

YGB - MPI PAPER Q - QUESTION 1

METHOD A

- $a^3 - a + 1 = a(a^2 - 1) + 1 = a(a+1)(a-1) + 1$
- AS $a(a+1)(a-1)$ CONTAINS CONSECUTIVE INTEGERS, AT LEAST ONE OF THEM WILL BE EVEN, SO $a(a+1)(a-1)$ WILL BE EVEN FOR ALL $a \in \mathbb{N}$
- HENCE $a(a+1)(a-1) + 1$ WILL BE ODD FOR ALL $a \in \mathbb{N}$

METHOD B (BY EXHAUSTION)

- LET a BE EVEN, $a = 2n$

$$\begin{aligned}(2n)^3 - 2n + 1 &= 8n^3 - 2n + 1 = 2(4n^3 - n) + 1 \\ &= 2m + 1 \\ &\therefore \text{ODD}\end{aligned}$$

- LET a BE ODD, $a = 2n + 1$

$$\begin{aligned}(2n+1)^3 - (2n+1) + 1 &= 8n^3 + 12n^2 + 6n + 1 - 2n - 1 + 1 \\ &= 8n^3 + 12n^2 + 4n + 1 \\ &= 2[4n^3 + 6n^2 + n] + 1 \\ &= 2m + 1 \\ &\therefore \text{ODD}\end{aligned}$$

Hence $a^3 - a + 1$ is odd for $a \in \mathbb{N}$

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PROCEED BY INTEGRATING THE LHS OF THE EQUATION

$$\Rightarrow \int_1^3 6x^2 + kx \, dx = 8$$

$$\Rightarrow \left[2x^3 + \frac{1}{2}kx^2 \right]_1^3 = 8$$

$$\Rightarrow \left(2 \times 3^3 + \frac{1}{2}k \times 3^2 \right) - \left(2 \times 1^3 + \frac{1}{2}k \times 1^2 \right) = 8$$

$$\Rightarrow 54 + \frac{9}{2}k - 2 - \frac{1}{2}k = 8$$

$$\Rightarrow 52 + 4k = 8$$

$$\Rightarrow 4k = -44$$

$$\Rightarrow \underline{k = -11}$$

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THE DEFINITION OF THE DERIVATIVE FOR $y = f(x)$ IS GIVEN BY

$$\frac{dy}{dx} = f'(x) = \lim_{h \rightarrow 0} \left[\frac{f(x+h) - f(x)}{h} \right]$$

$$f'(x) = \lim_{h \rightarrow 0} \left[\frac{(x+h)^2 - x^2}{h} \right]$$

$$f'(x) = \lim_{h \rightarrow 0} \left[\frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h} \right]$$

$$f'(x) = \lim_{h \rightarrow 0} \left[\frac{2xh + h^2}{h} \right]$$

DIVIDING h FROM ALL THE 3 TERMS

$$f'(x) = \lim_{h \rightarrow 0} [2x + h]$$

AS h TENDS TO ZERO THE ONLY TERM THAT SURVIVES IS $2x$

$$f'(x) = \underline{2x}$$

As required

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(a) FORM AN EQUATION USING THE POINT P (55, 1)

$$\Rightarrow y = 2\sin(2x+k)^\circ$$

$$\Rightarrow 1 = 2\sin(110+k)^\circ$$

$$\Rightarrow \sin(k+110)^\circ = \frac{1}{2}$$

$$\text{or } \sin\left(\frac{1}{2}\right) = 30^\circ$$

$$\Rightarrow \begin{cases} k+110 = 30 \pm 360n \\ k+110 = 150 \pm 360n \end{cases} \quad n=0,1,2,3,\dots$$

$$\Rightarrow \begin{cases} k = -80 \pm 360n \\ k = 40 \pm 360n \end{cases}$$

$$\Rightarrow k = \begin{cases} \dots, -440, -80, 280, 640, \dots \\ \dots, -320, 40, 400, 760, \dots \end{cases}$$

