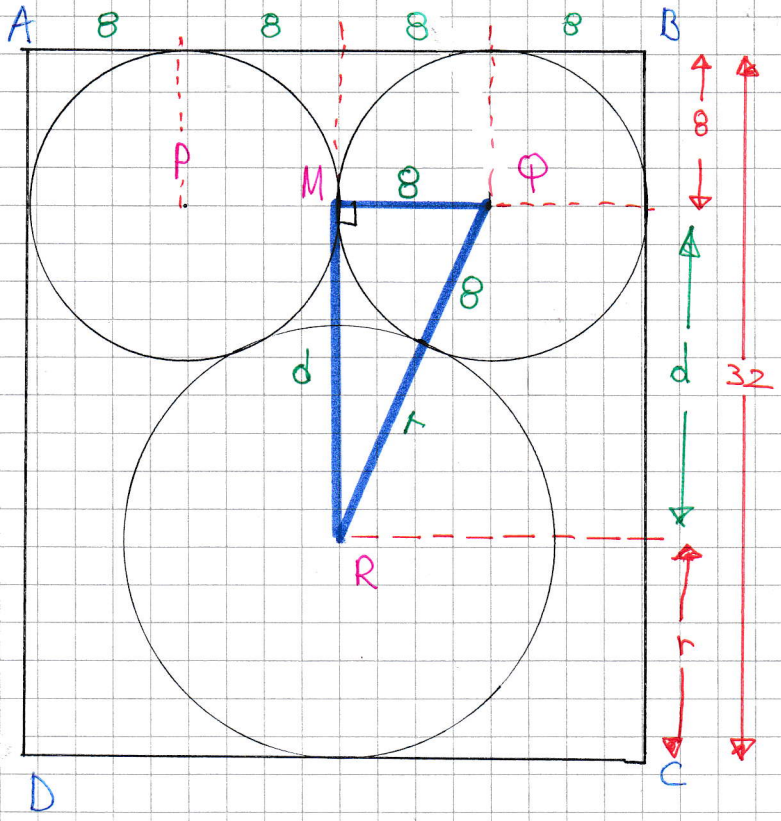
IYGB - MPI PAPER S - QUESTION 1

LOOKING AT THE DIAGRAM



$d = 32 - 8 - r$

$d = 24 - r$

BY PYTHAGORAS

$$|MQ|^2 + |MR|^2 = |QR|^2$$

$$8^2 + d^2 = (8+r)^2$$

$$64 + (24-r)^2 = (8+r)^2$$

$$64 + 576 - 48r + r^2 = 64 + 16r + r^2$$

$$576 = 64r$$

$$r = 9$$

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IYGB - MPI PAPER S - QUESTION 2

$$f(x) \equiv 2^{ax} + b \ , \ x \in \mathbb{R}$$

USING $f(2) = \frac{5}{2}$

$$\Rightarrow 2^{a\times 2} + b = \frac{5}{2}$$

$$\Rightarrow 2^{2a} + b = \frac{5}{2}$$

$$\Rightarrow b = \frac{5}{2} - 2^{2a}$$

USING $f(-2) = 4$

$$\Rightarrow 2^{a(-2)} + b = 4$$

$$\Rightarrow 2^{-2a} + b = 4$$

$$\Rightarrow b = 4 - 2^{-2a}$$

SOLVING SIMULTANEOUSLY

$$\Rightarrow 4 - 2^{-2a} = \frac{5}{2} - 2^{2a}$$

$$\Rightarrow 4 - \frac{1}{2^{2a}} = \frac{5}{2} - 2^{2a}$$

$$\Rightarrow 4 - \frac{1}{A} = \frac{5}{2} - A \qquad (A = 2^{2a})$$

$$\Rightarrow 4A - 1 = \frac{5}{2}A - A^2$$

$$\Rightarrow 8A - 2 = 5A - 2A^2$$

$$\Rightarrow 2A^2 + 3A - 2 = 0$$

$$\Rightarrow (2A - 1)(A + 2) = 0$$

$$\Rightarrow A = \begin{cases} \frac{1}{2} \\ \cancel{-2} \end{cases}$$

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IYGB - MPI PAPER S - QUESTION 2

$$\Rightarrow 2^{2a} = \frac{1}{2}$$

$$\Rightarrow 2^{2a} = 2^{-1}$$

$$\Rightarrow 2a = -1$$

$$\Rightarrow a = -\frac{1}{2}$$ //

Q USING FINALLY $b = 4 - 2^{-2a}$

$$\Rightarrow b = 4 - 2^{-2\left(-\frac{1}{2}\right)}$$

$$\Rightarrow b = 4 - 2^{1}$$

$$\Rightarrow b = 2$$ //

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IYGB - MPI PAPER S - QUESTION 3

