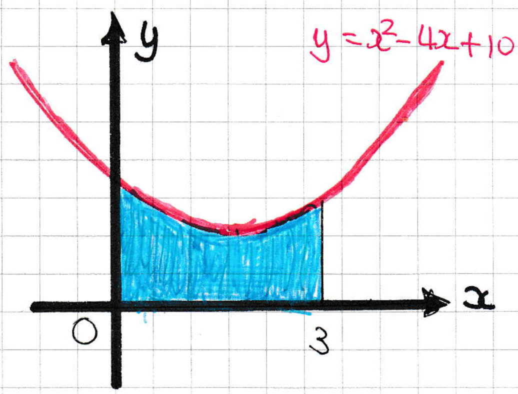
IYGB - MP1 PAPER C - QUESTION 1



$$\text{AREA} = \int_0^3 x^2 - 4x + 10 \, dx$$

$$= \left[\frac{1}{3}x^3 - 2x^2 + 10x\right]_0^3$$

$$= \left(\frac{1}{3}\times 3^3 - 2\times 3^2 + 10\times 3\right) - \cancel{\left(\frac{1}{3}\times 0^3 - 2\times 0^2 + 10\times 0\right)}$$

$$= 9 - 18 + 30$$

$$= 21$$

IYGB - MP1 PAPER C - QUESTION 2

a) COLLECTING ALL THE RELEVANT INFORMATION FIRST

- $+2x^2+\dots \Longrightarrow \cup$
- $x=0 \quad y=4 \Longrightarrow (0,4)$
- $y=0$

$2x^2-9x+4=0$

$(2x-1)(x-4)=0$

$x = \frac{1}{2}$ or 4 $\Longrightarrow$ $(\frac{1}{2},0)$, $(4,0)$

- $y = 2x^2-9x+4$

$\frac{1}{2}y = x^2 - \frac{9}{2}x + 2$

$\frac{1}{2}y = (x-\frac{9}{4})^2 - \frac{81}{16} + 2$

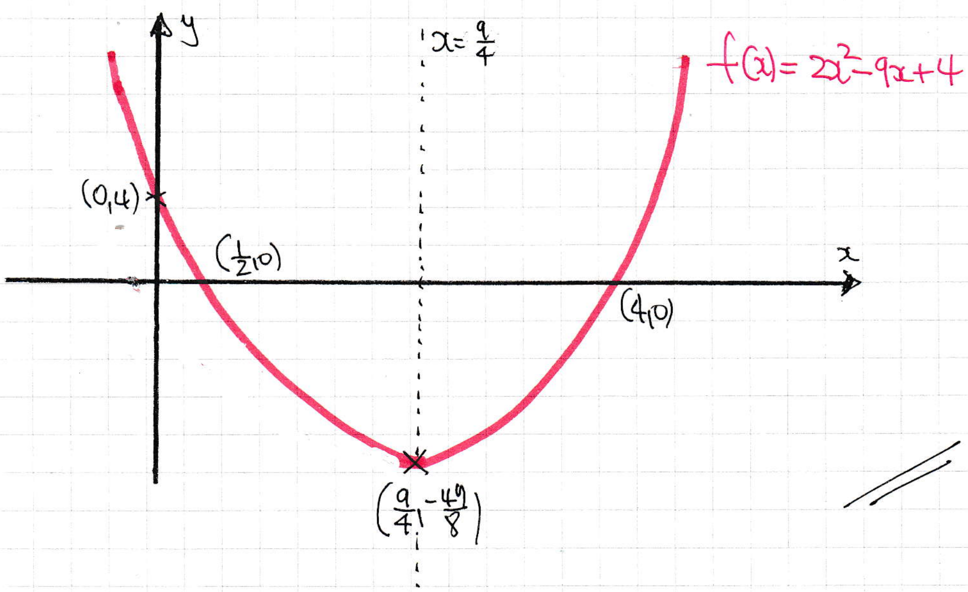
$y = 2(x-\frac{9}{4})^2 - \frac{81}{8} + 4$