∴ ONLY VALUE IN THE REQUIRED RANGE
 $0 < k < 90$ IS $k = 40$

b) USING $k = 40$ & THE POINT Q ($\alpha, \sqrt{3}$)

$$\Rightarrow y = 2\sin(2x+40)^\circ$$

$$\Rightarrow \sqrt{3} = 2\sin(2\alpha+40)^\circ$$

$$\Rightarrow \sin(2\alpha+40)^\circ = \frac{\sqrt{3}}{2}$$

$$\text{or } \sin\left(\frac{\sqrt{3}}{2}\right) = 60^\circ$$

$$\Rightarrow \begin{cases} 2\alpha+40 = 60 \pm 360n \\ 2\alpha+40 = 120 \pm 360n \end{cases} \quad n=0,1,2,3,\dots$$

$$\Rightarrow \begin{cases} 2\alpha = 20 \pm 360n \\ 2\alpha = 80 \pm 360n \end{cases}$$

$$\Rightarrow \begin{cases} \alpha = 10 \pm 180n \\ \alpha = 40 \pm 180n \end{cases}$$

IN THE RANGE $0 < \alpha < 360$
 $\alpha = 10, 190, 40, 220$

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a) "TAKING LOGS", BASE 10, FOR THE EQUATION

$$\Rightarrow y = a \times k^x$$

$$\Rightarrow \log_{10} y = \log_{10} (a \times k^x)$$

$$\Rightarrow \log_{10} y = \log_{10} a + \log_{10} k^x$$

$$\Rightarrow \log_{10} y = \log_{10} a + x \log_{10} k$$

$$\Rightarrow \log_{10} y = (\log_{10} k) x + (\log_{10} a)$$

$$\begin{matrix} \uparrow & & \uparrow & \uparrow & \uparrow \\ y & = & m & x & c \end{matrix}$$

\(\therefore\) A LINEAR RELATIONSHIP INDEED

b) LOOKING AT THE y INTERCEPT, B (0, 3)

$$\Rightarrow \log_{10} a = 3$$

$$\Rightarrow a = 10^3$$

$$\Rightarrow \underline{a = 1000}$$

LOOKING AT THE GRADIENT OF THE LINE THROUGH A (-3/2, 0) & (0, 3)

$$\Rightarrow \frac{y_2 - y_1}{x_2 - x_1} = \log_{10} k$$

$$\Rightarrow \frac{3 - 0}{0 - (-\frac{3}{2})} = \log_{10} k$$

$$\Rightarrow 2 = \log_{10} k$$

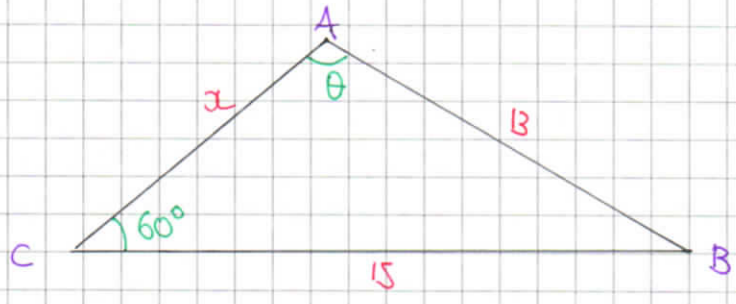
$$\Rightarrow k = 10^2$$

$$\Rightarrow \underline{k = 100}$$

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1YGB - MPI PAGE 0 - QUESTION 6

STARTING WITH A DIAGRAM



BY THE SINE RULE

$$\frac{13}{\sin 60^\circ} = \frac{15}{\sin \theta}$$

$$\sin \theta = \frac{15 \sin 60^\circ}{13}$$

$$\sin \theta = \frac{15\sqrt{3}}{26}$$

$$\theta = \begin{cases} 87.7957^\circ \dots \\ 92.2042^\circ \dots \end{cases}$$