APPLY THE DEFINITION OF A COMBINATION

$$\binom{n}{m}\binom{n-m}{r-m} = \frac{n!}{m!(n-m)!} \times \frac{(n-m)!}{(r-m)![(n-m)-(r-m)]!}$$

$$= \frac{n!}{m!(n-m)!} \times \frac{(n-m)!}{(r-m)!(n-r)!}$$

$$= \frac{n!}{m!} \times \frac{1}{(r-m)!(n-r)!}$$

$$= \frac{n!}{m!(r-m)!(n-r)!}$$

$$= \frac{n!\,r!}{m!(r-m)!(n-r)!\,r!}$$

$$= \frac{n!}{r!(n-r)!} \times \frac{r!}{m!(r-m)!}$$

$$= \binom{n}{r}\binom{r}{m}$$

// AS REQUIRED

IYGB – MPI PAPER 5 – QUESTION 4

TREATING A POINT ON l AS A LOCUS – LET $C(k,h)$ SO C MUST LIE ON THE PERPENDICULAR BISECTOR OF AB SO EQUIDISTANT FROM A & B

$A(3,2)$ $B(7,8)$ $C(k,h)$

- $|AC|^2 = (k-3)^2 + (h-2)^2$
- $|BC|^2 = (k-7)^2 + (h-8)^2$

THUS WE HAVE

$$(k-3)^2 + (h-2)^2 = (k-7)^2 + (h-8)^2$$
$$k^2 - 6k + 9 + h^2 - 4h + 4 = k^2 - 14k + 49 + h^2 - 16h + 64$$
$$-6k - 4h + 13 = -14k - 16h + 113$$
$$8k + 12h = 100$$
$$2k + 3h = 25$$

SOLVING SIMULTANEOUSLY WITH $|AC|^2 = 26$

$$\left.\begin{aligned}(k-3)^2 + (h-2)^2 &= 26 \\ 2k + 3h &= 25\end{aligned}\right\} \Rightarrow 4(k-3)^2 + 4(h-2)^2 = 104$$
$$\Rightarrow (2k-6)^2 + (2h-4)^2 = 104$$
$$\Rightarrow [(25-3h)-6]^2 + (2h-4)^2 = 104$$
$$\Rightarrow (19-3h)^2 + (2h-4)^2 = 104$$
$$\Rightarrow \begin{pmatrix} 361 - 114h + 9h^2 \\ 16 - 16h + 4h^2 \end{pmatrix} = 104$$
$$\Rightarrow 13h^2 - 130h + 377 = 104$$
$$\Rightarrow 13h^2 - 130h + 273 = 0 \quad \div 13$$
$$\Rightarrow h^2 - 10h + 21 = 0$$
$$\Rightarrow (h-3)(h-7) = 0$$

$h = 3$ $(2 < h < 5)$

$k = 8$ (USING $2k+3h=25$)

IYGB - MPI PAPER S - QUESTION 4

FINALLY THE EQUATION OF l IS THE WORDS FOUND IN x & y

I.E $2x + 3y = 25$

$2x + 3y = 25$

ALTERNATIVE APPROACH IS ALSO POSSIBLE

E.G
- GRAD $AB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-2}{7-3} = \frac{6}{4} = \frac{3}{2}$
- GRAD OF l MUST BE $-\frac{2}{3}$
- MIDPOINT OF AB MUST BE $M\left(\frac{7+3}{2}, \frac{8+2}{2}\right) = M(5,5)$
- EQUATION OF l MUST BE $y - y_0 = m(x - x_0)$

$y - 5 = -\frac{2}{3}(x-5)$

$3y - 15 = -2x + 10$

$2x + 3y = 25$

THEN SOLVE SIMULTANEOUS EQUATIONS

$|AC|$ OR $|BC|$ IS $\sqrt{26}$ & $2x + 3y = 25$

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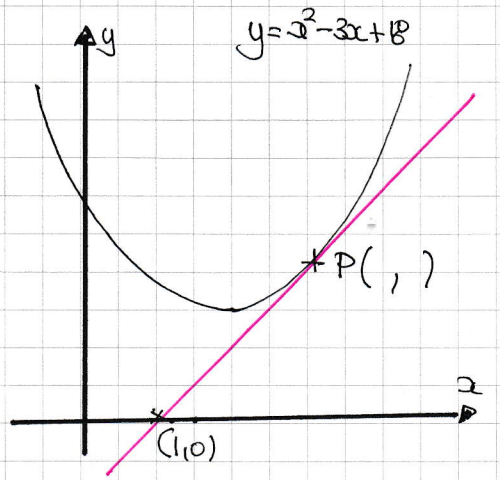
IYGB - MP1 PAPER S - QUESTION 5

FIRSTLY WE REQUIRE TO FIND THE COORDINATES OF THE POINT OF TANGENCY

METHOD A (PARAMETRIC APPROACH)