$y = 2(x-\frac{9}{4})^2 - \frac{81}{8} + \frac{32}{8}$

$y = 2(x-\frac{9}{4})^2 - \frac{49}{8}$

$\therefore$ minimum $(\frac{9}{4}, -\frac{49}{8})$

PRODUCING THE SKETCH

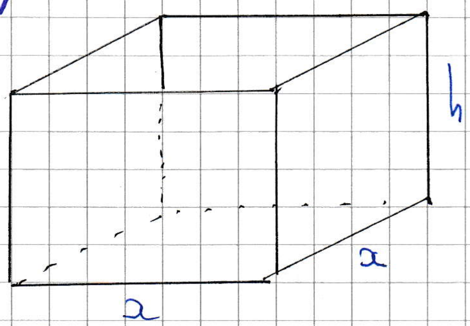


b) LOOKING AT THE GRAPH ABOVE

$f(x) > 0 \Longrightarrow x < \frac{1}{2}$ OR $x > 4$

IYGB - MP1 PAPER C - QUESTION 3

a)



CONSTRAINT ON VOLUME

$V = 500$

$x^2h = 500$

$x(xh) = 500$

$xh = \frac{500}{x}$

$A = x^2 + 4xh$

$A = x^2 + 4\left(\frac{500}{x}\right)$

$A = x^2 + \frac{2000}{x}$ // AS REQUIRED

b)

$A = x^2 + 2000x^{-1}$

$\Rightarrow \frac{dA}{dx} = 2x - 2000x^{-2}$

STATIONARY $\Rightarrow \frac{dA}{dx} = 0$

$\Rightarrow 2x - 2000x^{-2} = 0$

$\Rightarrow 2x - \frac{2000}{x^2} = 0$

$\Rightarrow 2x = \frac{2000}{x}$

$\Rightarrow 2x^3 = 2000$

$\Rightarrow x^3 = 1000$

$\Rightarrow x = 10 \text{ m}$

c)

- $A = x^2 + \frac{2000}{x}$

  $A_{MIN} = 10^2 + \frac{2000}{10}$

  $A_{MIN} = 300 \text{ m}^2$

- $\frac{dA}{dx} = 2x - 2000x^{-2}$

  $\frac{d^2A}{dx^2} = 2 + 4000x^{-3}$

  $\left.\frac{d^2A}{dx^2}\right|_{x=10} = 6 > 0$

INDEED A MINIMUM

IYGB – MP1 PAPER C – QUESTION 4

$$\tan(3x-75)^{\circ} = \tan 450^{\circ}, \quad 300^{\circ} \leq x < 500$$

SETTING UP A GENERAL SOLUTION IN DEGREES

$$\Longrightarrow \quad 3x - 75^{\circ} = 450^{\circ} \pm 180n \qquad n = 0,1,2,3,\ldots$$
$$\Longrightarrow \quad 3x = 525^{\circ} \pm 180n$$
$$\Longrightarrow \quad x = 175^{\circ} \pm 60n$$

COLLECTING THE SOLUTIONS IN THE REQUIRED INTERVAL

$$x = \ldots \cancel{175^{\circ}}, \cancel{235^{\circ}}, \cancel{295^{\circ}}, 355^{\circ}, 415^{\circ}, 475^{\circ}, \cancel{535^{\circ}}, \ldots$$

$$x = 355^{\circ}, 415^{\circ}, 475^{\circ}$$

IYGB - MP1 PAPER C - QUESTION 5

LET THE EQUATION OF THE CIRCLE IN EXPANDED FORM BE

$$x^2+y^2+Ax+By=C$$

- $(-1,0) \Rightarrow (-1)^2+0^2+A(-1)+B(0)=C$

  $\Rightarrow 1-A=C$

- $(7,0) \Rightarrow 7^2+0^2+7A+B(0)=C$

  $\Rightarrow 49+7A=C$

SOLVING SIMULTANEOUSLY

$$\left.\begin{array}{l} C=1-A \\ C=49+7A \end{array}\right\} \Rightarrow 49+7A=1-A$$

$\Rightarrow 8A=-48$

$\Rightarrow A=-6 \quad \& \quad C=7$

SUBSTITUTE THESE VALUES IN AND TRY THE POINT (3,8)