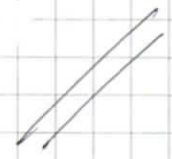
BY THE COSINE RULE

$$|AC|^2 = |AB|^2 + |BC|^2 - 2|AB||BC|\cos \theta$$

$$|AC|^2 = 13^2 + 15^2 - 2 \times 13 \times 15 \times \cos \theta$$

$$|AC| = \sqrt{394 - 390 \cos \theta}$$

$$|AC| = \begin{cases} \text{if } \theta = 87.7957^\circ \dots > \underline{x=8} \\ \text{if } \theta = 92.2042^\circ \dots > \underline{x=7} \end{cases}$$



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a) START WITH THE GRADIENT OF AB, A(2,1) & B(4,0)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 1}{4 - 2} = -\frac{1}{2}$$

EQUATION OF A LINE PARALLEL TO AB, THROUGH C(6,4)

$$y - y_0 = m(x - x_0)$$

$$y - 4 = -\frac{1}{2}(x - 6)$$

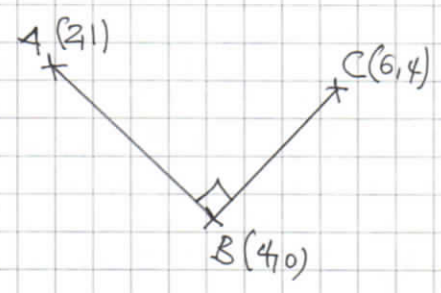
$$2y - 8 = -x + 6$$

$$\underline{2y + x = 14}$$

b) FIND THE GRADIENT OF BC

$$m_{BC} = \frac{4 - 0}{6 - 4} = \frac{4}{2} = 2$$

AS THE GRADIENTS OF BC & AB
ARE NEGATIVE RECIPROCAL OF ONE
ANOTHER $\hat{A}BC = 90^\circ$



c) USING THE DISTANCE FORMULA

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

$$|AC| = \sqrt{(4 - 1)^2 + (6 - 2)^2}$$

$$|AC| = \sqrt{9 + 16}$$

$$\underline{|AC| = 5}$$

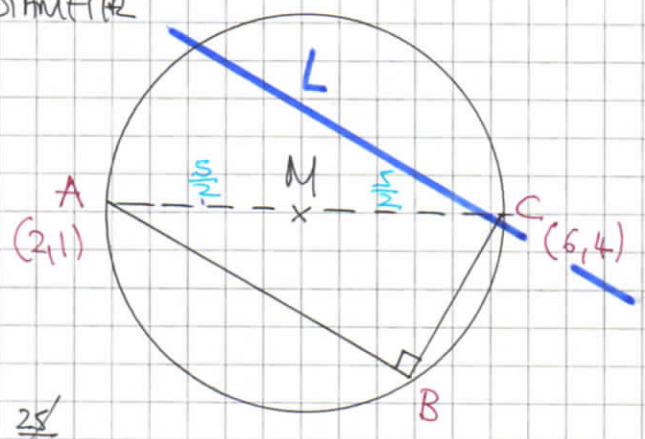
1YGB - MPI PAPER Q - QUESTION 7

d) LOOKING AT THE DIAGRAM BELOW

BY "CIRCLE THEOREM", AC IS A DIAMETER

$$M\left(\frac{6+2}{2}, \frac{1+4}{2}\right)$$

$$M\left(4, \frac{5}{2}\right)$$



$$\Rightarrow (x-4)^2 + (y-\frac{5}{2})^2 = (\frac{5}{2})^2$$

$$\Rightarrow x^2 - 8x + 16 + y^2 - 5y + \frac{25}{4} = \frac{25}{4}$$

$$\Rightarrow x^2 - 8x + 16 + y^2 - 5y = 0$$

$$\Rightarrow x^2 + y^2 - 8x - 5y + 16 = 0$$

* REQUIRED

e) LOOKING AT THE ABOVE DIAGRAM FOR L, & SOLVING SIMULTANEOUSLY

$$x + 2y = 14$$

$$x = 14 - 2y$$

$$\Rightarrow (14-2y)^2 + y^2 - 8(14-2y) - 5y + 16 = 0$$

$$\Rightarrow 196 - 56y + 4y^2 + y^2 - 112 + 16y - 5y + 16 = 0$$

$$\Rightarrow 5y^2 - 45y + 100 = 0$$

$$\Rightarrow y^2 - 9y + 20 = 0$$

$$\Rightarrow (y-4)(y-5) = 0$$

$$\Rightarrow y = \begin{cases} 4 \\ 5 \end{cases} \Rightarrow x = \begin{cases} 6 \\ 4 \end{cases}$$