- LET THE COORDINATES OF P BE $(a, a^2-3a+18)$, FOR SOME $a>0$
- THEN WE HAVE

$$\Rightarrow \frac{dy}{dx} = 2x-3$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{x=a} = 2a-3$$

- EQUATION OF THE TANGENT AT P WILL BE

$$\Rightarrow y-(a^2-3a+18) = (2a-3)(x-a)$$

- AS THE TANGENT PASSES THROUGH $(1,0)$ WE HAVE

$$\Rightarrow -(a^2-3a+18) = (2a-3)(1-a)$$

$$\Rightarrow a^2-3a+18 = (2a-3)(a-1)$$

$$\Rightarrow a^2-3a+18 = 2a^2-5a+3$$

$$\Rightarrow 0 = a^2-2a-15$$

$$\Rightarrow (a+3)(a-5)=0$$

$$\Rightarrow a = \cancel{-3}, 5$$

IE $P(5,28)$

METHOD B (DISCRIMINANT APPROACH)

- LET THE EQUATION OF THE TANGENT BE $y=mx+c$
- USING THE CONSTRAINT $(1,0)$ WE HAVE $0=m+c$
- SOLVING SIMULTANEOUSLY

$$\left.\begin{array}{l} y=x^2-3x+18 \\ y=mx+c \end{array}\right\} \Rightarrow \left.\begin{array}{l} y=x^2-3x+18 \\ y=mx-m \end{array}\right\} \Rightarrow x^2-3x+18 = mx-m$$

$$\Rightarrow x^2-(3+m)x+(m+18)=0$$

LOOKING FOR REPEATED ROOTS

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IYGB – MPI PAPER S – QUESTION 5

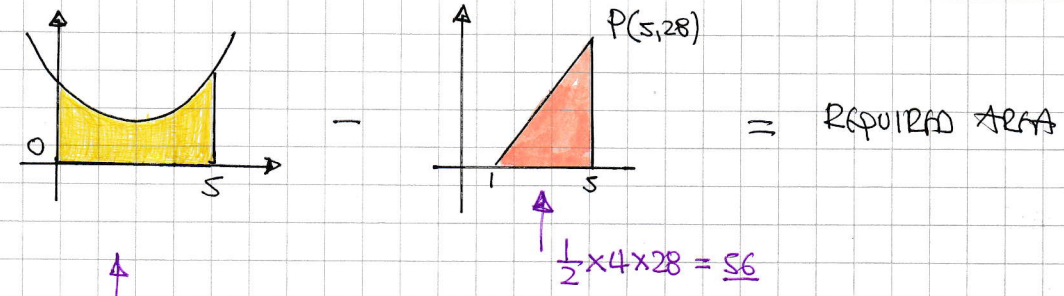
$$\Rightarrow [-(3+m)]^2 - 4\times 1\times(m+18) = 0$$
$$\Rightarrow m^2+6m+9-4m-72=0$$
$$\Rightarrow m^2+2m-63=0$$
$$\Rightarrow (m-7)(m+9)=0$$
$$\Rightarrow m = 7 \text{ (POSITIVE GRADIENT)}, \not{-9}$$

- HENCE WE HAVE

$$x^2-(3+m)x+(m+18)=0$$
$$x^2-10x+25=0$$
$$(x-5)^2=0$$

$x=5$ & $y=28$ AS BEFORE

- NOW WE HAVE



$$\int_0^5 x^2-3x+18\,dx = \left[\frac{1}{3}x^3-\frac{3}{2}x^2+18x\right]_0^5 = \left(\frac{125}{3}-\frac{75}{2}+90\right)-(0) = \frac{565}{6}$$

- THE REQUIRED AREA IS

$$\frac{565}{6}-56 = \frac{229}{6}$$

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IYGB - MPI PAPER S - QUESTION 6

$$\frac{4\sqrt{2}-3\sqrt{3}-1}{1-2\sqrt{2}+\sqrt{3}}=\frac{(4\sqrt{2}-3\sqrt{3}-1)[1-2\sqrt{2}-\sqrt{3}]}{[(1-2\sqrt{2})+\sqrt{3}][(1-2\sqrt{2})-\sqrt{3}]}$$

$$=\frac{(4\sqrt{2}-3\sqrt{3}-1)(1-2\sqrt{2}-\sqrt{3})}{(1-2\sqrt{2})^2-(\sqrt{3})^2}$$ ← $1-4\sqrt{2}+8-3=6-4\sqrt{2}$

NOW MULTIPLYING & TIDYING UP THE NUMERATOR

$$\cdots=\begin{array}{r} 4\sqrt{2}-16-4\sqrt{6}-3\sqrt{3} \\ 2\sqrt{2}+9+6\sqrt{6}+\sqrt{3} \\ -1 \\ \hline \end{array}$$

$$\cdots=\frac{6\sqrt{2}+2\sqrt{6}-2\sqrt{3}-8}{6-4\sqrt{2}}=\frac{3\sqrt{2}+\sqrt{6}-\sqrt{3}-4}{3-2\sqrt{2}}$$

