

IYGB - MPI PAPER K - QUESTION 1

REWRITE IN INDEX NOTATION

$$y = \frac{8}{x} + 3\sqrt{x}$$

$$y = 8x^{-1} + 3x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = -8x^{-2} + \frac{3}{2}x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = -\frac{8}{x^2} + \frac{3}{2\sqrt{x}}$$

SUBSTITUTING $x=4$

$$\left. \frac{dy}{dx} \right|_{x=4} = -\frac{8}{4^2} + \frac{3}{2\sqrt{4}}$$

$$= -\frac{8}{16} + \frac{3}{4}$$

$$= -\frac{1}{2} + \frac{3}{4}$$

$$= \frac{1}{4}$$

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1YGB - MPI PART K - QUESTION 2

a) COMPLETING THE SQUARE

$$f(x) = 4x^2 + 4x - 1$$

$$f(x) = 4\left[x^2 + x - \frac{1}{4}\right]$$

$$f(x) = 4\left[\left(x + \frac{1}{2}\right)^2 - \frac{1}{4} - \frac{1}{4}\right]$$

$$f(x) = 4\left(x + \frac{1}{2}\right)^2 - 1 - 1$$

$$f(x) = 4\left(x + \frac{1}{2}\right)^2 - 2$$

ALTERNATIVE

$$f(x) = 4x^2 + 4x - 1$$

$$f(x) = (4x^2 + 4x + 1) - 2$$

$$f(x) = (2x + 1)^2 - 2$$

b) SOLVING THE EQUATION USING PART (a)

$$f(x) = 0$$

$$4x^2 + 4x - 1 = 0$$

$$4\left(x + \frac{1}{2}\right)^2 - 2 = 0$$

$$4\left(x + \frac{1}{2}\right)^2 = 2$$

$$\left(x + \frac{1}{2}\right)^2 = \frac{1}{2}$$

$$x + \frac{1}{2} = \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}}$$

$$x + \frac{1}{2} = \pm\frac{\sqrt{2}}{2}$$

$$x = -\frac{1}{2} \pm \frac{\sqrt{2}}{2}$$

ALTERNATIVE

$$f(x) = 0$$

$$(2x + 1)^2 - 2 = 0$$

$$(2x + 1)^2 = 2$$

$$2x + 1 = \pm\sqrt{2}$$

$$2x = -1 \pm \sqrt{2}$$

$$x = -\frac{1}{2} \pm \frac{\sqrt{2}}{2}$$

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IYGB - MPI PAPER K - QUESTION 3

PROCESS AS FOLLOWS

$$f(x) = x^{\frac{3}{2}} - 8x^{-\frac{1}{2}}$$

$$f(3) = 3^{\frac{3}{2}} - 8 \times 3^{-\frac{1}{2}} = (\sqrt{3})^3 - \frac{8}{\sqrt{3}} = 3\sqrt{3} - \frac{8}{\sqrt{3}}$$

$$= 3\sqrt{3} - \frac{8\sqrt{3}}{\sqrt{3}\sqrt{3}} = 3\sqrt{3} - \frac{8\sqrt{3}}{3} = 3\sqrt{3} - \frac{8}{3}\sqrt{3}$$

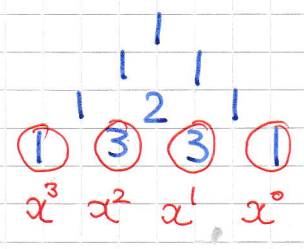
$$= \frac{1}{3}\sqrt{3} \quad \text{i.e. } k = \frac{1}{3}$$

1YGB - MPI PAPER K - QUESTION 4

a) EXPANDING USING PASCAL'S TRIANGLE OR CALCULATOR

$$(2x+4)^3 = 1(2x)^3(4)^0 + 3(2x)^2(4)^1 + 3(2x)^1(4)^2 + 1(2x)^0(4)^3$$

$$(2x+4)^3 = 8x^3 + 48x^2 + 96x + 64$$



b) USING PART (a)

$$\begin{aligned} (2x-1)(2x+4)^3 &= (2x-1)(8x^3 + 48x^2 + 96x + 64) \\ &= 16x^4 + 96x^3 + 192x^2 + 128x \\ &\quad - 8x^3 - 48x^2 - 96x - 64 \\ \hline &= 16x^4 + 88x^3 + 144x^2 + 32x - 64 \end{aligned}$$