\therefore ~~(6, 4)~~ & (4, 5)
[POINT C]

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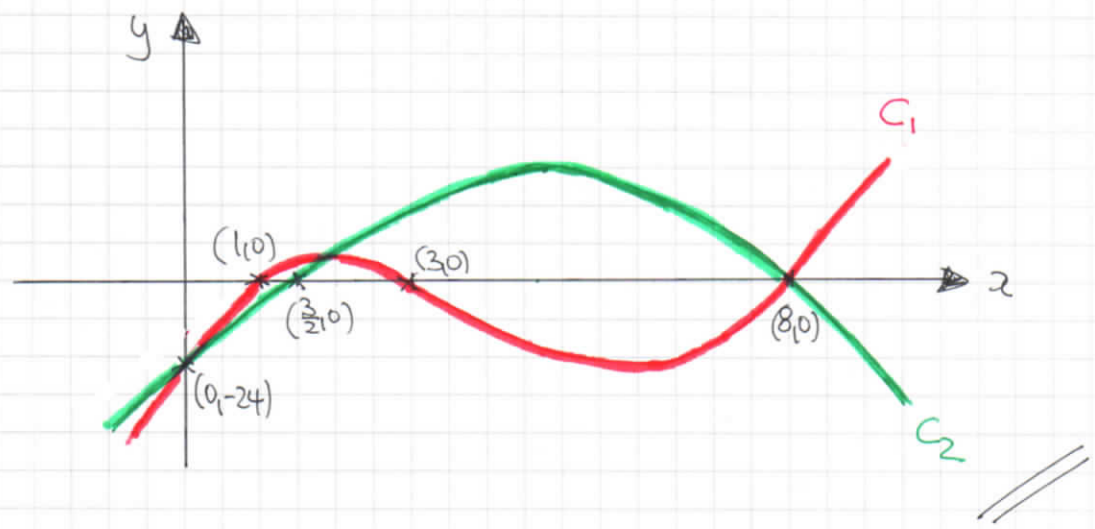
a) $C_1: y = (x-8)(x^2-4x+3)$

$C_2: y = (2x-3)(8-x)$

$y = (x-8)(x-3)(x-1)$

$(1,0), (3,0), (8,0), (0,-24)$

$(\frac{3}{2}, 0), (8,0), (0,-24)$



b) SOLVING AS FOLLOWS

$\Rightarrow (x-8)(x^2-4x+3) = (2x-3)(8-x)$

$\Rightarrow (x-8)(x^2-4x+3) - (2x-3)(8-x) = 0$

$\Rightarrow (x-8)(x^2-4x+3) + (x-8)(2x-3) = 0$

$\Rightarrow (x-8)[(x^2-4x+3) + (2x-3)] = 0$

$\Rightarrow (x-8)(x^2-2x) = 0$

$\Rightarrow x(x-2)(x-8)$

$\Rightarrow x = \begin{cases} 0 \\ 2 \\ 8 \end{cases}$

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ALTERNATIVE SOLUTION TO (b)

$$\Rightarrow (x-8)(x^2-4x+3) = (2x-3)(8-x)$$

$$\Rightarrow \begin{array}{l} x^3 - 4x^2 + 3x \\ - 8x^2 + 32x - 24 \end{array} = 16x - 2x^2 - 24 + 3x$$