RATIONALIZING ONCE MORE

$$=\frac{(3+2\sqrt{2})(3\sqrt{2}+\sqrt{6}-\sqrt{3}-4)}{(3+2\sqrt{2})(3-2\sqrt{2})}$$ ← $9-8=1$

TIDY THE NUMERATOR

$$\begin{array}{r} \cdots 9\sqrt{2}+3\sqrt{6}-3\sqrt{3}-12 \\ -8\sqrt{2}-2\sqrt{6}+2\sqrt{12}+12 \\ \hline \sqrt{2}+\sqrt{6}+\sqrt{3} \end{array}$$

$$\cdots=\frac{\sqrt{6}+\sqrt{3}+\sqrt{2}}{1}=\sqrt{6}+\sqrt{3}+\sqrt{2}$$

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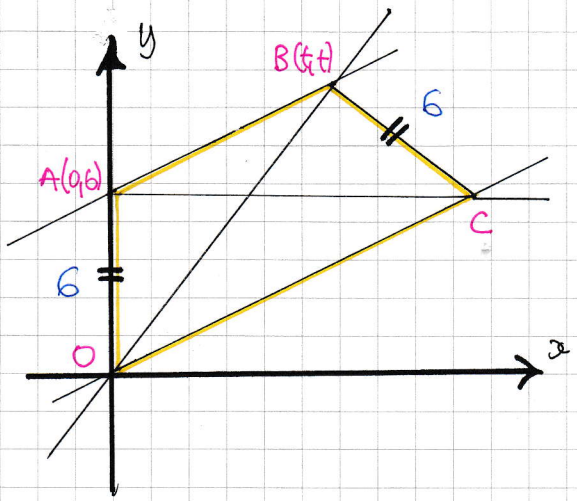
IYGB - MPI PAPER S - QUESTION 1

AS B LIES ON THE LINE $y=x$, LET $B(t,t)$



- GRADIENT "AB" $= \frac{t-6}{t-0} = \frac{t-6}{t}$
- EQUATION OF "OC" (PARRALEL TO "AB")

$$y = \frac{t-6}{t}x$$

- AS "AC" IS HORIZONTAL, $y=6$

$$6 = \frac{t-6}{t}x$$

$$x = \frac{6t}{t-6}$$

- DISTANCE BC MUST BE 6, $B(t,t)$, $C\left(\frac{6t}{t-6},6\right)$

$$\sqrt{\left(\frac{6t}{t-6}-t\right)^2+(6-t)^2} = 6$$

SOLVING THE ABOVE EQUATION

$$\Rightarrow \left(\frac{6t}{t-6}-t\right)^2+(6-t)^2 = 36$$

$$\Rightarrow \left[\frac{6t-t(t-6)}{t-6}\right]^2+(t-6)^2 = 36$$

$$\Rightarrow \left[\frac{12t-t^2}{t-6}\right]^2+(t-6)^2 = 36$$

$$\Rightarrow \frac{t^2(12-t)^2}{(t-6)^2}+(t-6)^2 = 36$$

$$\Rightarrow t^2(t-12)^2+(t-6)^4 = 36(t-6)^2$$

IYGB - MPI PAPER S - QUESTION 7

SOLVING BY A SUBSTITUTION FOR SIMPLICITY, LET $w = t-6$

$$\Rightarrow \underbrace{(w+6)^2}_{t^2}\underbrace{(w-6)^2}_{t-12} + w^4 = 36w^2$$

$$\Rightarrow (w^2-36)^2 + w^4 = 36w^2$$

$$\Rightarrow w^4 - 72w^2 + 1296 + w^4 - 36w^2 = 0$$

$$\Rightarrow 2w^4 - 108w^2 + 1296 = 0$$

$$\Rightarrow w^4 - 54w^2 + 648 = 0$$

BY QUADRATIC FORMULA, COMPLETING THE SQUARE, OR FACTORIZATION

$$\Rightarrow (w^2-36)(w^2-18) = 0$$

$$\Rightarrow w^2 = 36, \ 18$$

$$\Rightarrow w = 6, \ -6, \ 3\sqrt{2}, \ -3\sqrt{2}$$

$$t-6 = 6, \ -6, \ 3\sqrt{2}, \ -3\sqrt{2}$$

$$\Rightarrow t = 12, \ \cancel{0}, \ 6+3\sqrt{2}, \ \cancel{6-3\sqrt{2}}$$

$t>6$ DUE TO POINT C

$t>6$ DUE TO POINT C

NOW WE HAVE TO CHECK AGAINST THE x CO-ORDINATE OF C

- IF $B(12,12) \Rightarrow x_c = \dfrac{6t}{t-6} = \dfrac{6\times 12}{12-6} = \dfrac{72}{6} = \cancel{12}$