$\Rightarrow x^2+y^2-6x+By=7$

$\Rightarrow 3^2+8^2-6(3)+B(8)=7$

$\Rightarrow 9+64-18+8B=7$

$\Rightarrow 8B=-48$

$\Rightarrow B=-6$

FINALLY WE HAVE THE EQUATION

$\Rightarrow x^2+y^2-6x-6y=7$

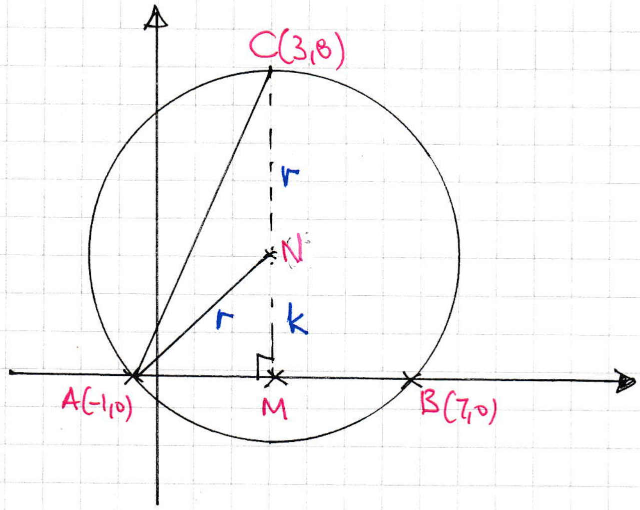
$\Rightarrow (x-3)^2-9+(y-3)^2-9=7$

$\Rightarrow (x-3)^2+(y-3)^2=25$

$\therefore$ CENTRE AT (3,3), RADIUS 5

IYGB - MP1 PAPER C - QUESTION 5

ALTERNATIVE USING CIRCLE GEOMETRY



- $M(3,0)$ MIDPOINT OF AB (CIRCLE THEOREM)
- $N(3,k)$ THE CENTRE OF THE CIRCLE HAS x CO-ORDINATE 3 (CIRCLE THEOREM)
- DISTANCE OF AC IS $\sqrt{(8-0)^2+(3+1)^2}$
  $\sqrt{64+16} = \sqrt{80}$

LOOKING AT $A\hat{M}N$

$$|AM|^2 + |MN|^2 = |AN|^2$$
$$4^2 + k^2 = r^2$$
$$16 + k^2 = r^2$$

LOOKING AT $A\hat{M}C$

$$|AM|^2 + |MC|^2 = |AC|^2$$
$$4^2 + (r+k)^2 = (\sqrt{80})^2$$
$$16 + (r+k)^2 = 80$$
$$(r+k)^2 = 64$$
$$r+k = +8$$

COMBINING EQUATIONS

$$\left.\begin{array}{l} 16+k^2=r^2 \\ r+k=8 \end{array}\right\} \Rightarrow r = 8-k$$

$$\Rightarrow 16+k^2 = (8-k)^2$$
$$\Rightarrow 16+\cancel{k^2} = 64-16k+\cancel{k^2}$$
$$\Rightarrow 16k = 48$$
$$\Rightarrow k=3$$
$$\Rightarrow r=5$$

$$\therefore N(3,3) \quad \& \quad r=5$$

IYGB - MP1 PAPER C - QUESTION 6

a) MANIPULATING THE EXPONENTIAL AS FOLLOWS

$$2^{3x+4} = 2^{3\left(x+\frac{4}{3}\right)} \quad \text{ie } "f\left(x+\tfrac{4}{3}\right)"$$

$\therefore$ TRANSLATION BY THE VECTOR $\begin{pmatrix} -\frac{4}{3} \\ 0 \end{pmatrix}$

b) CREATING AN ENLARGEMENT AS FOLLOWS

$$2^{3x+4} = 2^{3x} \times 2^{4} = 16 \times 2^{3x} \quad \text{ie } "16f(x)"$$

$\therefore$ STRETCH PARALLEL TO THE y AXIS BY SCALE FACTOR 16

IYGB - MP1 PAPER C - QUESTION 7

IF THE POSITIVE INTEGER n, IS NOT DIVISIBLE BY 3, THEN IT MUST BE OF ONE OF THE FOLLOWING FORMS

- $n = 3k+1, \ k \in \mathbb{N}$
- $n^2 - 1 = (3k+1)^2 - 1$
  $= 9k^2 + 6k + 1 - 1$
  $= 9k^2 + 6k$
  $= 3(3k^2 + 2k)$

  I.E. DIVISIBLE BY 3

- $n = 3k+2, \ k \in \mathbb{N}$
- $n^2 - 1 = (3k+2)^2 - 1$
  $= 9k^2 + 12k + 4 - 1$
  $= 9k^2 + 12k + 3$
  $= 3(3k^2 + 4k + 1)$