$$\Rightarrow x^3 - 12x^2 + 35x - 24 = -2x^2 + 19x - 24$$

$$\Rightarrow x^3 - 10x^2 + 16x = 0$$

$$\Rightarrow x(x^2 - 10x + 16) = 0$$

$$\Rightarrow x(x-2)(x-8) = 0$$

$$\Rightarrow x = \begin{array}{l} 0 \\ 2 \\ 8 \end{array}$$



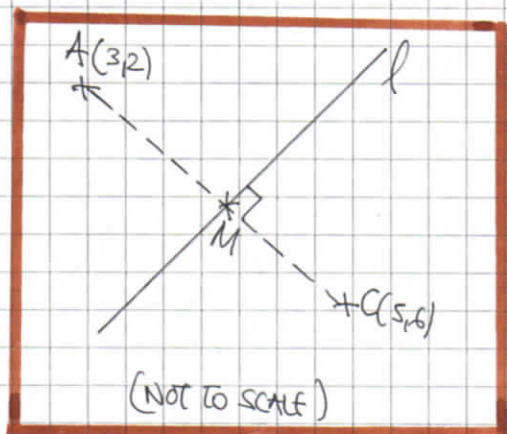
LYGB - MPI PAPER Q - QUESTION 9

a) GRADIENT OF AC

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 2}{5 - 3} = \frac{4}{2} = 2$$

MIDPOINT OF AC

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = M\left(\frac{3 + 5}{2}, \frac{2 + 6}{2}\right) \\ = M(4, 4)$$



EQUATION OF THE PERPENDICULAR BISECTOR l

$$y - y_0 = m(x - x_0)$$

$$y - 4 = -\frac{1}{2}(x - 4)$$

← GRADIENT OF l IS NEGATIVE RECIPROCAL OF THE GRADIENT OF AC

$$2y - 8 = -(x - 4)$$

$$2y - 8 = -x + 4$$

$$\underline{x + 2y = 12}$$

b) FIRST FIND THE COORDINATES OF B

$$\text{when } x = 0 \Rightarrow 2y = 12$$

$$\Rightarrow y = 6$$

$$\therefore \underline{B(0, 6)}$$

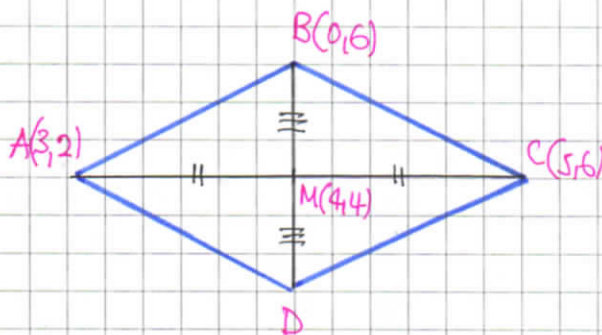
NEXT DRAW DIAGRAM - M(4,4) MUST ALSO BE THE MIDPOINT OF BD

$$\begin{pmatrix} 0 \\ 6 \end{pmatrix} \xrightarrow{+4} \begin{pmatrix} 4 \\ 4 \end{pmatrix} \xrightarrow{+4} \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 6 \end{pmatrix} \xrightarrow{-2} \begin{pmatrix} 4 \\ 4 \end{pmatrix} \xrightarrow{-2} \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\therefore a = 8 \text{ \& } b = 2$$

$$\text{i.e. } \underline{D(8, 2)}$$



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c) USING DISTANCE FORMULA FOR $P(x_1, y_1)$ & $Q(x_2, y_2)$

$$d = |PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- $|AM| = \sqrt{(3-4)^2 + (2-4)^2} = \sqrt{1+4} = \sqrt{5}$
- $|BM| = \sqrt{(0-4)^2 + (6-4)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$

AREA OF RHOMBUS CONSISTS OF 4 IDENTICAL RIGHT ANGLED TRIANGLES

$$\text{AREA} = 4 \times \frac{1}{2} |AM| |BM| = 2 \times \sqrt{5} \times 2\sqrt{5} = 4 \times 5 = \underline{20}$$