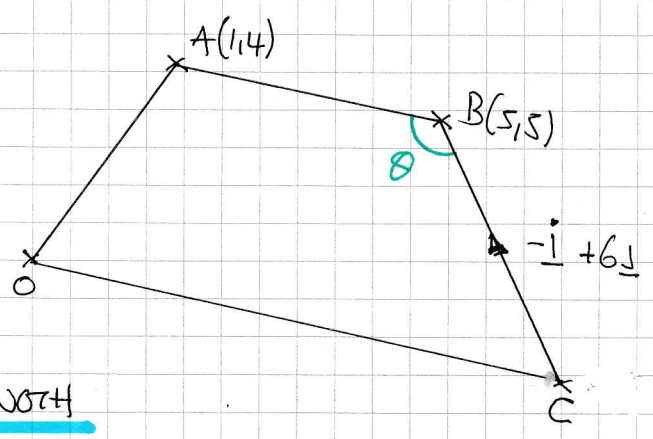
1YGB - MPI PAPER K - QUESTION 5

a) START WITH A DIAGRAM

$$\vec{OA} = \underline{1} + \underline{4j}$$

$$\vec{OB} = \underline{5} + \underline{5j}$$

$$\vec{CB} = \underline{-1} + \underline{6j}$$



FIND \vec{AB} , FOLLOWED BY ITS LENGTH

$$\vec{AB} = \vec{AO} + \vec{OB} = -(\underline{1} + \underline{4j}) + (\underline{5} + \underline{5j}) = \underline{4} + \underline{j}$$

$$|\vec{AB}| = |\underline{4} + \underline{j}| = \sqrt{4^2 + 1^2} = \underline{\underline{\sqrt{17}}}$$

b) NEXT FIND THE VECTOR \vec{AC}

$$\vec{AC} = \vec{AB} + \vec{BC} = \vec{AB} - \vec{CB} = (\underline{4} + \underline{j}) - (\underline{-1} + \underline{6j}) = \underline{5} - \underline{5j}$$

NEXT THE MODULI OF \vec{AC}

$$|\vec{AC}| = |\underline{5} - \underline{5j}| = \sqrt{5^2 + (-5)^2} = \sqrt{25 + 25} = \underline{\underline{\sqrt{50}}}$$

c) FINALLY THE LENGTH OF CB

$$|\vec{CB}| = |\underline{-1} + \underline{6j}| = \sqrt{(-1)^2 + 6^2} = \sqrt{1 + 36} = \underline{\underline{\sqrt{37}}}$$

BY THE COSINE RULE

$$\Rightarrow |\vec{AC}|^2 = |\vec{AB}|^2 + |\vec{CB}|^2 - 2|\vec{AB}||\vec{CB}|\cos\theta$$

$$\Rightarrow (\underline{\sqrt{50}})^2 = (\underline{\sqrt{17}})^2 + (\underline{\sqrt{37}})^2 - 2\underline{\sqrt{17}}\underline{\sqrt{37}}\cos\theta$$