  I.E. DIVISIBLE BY 3

HENCE, BY EXHAUSTION, THE RESULT HOLDS

IYGB – MP1 PAPER C – QUESTION 8

a) WHEN $n=0$ (START), $V=40$

$\therefore$ £40,000

b) USING LOGARITHMS OF ANY BASE INCLUDING NATURAL

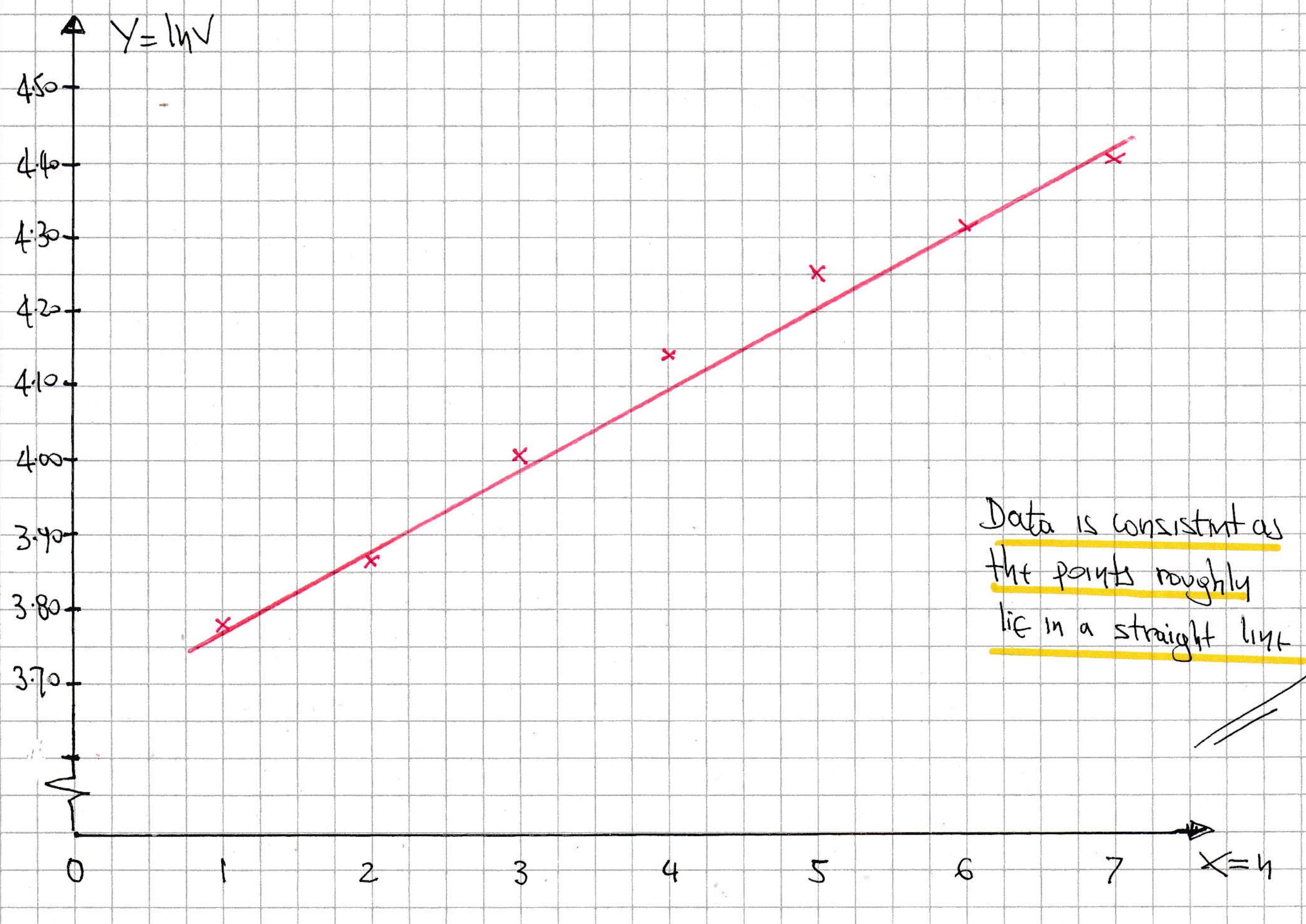
$$V = 40\left(1+\frac{r}{100}\right)^n \implies \ln V = \ln\left[40\left(1+\frac{r}{100}\right)^n\right]$$

$$\implies \ln V = \ln 40 + \ln\left(1+\frac{r}{100}\right)^n$$

$$\implies \ln V = \ln 40 + n\ln\left(1+\frac{r}{100}\right)$$

$$\implies \underbrace{\ln V}_{Y} = \underbrace{n}_{X}\underbrace{\ln\left(1+\frac{r}{100}\right)}_{\text{GRADIENT}} + \underbrace{\ln 40}_{\text{"Y INTERCEPT"}}$$