  (AS THIS YIELDS A PARALLELOGRAM)

- IF $B(6+3\sqrt{2}, 6+3\sqrt{2}) \Rightarrow x_c = \dfrac{6(6+3\sqrt{2})}{6+3\sqrt{2}-6} = \dfrac{6(6+3\sqrt{2})}{3\sqrt{2}} = \dfrac{12+6\sqrt{2}}{\sqrt{2}} = 6+6\sqrt{2}$

  WHICH IS OK

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IYGB - MPI PAPER S - QUESTION 8

$$(19+2\sin^2 2\theta)\tan 2\theta = \frac{3}{\cos 2\theta} - 17\cos 2\theta$$

- START MANIPULATING THE EQUATION BY SWITCHING THE $\tan 2\theta$ INTO SINES AND COSINES AND THEN GETTING RID OF THE DENOMINATORS

$$\Rightarrow (19+2\sin^2 2\theta)\times\frac{\sin 2\theta}{\cos 2\theta} = \frac{3}{\cos 2\theta} - 17\cos 2\theta$$

$$\Rightarrow (19+2\sin^2 2\theta)\times\sin 2\theta = 3 - 17\cos^2 2\theta \qquad \times\cos 2\theta$$

$$\Rightarrow (19+2\sin^2 2\theta)\sin 2\theta = 3 - 17(1-\sin^2 2\theta)$$

$$\Rightarrow 19\sin 2\theta + 2\sin^3 2\theta = -14 + 17\sin^2 2\theta$$

$$\Rightarrow 2\sin^3 2\theta - 17\sin^2 2\theta + 19\sin 2\theta + 14 = 0$$

- THIS IS A CUBIC IN $\sin 2\theta$, SO LOOK FOR SOME SENSIBLE LINEAR FACTORS

$$f(x) = 2x^3 - 17x^2 + 19x + 14$$

- $f(1) = 2 - 17 + 19 + 14 \neq 0$
- $f(-1) = -2 - 17 - 19 + 14 \neq 0$
- $f(2) = 16 - 68 + 38 + 14 = 0$

$\therefore (x-2)$ IS A FACTOR

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IYGB – MPI PAPER S – QUESTION 8

- FACTORIZE THE CUBIC BY USING LONG DIVISION, INSPECTION OR ANY OTHER SENSIBLE METHOD

$$\begin{aligned} f(x) &= 2x^3-17x^2+19x+14 \\ &= 2x^2(x-2)-13x(x-2)-7(x-2) \\ &= (x-2)(2x^2-13x-7) \\ &= (x-2)(2x+1)(x-7) \end{aligned}$$

$$\therefore f(x)=0 \Rightarrow x= \begin{cases} 2 \\ 7 \\ -\frac{1}{2} \end{cases}$$

$$f(\sin 2\theta)=0 \Rightarrow \sin 2\theta = \begin{cases} 2 \\ 7 \\ -\frac{1}{2} \end{cases}$$

- SOLVING THE EQUATION $\sin 2\theta = -\frac{1}{2}$ FOR $0 \leq \theta < 360^\circ$

$$\Rightarrow \arcsin\left(-\tfrac{1}{2}\right) = -30^\circ$$

$$\Rightarrow \begin{cases} 2\theta = -30^\circ \pm 360n \\ 2\theta = 210^\circ \pm 360n \end{cases} \quad n=0,1,2,3,\ldots$$

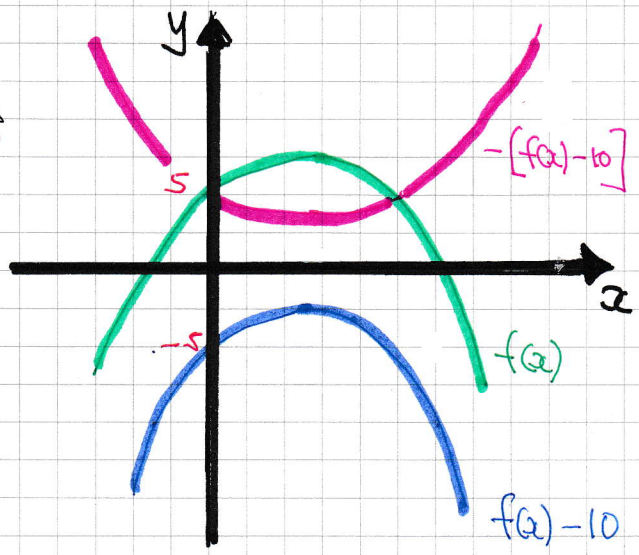
$$\Rightarrow \begin{cases} \theta = -15^\circ \pm 180n \\ \theta = 105^\circ \pm 180n \end{cases}$$