$$\Rightarrow 50 = 17 + 37 - 2\underline{\sqrt{629}}\cos\theta$$

$$\Rightarrow 2\underline{\sqrt{629}}\cos\theta = 4$$

$$\Rightarrow \cos\theta = 0.079745\dots$$

$$\therefore \underline{\underline{\theta = 85.4^\circ}}$$

YGB-MPI PAPER K-QUESTION 6

a) LET $f(x) = x^3 + x^2 - 10x + 8$

looking for factors, trying $\pm 1, \pm 2, \pm 4, \pm 8$

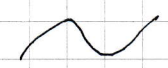
$f(1) = 1 + 1 - 10 + 8 = 0$

$\therefore (x-1)$ IS A FACTOR OF $f(x)$

BY LONG DIVISION OR MANIPULATIONS

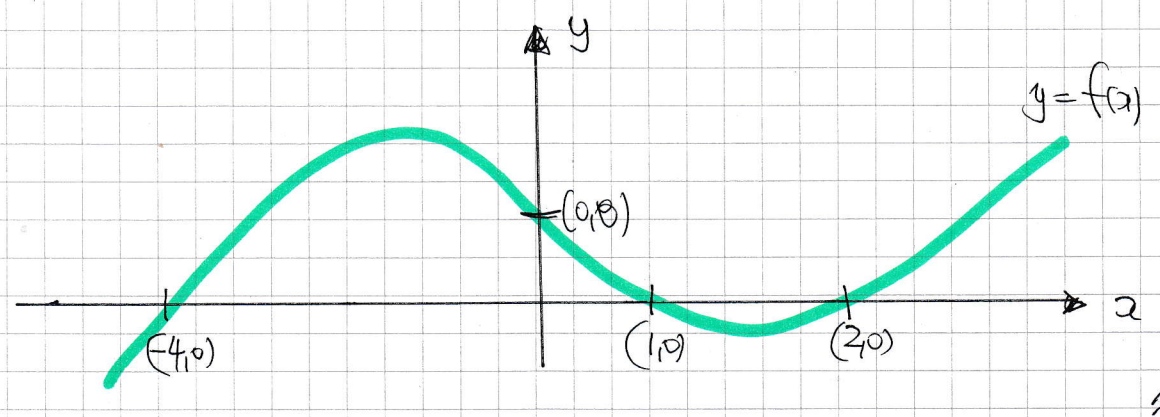
$$\begin{aligned} f(x) &= x^3 + x^2 - 10x + 8 = x^2(x-1) + 2x(x-1) - 8(x-1) \\ &= (x-1)(x^2 + 2x - 8) \\ &= \underline{(x-1)(x-2)(x+4)} \end{aligned}$$

b) COLLECTING ALL INFORMATION

$+x^3 \Rightarrow$ 

$x=0, y=8 \Rightarrow (0, 8)$

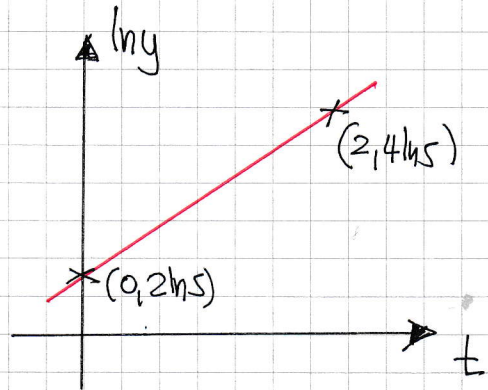
$y=0, x = \begin{matrix} 1 \\ 2 \\ -4 \end{matrix} \Rightarrow \begin{matrix} (1, 0) \\ (2, 0) \\ (-4, 0) \end{matrix}$



IYGB - MPI PAPER K - QUESTION 7

Proceed as follows

$$\begin{aligned} \text{Gradient} &= \frac{4\ln 5 - 2\ln 5}{2 - 0} \\ &= \frac{2\ln 5}{2} \\ &= \ln 5 \end{aligned}$$



THE EQUATION OF THE STRAIGHT LINE IS

$$\begin{aligned} \Rightarrow \ln y - 2\ln 5 &= (\ln 5)(t - 0) \\ \Rightarrow \ln y - 2\ln 5 &= t\ln 5 \\ \Rightarrow \ln y &= t\ln 5 + 2\ln 5 \\ \Rightarrow \ln y &= \ln 5^t + \ln 5^2 \\ \Rightarrow \ln y &= \ln(5^t \times 5^2) \\ \Rightarrow \ln y &= \ln(5^{t+2}) \\ \Rightarrow \underline{y} &= \underline{5^{t+2}} \end{aligned}$$