$X = n$	1	2	3	4	5	6	7
$Y = \ln V$	3.78	3.87	4.01	4.14	4.20	4.32	4.41



Data is consistent as the points roughly lie in a straight line

IYGB — MP1 PAPER C — QUESTION 8

c) THE ANNUAL % GROWTH IS PART OF THE GRADIENT

USING TWO POINTS ON THE LINE (2, 3.88) & (7, 4.42)

$$\Rightarrow \quad \text{GRADIENT} = \frac{4.42 - 3.88}{7-2}$$

$$\ln\left(1 + \frac{r}{100}\right) = 0.108$$

$$1 + \frac{r}{100} = e^{0.108}$$

$$1 + \frac{r}{100} = 1.11404\ldots$$

$$100 + r = 111.404\ldots$$

$$r = 11.1404$$

$$r \simeq 11$$

I.E 11%

d) USING THE FORMULA NOW

$$V = 40\left(1 + \frac{11}{100}\right)^n$$

$$V = 40 \times 1.11^{10}$$

$$V = 113.576\ldots$$

$$\therefore V \simeq £113.576\ldots$$

$$V \simeq £114{,}000$$

NOT RELIABLE AS THERE IS NO EVIDENCE THAT THIS TREND WILL CONTINUE

IYGB - MP1 PAPER C - QUESTION 9

- IF $f(x) = \frac{1}{x^3}$ THEN $f(x+h) = \frac{1}{(x+h)^3}$

- $f(x+h) - f(x) = \frac{1}{(x+h)^3} - \frac{1}{x^3} = \frac{x^3 - (x+h)^3}{x^3(x+h)^3}$

$$= \frac{\cancel{x^3} - (\cancel{x^3} + 3x^2h + 3xh^2 + h^3)}{x^3(x+h)^3}$$

$$= -\frac{3x^2h + 3xh^2 + h^3}{x^3(x+h)^3}$$

FROM THE FORMAL DEFINITION OF THE DERIVATIVE AS A LIMIT

$$f'(x) = \lim_{h \to 0}\left[\frac{f(x+h) - f(x)}{h}\right] = \lim_{h \to 0}\left[\left[f(x+h) - f(x)\right] \div h\right]$$

$$= \lim_{h \to 0}\left[-\frac{3x^2h + 3xh^2 + h^3}{x^3(x+h)^3} \times \frac{1}{h}\right]$$

$$= \lim_{h \to 0}\left[-\frac{(3x^2 + 3xh + h^2)\cancel{h}}{x^3(x+h)^3} \times \frac{1}{\cancel{h}}\right]$$

$$= \lim_{h \to 0}\left[-\frac{3x^2 + 3xh + h^2}{x^3(x+h)^3}\right]$$

TAKING LIMITS YIELDS

$$= -\frac{3x^2}{x^3 \times x^3} = -\frac{3x^2}{x^6} = -\frac{3}{x^4}$$

IYGB – MP1 PAPER C – QUESTION 10

a) $A(-4,-7) \quad B(4,9)$

$\Rightarrow$ GRAD AB $= \dfrac{9-(-7)}{4-(-4)} = \dfrac{16}{8} = 2$

$\Rightarrow y - y_0 = m(x - x_0)$

$\Rightarrow y - 9 = 2(x-4)$

$\Rightarrow y - 9 = 2x - 8$

$\Rightarrow y = 2x + 1$

b) SOLVING SIMULTANEOUSLY

$$\left.\begin{array}{l} y = 2x+1 \\ y = \frac{1}{2}x + 4 \end{array}\right\} \Rightarrow 2x+1 = \frac{1}{2}x + 4$$