$$\Rightarrow \theta = 165^\circ, 345^\circ, 105^\circ, 285^\circ$$

$$\Rightarrow \theta = 105^\circ, 165^\circ, 285^\circ, 345^\circ$$

IYGB - MPI PAPER S - QUESTION 9

FIRSTLY, BY INSPECTION, THE y INTERCEPT OF $f(x)$ IS 5, BY SETTING $x=0$



THEN WE TRANSFORM AS FOLLOWS

- TRANSLATE $f(x)$, "DOWN" BY 10 UNITS, I.E ONTO THE "BLUE" GRAPH $f(x)-10$
- THEN REFLECT THE GRAPH OF $f(x)-10$ ABOUT THE x AXIS ONTO THE "PINK" GRAPH WITH EQUATION $y=-[f(x)-10]$ OR $y=10-f(x)$

HENCE WE CAN OBTAIN THE EQUATION OF $g(x)$

$$\Rightarrow g(x) = 10 - f(x)$$
$$\Rightarrow g(x) = 10 - (x+1)(5-x)$$
$$\Rightarrow g(x) = 10 + (x+1)(x-5)$$
$$\Rightarrow g(x) = 10 + x^2 - 4x - 5$$
$$\Rightarrow g(x) = x^2 - 4x + 5$$

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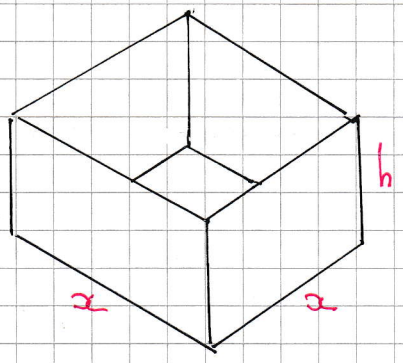
IYGB – MPI PAPER S – QUESTION 10



THE BOX HAS TO HAVE A FIXED VOLUME

$$x^2h = \text{constant volume}$$
$$x^2h = V$$

THE SURFACE AREA OF THE BOX IS GIVEN BY

$$A = x^2 + 4xh$$
$$A = x^2 + \frac{4x^2h}{x}$$
$$A = x^2 + \frac{4V}{x}$$
$$A = x^2 + 4Vx^{-1}$$

DIFFERENTIATING & SETTING TO ZERO

$$\frac{dA}{dx} = 2x - 4Vx^{-2}$$
$$0 = 2x - \frac{4V}{x^2}$$
$$\frac{4V}{x^2} = 2x$$
$$2x^3 = 4V$$
$$x^3 = 2V$$
$$x = \sqrt[3]{2V}$$

CHECK WHETHER THIS VALUE OF x PRODUCES A MINIMUM OR MAXIMUM

$$\frac{d^2A}{dx^2} = 2 + 8Vx^{-3} = 2 + \frac{8V}{x^3}$$
$$\left.\frac{d^2A}{dx^2}\right|_{x=\sqrt[3]{2V}} = 2 + \frac{8V}{2V} = 6 > 0$$

INDEED THIS VALUE OF x MINIMIZES

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IYGB – MPI PAPER S – QUESTION 10

USING $a^2h = V$

$$\Rightarrow h = \frac{V}{x^2}$$

$$\Rightarrow h = \frac{V}{\left(\sqrt[3]{2V^1}\right)^2} = \frac{V}{\left[(2V)^{\frac{1}{3}}\right]^2} = \frac{V}{(2V)^{\frac{2}{3}}}$$

$$\Rightarrow h = \frac{V}{2^{\frac{2}{3}}V^{\frac{2}{3}}} = \frac{V^{\frac{1}{3}}}{2^{\frac{2}{3}}} = \frac{V^{\frac{1}{3}} \times 2^{\frac{1}{3}}}{2^{\frac{2}{3}} \times 2^{\frac{1}{3}}}$$

$$\Rightarrow h = \frac{2^{\frac{1}{3}}V^{\frac{1}{3}}}{2} = \frac{(2V)^{\frac{1}{3}}}{2}$$

HENCE $h = \frac{x}{2}$

$\therefore$ $x = 2h$ WITH SURFACE IS MINIMUM

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IYGB - MPI PAPER S - QUESTION 11