1YGB-MPI PAPER K - QUESTION 8

SOLVING THE FIRST INEQUALITY

$$\Rightarrow (x+2)(x+4) > 10x+7$$

$$\Rightarrow x^2+6x+8 > 10x+7$$

$$\Rightarrow x^2-4x+1 > 0$$

THIS DOES NOT FACTORIZE NICELY - OBTAIN CRITICAL VALUES BY COMPLETING THE SQUARE IN THE CORRESPONDING EQUATION - OR USE QUADRATIC FORMULA

$$\Rightarrow "x^2-4x+1=0"$$

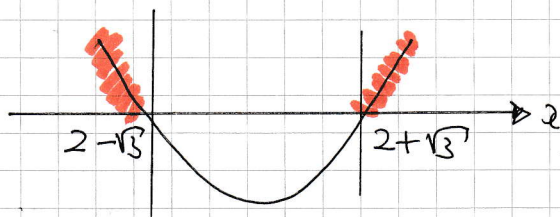
$$\Rightarrow (x-2)^2-4+1=0$$

$$\Rightarrow (x-2)^2=3$$

$$\Rightarrow x-2 = \pm\sqrt{3}$$

$$\Rightarrow x = 2 \pm \sqrt{3}$$

LOOKING AT THE DIAGRAM



$$x < 2 - \sqrt{3} \text{ or } x > 2 + \sqrt{3}$$

SOLVING THE SECOND INEQUALITY

$$\Rightarrow x\sqrt{3} < 2 + \frac{2(2x-1)}{\sqrt{3}}$$

$$\Rightarrow 3x < 2\sqrt{3} + 2(2x-1)$$

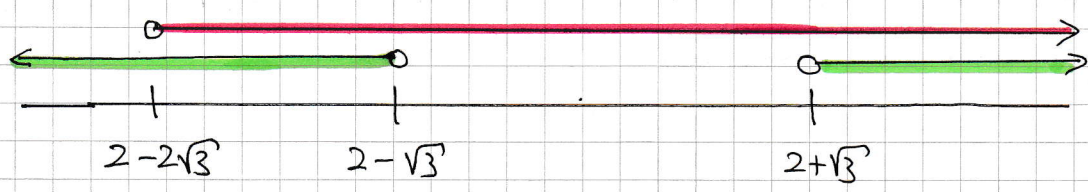
$$\Rightarrow 3x < 2\sqrt{3} + 4x - 2$$

-2-

1YGB - MPI PAPER K - QUESTION 8

$$\begin{aligned} \Rightarrow -x &< -2 + 2\sqrt{3} \\ \Rightarrow \underline{x} &> \underline{2 - 2\sqrt{3}} \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{MULTIPLIED BY NEGATIVE}$$

FINALLY TO OBTAIN THE COMMON SOLUTION INTERVAL



$\therefore \underline{2-2\sqrt{3} < x < 2-\sqrt{3} \quad \text{OR} \quad x > 2+\sqrt{3}}$

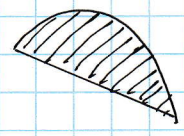
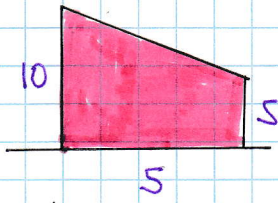
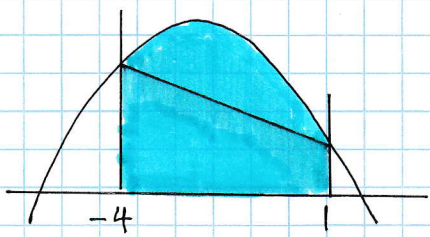
1YGB - MPI PAPER K - QUESTION 9

OBTAIN THE COORDINATES OF A & B

$$\left. \begin{aligned} y &= 10 - 4x - x^2 \\ y &= 6 - x \end{aligned} \right\} \Rightarrow \begin{aligned} 6 - x &= 10 - 4x - x^2 \\ \Rightarrow x^2 + 3x - 4 &= 0 \\ \Rightarrow (x + 4)(x - 1) &= 0 \\ \Rightarrow x &= \begin{cases} -4 \\ 1 \end{cases} & y = \begin{cases} 10 \\ 5 \end{cases} \end{aligned}$$