$\Rightarrow 4x + 2 = x + 8$

$\Rightarrow 3x = 6$

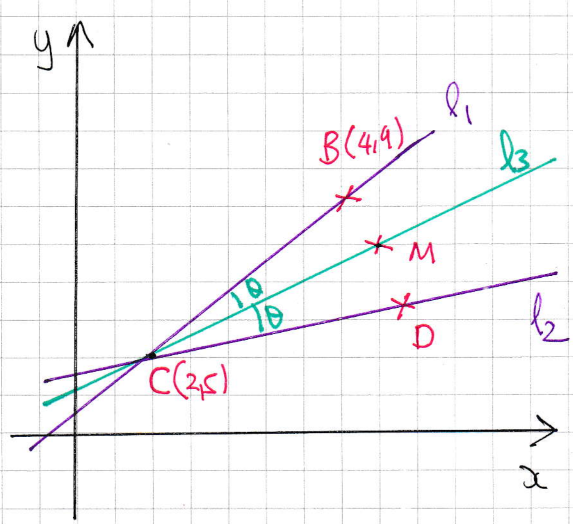
$\Rightarrow x = 2$

$\Rightarrow y = 5$

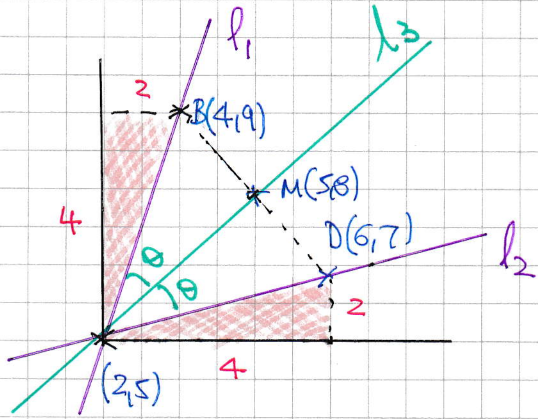
$\therefore C(2,5)$

c) DRAW A DIAGRAM

- FIND A POINT D ON l_2 SO THAT $|BC| = |DC|$
- FIND THE MIDPOINT OF BD, CALL IT M
- FIND THE GRADIENT MC
- DETERMINE THE EQUATION OF THE ANGLE BISECTOR l_3



IYGB - MP1 PAPER C - QUESTION 10



BY INSPECTION

$D(6,7)$

$M(5,8)$

GRAD MC $= \frac{8-5}{5-2} = 1$

$\therefore$ l_3 HAS GRADIENT 1

$\therefore$ $y-5 = 1(x-2)$

$y = x+3$

IYGB – MP1 PAPER C – QUESTION 11

a) USING THE STANDARD FORMULA FOR EXPANDING $(1+2x)^n$

$$\Longrightarrow (1+ax)^n = 1+\frac{n}{1}(ax)^1+\frac{n(n-1)}{1\times 2}(ax)^2+\frac{n(n-1)(n-2)}{1\times 2\times 3}(ax)^3+\dots$$

$$\Longrightarrow (1+2x)^7 = 1+\frac{7}{1}(2x)^1+\frac{7\times 6}{1\times 2}(2x)^2+\frac{7\times 6\times 5}{1\times 2\times 3}(2x)^3+\dots$$

$$\Longrightarrow (1+2x)^7 = 1+14x+21(4x^2)+35(8x^3)+\dots$$

$$\Longrightarrow (1+2x)^7 = 1+14x+84x^2+280x^3+\dots$$

b) LOOKING THE EXPRESSION GIVEN, PROCEED AS FOLLOWS

$$\begin{aligned}(3+4x-4x^2)(1+2x)^6 &= -(4x^2-4x-3)(1+2x)^6\\ &= -(2x+1)(2x-3)(1+2x)^6\\ &= -(2x-3)(2x+1)^7\\ &= (3-2x)(1+2x)^7\end{aligned}$$

USING PART (a)

$$\begin{aligned}(3+4x-4x^2)(1+2x)^6 &= (3-2x)(1+14x+84x^2+280x^3+\dots)\\ &= 3+42x+252x^2+840x^3+\dots\\ &\qquad -2x-28x^2-168x^3+\dots\\ \hline &= 3+40x+224x^2+672x^3+\dots\end{aligned}$$