ASSERTION: THE SQUARE OF AN ODD POSITIVE INTEGER IS ALWAYS OF THE FORM $8T+1$, WHERE "T" IS A TRIANGLE NUMBER

PROOF BY EXHUSTION

- LET n BE ODD

$\Rightarrow n = 2m+1$

$\Rightarrow 2n+1 = 2(2m+1)+1$

$\Rightarrow 2n+1 = 4m+3$

e.g. $7 = (4\times 1)+3$
$35 = (4\times 8)+3$
$47 = (4\times 11)+3$
ETC

- LET n BE EVEN

$\Rightarrow n = 2m$

$\Rightarrow 2n+1 = 2(2m)+1$

$\Rightarrow 2n+1 = 4m+1$

e.g. $5 = (4\times 1)+1$
$21 = (4\times 5)+1$
$33 = (4\times 8)+1$
ETC

- SQUARING THE ODD NUMBER IN EACH CASE YIELDS

$(2n+1)^2 = (4m+3)^2$
$= 16m^2+24m+9$
$= 8(2m^2+3m+1)+1$
$= 8(2m+1)(m+1)+1$

$(2n+1)^2 = (4m+1)^2$
$= 16m^2+8m+1$
$= 8m(2m+1)+1$

I.E IN BOTH CASES THE OUTCOME IS OF THE FORM $8f(m)+1$

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IYGB - MPI PAPER S - QUESTION 11

NOW TO PROVE THAT $f(m)$ IS A TRIANGLE NUMBER

- TRIANGLE NUMBERS ARE $1, 3, 6, 10, 15, 21, 28, 36, \ldots$

- $1, 6, 15, 28$

  $5, 9, 13$

  $4, 4$

$u_n = 2n^2 + \alpha n + b$

$2n^2$: $2, 8, 18, 32, \ldots$

HERE: $1, 6, 15, 28$

$-1, -2, -3, -4$

$\therefore u_n = 2n^2 - n$

$\Rightarrow u_n = n(2n-1)$

$n \longmapsto m+1$

$\Rightarrow u_m = (m+1)[2(m+1)-1]$

$\Rightarrow u_m = (m+1)(2m+1)$

WHICH WE OBTAINED

- $3, 10, 21, 36$

  $7, 11, 15$

  $4, 4$

$u_n = 2n^2 + \alpha n + b$

$2n^2$: $2, 8, 18, 28$

HERE: $3, 10, 21, 36$

$+1, +2, +3, +4$

$\therefore u_n = 2n^2 + n$

$\Rightarrow u_n = n(2n+1)$

OR

$\Rightarrow u_m = m(2m+1)$

WHICH WE OBTAINED

EVERY SQUARE OF AN ODD NATURAL NUMBER GREATER THAN 3, IS OF THE FORM $8T+1$, WHERE T IS A TRIANGLE NUMBER

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IYGB - MPI PAPER S - QUESTION 12

$$3e^{2(x+1)} - (2e)^x(e^4+9) + 3e^2 4^x = 0$$

- PROCEED AS FOLLOWS

$$\Rightarrow 3e^{2x+2} - 2^x e^x (e^4+9) + 3e^2 4^x = 0$$

$$\Rightarrow 3e^{2x}e^2 - 2^x e^x (e^4+9) + 3e^2(2^x)^2 = 0$$

- DIVIDE THE EQUATION THROUGH BY $3e^2e^x(2^x) \neq 0$

$$\Rightarrow \frac{3e^{2x}e^2}{3e^2e^x(2^x)} - \frac{2^x e^x(e^4+9)}{3e^2e^x(2^x)} + \frac{3e^2(2^x)^2}{3e^2e^x(2^x)} = 0$$

$$\Rightarrow \frac{e^x}{2^x} - \frac{e^4+9}{3e^2} + \frac{2^x}{e^x} = 0$$

- LET $A = \frac{e^x}{2^x}$ TO TRANSFORM THE ABOVE EQUATION INTO A QUADRATIC IN $\frac{e^x}{2^x}$

$$\Rightarrow A - \frac{e^4+9}{3e^2} + \frac{1}{A} = 0$$

$$\Rightarrow A^2 - A\left(\frac{e^4+9}{3e^2}\right) + 1 = 0$$

- COMPLETING THE SQUARE (OR ATTEMPT TO FACTORIZE)

$$\Rightarrow \left[A - \frac{e^4+9}{6e^2}\right]^2 - \frac{(e^4+9)^2}{36e^4} + 1 = 0$$

$$\Rightarrow \left[A - \frac{e^4+9}{6e^2}\right]^2 = \frac{(e^4+9)^2}{36e^4} - 1$$