$\therefore A(-4, 10) \quad B(1, 5)$

LOOKING AT THE PICTORIAL EQUATION BELOW



$$\begin{aligned} & \int_{-4}^1 (10 - 4x - x^2) dx \\ &= \left[10x - 2x^2 - \frac{1}{3}x^3 \right]_{-4}^1 \\ &= \left(10 - 2 - \frac{1}{3} \right) - \left(-40 - 32 + \frac{64}{3} \right) \\ &= \frac{23}{3} - \left(-\frac{152}{3} \right) \\ &= \frac{175}{3} \end{aligned}$$

$$\frac{1}{2}(10 + 5) \times 5 = \frac{75}{2}$$

REQUIRED AREA = $\frac{175}{3} - \frac{75}{2} = \frac{125}{6}$

1YGB - MPI PAPER K - QUESTION 10

SOLVING SIMULTANEOUSLY

$$y = 9^x \quad \& \quad y = 6 \times 5^x$$

↙ ↘

$$9^x = 6 \times 5^x$$

TAKING LOGARITHMS BASE 3

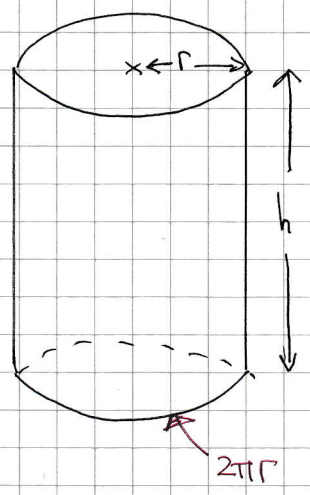
$$\Rightarrow \log_3 9^x = \log_3 (6 \times 5^x)$$
$$\Rightarrow x \log_3 9 = \log_3 6 + \log_3 5^x$$
$$\Rightarrow x \log_3 9 = \log_3 6 + x \log_3 5$$
$$\Rightarrow x \log_3 9 - x \log_3 5 = \log_3 6$$
$$\Rightarrow x [\log_3 9 - \log_3 5] = \log_3 6$$
$$\Rightarrow x = \frac{\log_3 6}{\log_3 9 - \log_3 5}$$

TIDY FURTHER AS FOLLOWS

$$\Rightarrow x = \frac{\log_3 3 + \log_3 2}{\log_3 3^2 - \log_3 5}$$
$$\Rightarrow x = \frac{1 + \log_3 2}{2 \log_3 3 - \log_3 5}$$
$$\Rightarrow x = \frac{1 + \log_3 2}{2 - \log_3 5}$$

IYGB - MPI PAPER K - QUESTION 11

a)



CONSTRAINT SURFACE AREA = 192π

⇒ $h \rightarrow \boxed{} + \bigcirc = 192\pi$
↑
2πr

⇒ ~~2πrh~~ + ~~2 × πr²~~ = ~~192π~~

⇒ $rh + r^2 = 96$

VOLUME = πr² × h

⇒ $V = \pi r(rh)$

⇒ $V = \pi r(96 - r^2)$

⇒ $V = 96\pi r - \pi r^3$

← $rh = 96 - r^2$

AS REQUIRED

b)

DIFFERENTIATE & SOLVE FOR ZERO

⇒ $V = 96\pi r - \pi r^3$

⇒ $\frac{dV}{dr} = 96\pi - 3\pi r^2$

⇒ $0 = 96\pi - 3\pi r^2$

⇒ ~~3πr²~~ = ~~96π~~

⇒ $r^2 = 32$

⇒ $r = +\sqrt{32} \approx 5.66 \text{ cm}$

LYGB - MPI PAPER K - QUESTION 11

c) CHECKING WITH THE 2ND DERIVATIVE

$$\Rightarrow \frac{dV}{dr} = 96\pi - 3\pi r^2$$

$$\Rightarrow \frac{d^2V}{dr^2} = -6\pi r$$

$$\Rightarrow \left. \frac{d^2V}{dr^2} \right|_{r=5.68} = -106.62... < 0$$

INDEED IT WILL GIVE
THE MAXIMUM VALUE FOR V

d) $V = 96\pi r - \pi r^3$

$$\Rightarrow V_{\text{MAX}} = 96\pi(4\sqrt{2}) - \pi(4\sqrt{2})^3$$

$$\Rightarrow V_{\text{MAX}} = 4\pi\sqrt{2} [96 - (4\sqrt{2})^2]$$

$$\Rightarrow V_{\text{MAX}} = 4\pi\sqrt{2} \times 64$$

$$\Rightarrow V_{\text{MAX}} = 256\pi\sqrt{2}$$

$$\Rightarrow \underline{V_{\text{MAX}} \approx 1137}$$

1YGB - MPI PAPER K - QUESTION 12

a) PROCEED AS FOLLOWS

- GRADIENT AB = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 1}{6 - 0} = \frac{4}{6} = \frac{2}{3}$
- OUR LINE (PERPENDICULAR BISECTOR) = $-\frac{3}{2}$
- FINDING THE MIDPOINT OF AB : $(\frac{0+6}{2}, \frac{1+5}{2}) = (3, 3)$
- REQUIRED LINE HAS EQUATION