ALTERNATIVE TO PART (b)

$$(1+2x)^6 = 1+\frac{6}{1}(2x)^1+\frac{6\times 5}{1\times 2}(2x)^2+\frac{6\times 5\times 4}{1\times 2\times 3}(2x)^3+\dots$$

$$(1+2x)^6 = 1+12x+15(4x^2)+20(8x^3)+\dots$$

IYGB - MP1 PAPER C - QUESTION 11

$$\Rightarrow (1+2x)^6 = 1 + 12x + 60x^2 + 160x^3 + \ldots$$

NOW MULTIPLY THE EXPANSION BY THE GIVEN QUADRATIC

$$(3+4x-4x^2)(1+2x)^6 = (3+4x-4x^2)(1+12x+60x^2+160x^3+\ldots)$$

$$\begin{aligned} = 3 + 36x &+ 180x^2 + 480x^3 + \ldots \\ 4x &+ 48x^2 + 240x^3 + \ldots \\ &- 4x^2 - 48x^3 - \ldots \end{aligned}$$

$$= 3 + 40x + 224x^2 + 672x^3 + \ldots$$

AS BEFORE

IYGB – MP1 PAPER C – QUESTION 12

$$P = \frac{125ka^t}{k+2a^t}, \quad t \geqslant 0$$

P = POPULATION (NUMBER)
t = TIME (IN YEARS)

t = 0 , P = 100
t = 5 , P = 200

a) USING $t=0$, $P=100$ IN THE ABOVE FORMULA

$$\Rightarrow 100 = \frac{125k \times a^0}{k + 2 \times a^0}$$

$$\Rightarrow 100 = \frac{125k}{k+2}$$

$$\Rightarrow 100(k+2) = 125k$$

$$\Rightarrow 100k + 200 = 125k$$

$$\Rightarrow 200 = 25k$$

$$\Rightarrow k = 8$$

b) USING $t=5$, $P=200$ IN THE REVISED FORMULA

$$\Rightarrow P = \frac{125 \times 8 \times a^t}{8 + 2 \times a^t}$$

$$\Rightarrow 200 = \frac{125 \times 8 \times a^5}{8 + 2 \times a^5}$$

$$\Rightarrow 200 = \frac{1000a^5}{8 + 2a^5} \quad \div 200$$

$$\Rightarrow 1 = \frac{5a^5}{8 + 2a^5}$$

$$\Rightarrow 8 + 2a^5 = 5a^5$$

$$\Rightarrow 8 = 3a^5$$

$$\Rightarrow a^5 = \frac{8}{3}$$

$$\Rightarrow a = \sqrt[5]{8/3} = 1.216728684\ldots \qquad \therefore a \approx 1.217$$

IYGB - MP1 PAPER C - QUESTION 12

c) STARTING AGAIN WITH A YET REVISED VERSION OF THE FORMULA

$$P = \frac{125 \times 8 \times a^t}{8 + 2a^t} \implies P = \frac{\overset{500}{\cancel{1000}}a^t}{{}_4\cancel{8+2a^t}} \qquad [a = 1.216720684...]$$

$$\implies P = \frac{500a^t}{4 + a^t}$$

$$\implies 400 = \frac{500a^t}{4 + a^t}$$

$$\div 100$$

$$\implies 4 = \frac{5a^t}{4 + a^t}$$

$$\implies 16 + 4a^t = 5a^t$$

$$\implies 16 = a^t$$

$$\implies \log 16 = \log a^t$$

$$\implies \log 16 = t \log a$$

$$\implies t = \frac{\log 16}{\log a} = \frac{\log 16}{\log(1.2167...)} = 14.13390...$$

$$\therefore \underline{t \approx 14.13}$$

d) LOOKING AT THE FORMULA $P = \dfrac{1000a^t}{8 + 2a^t}$ WHICH CAN BE REDUCED

TO $P = \dfrac{500a^t}{4 + a^t}$

DIVIDE TOP & BOTTOM OF THE FRACTION BY a^t TO GIVE

$$P = \frac{\frac{500a^t}{a^t}}{\frac{4}{a^t} + \frac{a^t}{a^t}} \implies P = \frac{500}{\frac{4}{a^t} + 1}$$

AS t GETS VERY LARGE a^t ALSO GETS VERY LARGE (AS $a > 1$)

SO $\frac{4}{a^t}$ BECOMES PRACTICALLY ZERO, WHICH MEANS THE FORMULA GIVES $P = \frac{500}{1}$

AS THE POPULATION STARTS FROM 100 THE LIMITING VALUE IS 500 & CANNOT EXCEED IT