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IYGB - MPI PAPER A - QUESTION 10

a) TIDY THE EQUATION INTO INDICIAL FORM AND DIFFERENTIATE

$$\bullet y = \frac{4x\sqrt{x} + k}{7x} = \frac{4x\sqrt{x}}{7x} + \frac{k}{7x} = \frac{4}{7}x^{\frac{1}{2}} + \frac{k}{7}x^{-1}$$

$$\bullet \frac{dy}{dx} = \frac{2}{7}x^{-\frac{1}{2}} - \frac{k}{7}x^{-2}$$

$$\bullet \left. \frac{dy}{dx} \right|_{x=\frac{1}{4}} = \frac{2}{7}\left(\frac{1}{4}\right)^{-\frac{1}{2}} - \frac{k}{7}\left(\frac{1}{4}\right)^{-2} = \frac{2}{7} \times 2 - \frac{k}{7} \times 16 = \frac{4}{7} - \frac{16}{7}k$$

b) REARRANGE THE EQUATION OF THE LINE TO "READ" THE GRADIENT

$$\Rightarrow 7y + 44x - 5 = 0$$

$$\Rightarrow 7y = -44x + 5$$

$$\Rightarrow y = -\frac{44}{7}x + \frac{5}{7}$$

HENCE THE GRADIENT AT P WILL BE $-\frac{44}{7}$ (PARALLEL)

$$\Rightarrow \frac{4}{7} - \frac{16}{7}k = -\frac{44}{7}$$

$$\Rightarrow 4 - 16k = -44$$

$$\Rightarrow 48 = 16k$$

$$\Rightarrow k = 3$$

FIND THE y CO-ORDINATE OF P

$$y = \frac{4x\sqrt{x} + 3}{7x} = \frac{4 \times \frac{1}{4} \times \sqrt{\frac{1}{4}} + 3}{7 \times \frac{1}{4}} = \frac{\left(\frac{1}{2} + 3\right)^{\times 4}}{\left(\frac{7}{4}\right)^{\times 4}} = \frac{2+12}{7} = 2$$

EQUATION OF TANGENT AT P($\frac{1}{4}$, 2)

$$y - y_0 = m(x - x_0)$$

$$y - 2 = -\frac{44}{7}(x - \frac{1}{4})$$

$$7y - 14 = -44x + 11$$

$$\therefore 44x + 7y = 25$$

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LYGB - MPI PAPER Q - QUESTION 11

FORM A CUBIC IN EXPONENTIALS

$$\Rightarrow 2e^{2x} - 5e^x + 3e^{-x} = 4$$

$$\Rightarrow 2e^{2x} - 5e^x + \frac{3}{e^x} = 4$$

$$\Rightarrow 2e^{3x} - 5e^{2x} + 3 = 4e^x$$

$$\Rightarrow 2e^{3x} - 5e^{2x} - 4e^x + 3 = 0$$

) MULTIPLY THROUGH BY e^x

LET $y = e^x$

$$\Rightarrow 2y^3 - 5y^2 - 4y + 3 = 0$$

LET $f(y) = 2y^3 - 5y^2 - 4y + 3$ & LOOK FOR FACTORS

- $f(1) = 2 - 5 - 4 + 3 \neq 0$
- $f(-1) = -2 - 5 + 4 + 3 = 0 \quad \therefore (y+1) \text{ IS A FACTOR}$

BY LONG DIVISION OR ALGEBRAIC MANIPULATION WE OBTAIN

$$\Rightarrow 2y^2(y+1) - 7y(y+1) + 3(y+1) = 0$$

$$\Rightarrow (y+1)(2y^2 - 7y + 3) = 0$$

$$\Rightarrow (y+1)(2y-1)(y-3)$$

$$\Rightarrow y = \begin{cases} -1 \\ \frac{1}{2} \\ 3 \end{cases} \quad \text{OR} \quad e^x = \begin{cases} -1 \\ \frac{1}{2} \\ 3 \end{cases} \quad \text{X } (e^x > 0)$$

$$\Rightarrow x = \begin{cases} \ln \frac{1}{2} = -\ln 2 \\ \ln 3 \end{cases}$$

$\therefore \underline{x = -\ln 2 \text{ OR } x = 3}$