$$y - y_0 = m(x - x_0)$$

$$y - 3 = -\frac{3}{2}(x - 3)$$

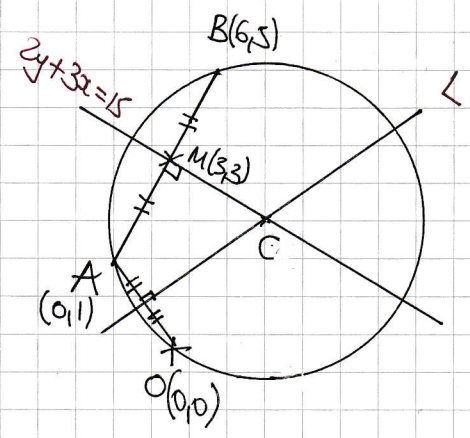
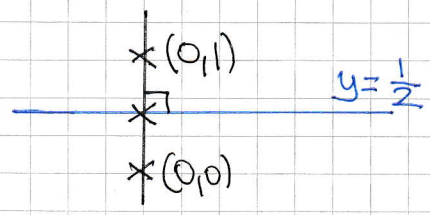
$$2y - 6 = -3x + 9$$

$$2y + 3x = 15$$

b) LOOKING AT A DIAGRAM - NOT DRAWN TO SCALE

GRADIENT OA = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 0}{0 - 0} = \frac{1}{0}$

i.e INFINITE GRADIENT
i.e LINE IS VERTICAL



∴ PERPENDICULAR BISECTOR HAS EQUATION $y = \frac{1}{2}$

USING $2y + 3x = 15$ WITH $y = \frac{1}{2}$

$$2 \times \frac{1}{2} + 3x = 15$$

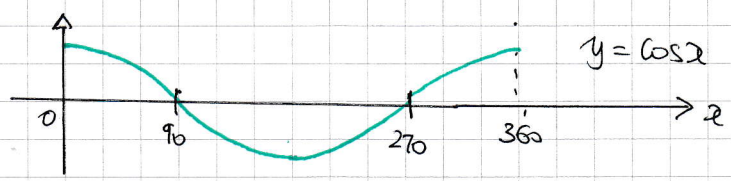
$$3x = 14$$

$$x = \frac{14}{3}$$

∴ C $(\frac{14}{3}, \frac{1}{2})$

1YGB - MPI PAPER K - QUESTION 13

a) LOOKING AT A SKETCH OF THE GRAPH OF $y = \cos x$



TRANSLATING BY 60° TO THE "LEFT"

$\therefore (270, 0) \mapsto \underline{B(210, 0)}$

$(360, 1) \mapsto \underline{C(300, 1)}$

& SUBSTITUTING $x=0$ INTO $y = \cos(x+60)$ GIVES $y = \frac{1}{2}$

$\therefore \underline{A(0, \frac{1}{2})}$

b) SOLVING THE EQUATION $y = -\frac{1}{2}$

$\Rightarrow \cos(x+60) = -\frac{1}{2}$

$\arccos(-\frac{1}{2}) = 120^\circ$

$\Rightarrow \begin{cases} x+60^\circ = 120^\circ \pm 360n \\ x+60^\circ = 240^\circ \pm 360n \end{cases} \quad n=0,1,2,3,\dots$

$\Rightarrow \begin{cases} x = 60^\circ \pm 360n \\ x = 180^\circ \pm 360n \end{cases}$

$\therefore x_1 = 60^\circ$

$x_2 = 180^\circ$

$\in P(60, -\frac{1}{2})$ & $Q(180, -\frac{1}{2})$