IYGB – MPI PAPER S – QUESTION 12

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- TIDY THE R.H.S FIRST

$$\frac{(e^4+9)^2}{36e^4}-1=\frac{e^8+18e^4+81}{36e^4}-\frac{36e^4}{36e^4}=\frac{e^8-18e^4+81}{36e^4}$$

$$=\frac{(e^4-9)^2}{36e^4}=\left(\frac{e^4-9}{6e^2}\right)^2$$

- THUS WE HAVE

$$\Rightarrow\left[A-\frac{e^4+9}{6e^2}\right]^2=\left(\frac{e^4-9}{6e^2}\right)^2$$

$$\Rightarrow A-\frac{e^4+9}{6e^2}=\begin{cases}\frac{e^4-9}{6e^2}\\ \frac{9-e^4}{6e^2}\end{cases}$$

$$\Rightarrow A=\begin{cases}\frac{2e^4}{6e^2}\\ \frac{18}{6e^2}\end{cases}$$

$$\Rightarrow A=\begin{cases}\frac{1}{3}e^2\\ 3e^{-2}\end{cases}$$

- SOLVING EACH CASE SEPARATELY

$$\Rightarrow\frac{e^x}{2^x}=\frac{1}{3}e^2 \qquad \Rightarrow\frac{e^x}{2^x}=\frac{3}{e^2}$$

$$\Rightarrow\left(\frac{1}{2}e\right)^x=\frac{1}{3}e^2 \qquad \Rightarrow\left(\frac{e}{2}\right)^x=\frac{3}{e^2}$$

$$\Rightarrow\ln\left(\frac{1}{2}e\right)^x=\ln\left(\frac{1}{3}e^2\right) \qquad \Rightarrow\ln\left(\frac{1}{2}e\right)^x=\ln\left(\frac{3}{e^2}\right)$$

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$$\Rightarrow x\ln\left(\tfrac{1}{2}e\right) = \ln\tfrac{1}{3}e^2$$

$$\Rightarrow x\left[\ln\tfrac{1}{2}+\ln e\right] = \ln\tfrac{1}{3}+\ln e^2$$

$$\Rightarrow x\left[-\ln 2+1\right] = -\ln 3+2$$

$$\Rightarrow x = \frac{2-\ln 3}{1-\ln 2}$$

$$\Rightarrow x\ln\left(\tfrac{1}{2}e\right) = \ln\left(\tfrac{3}{e^2}\right)$$

$$\Rightarrow x\left[\ln\tfrac{1}{2}+\ln e\right] = \ln 3-\ln e^2$$

$$\Rightarrow x\left[-\ln 2+1\right] = \ln 3-2$$

$$\Rightarrow x = \frac{\ln 3-2}{1-\ln 2}$$

$$\Rightarrow x = -\frac{2-\ln 3}{1-\ln 2}$$

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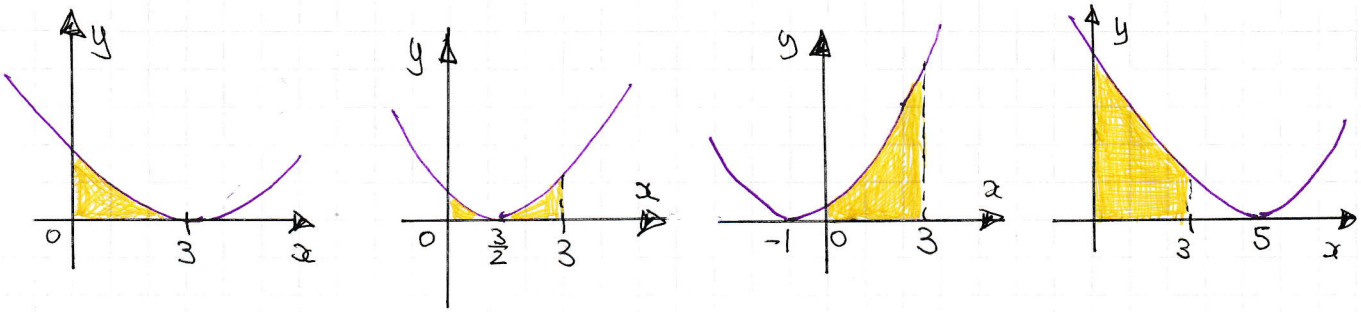
- START WITH SOME DIAGRAMS SHOWING SOME POSSIBLE CONFIGURATIONS



- GET AN EXPRESSION FOR THE AREA IN TERMS OF t, FOR $y=(x-t)^2$

$$\text{AREA}=\int_0^3 (x-t)^2\,dx = \int_0^3 x^2-2tx+t^2\,dx$$

$$=\left[\tfrac{1}{3}x^3-tx^2+t^2x\right]_0^3 = (9-9t+3t^2)-(0)$$

- LET $f(t)=3t^2-9t+9$
- BY COMPLETING THE SQUARE (OR CALCULUS)

$$\text{AREA}=f(t)=3\left[t^2-3t+3\right]$$

$$=3\left[\left(t-\tfrac{3}{2}\right)^2-\tfrac{9}{4}+3\right]$$

$$=3\left[\left(t-\tfrac{3}{2}\right)^2+\tfrac{3}{4}\right]$$

$$=3\left(t-\tfrac{3}{2}\right)^2+\tfrac{9}{4}$$

$\therefore$ MINIMUM AREA IS $\frac{9}{4}$

(IT OCCURS WHEN $t=\frac{3}{2}$ IE DIAGRAM 2)