IYGB - SYNOPTIC PAPER Q - QUESTION 1

a) LOOKING AT $\sqrt{28}$

$$\sqrt{28} = \sqrt{4\times7} = \sqrt{4}\sqrt{7} = 2\sqrt{7}$$

IF THIS IS TO SIMPLIFY IT MUST CONTAIN $\sqrt{7}$, SO 7 MUST PERHAPS DIVIDE 343 INTO A SQUARE NUMBER

$$\begin{array}{r} 25 \\ 7 \\ \hline 175 \end{array} \qquad \begin{array}{r} 36 \\ 7 \\ \hline 252 \end{array} \qquad \begin{array}{r} 49 \\ 7 \\ \hline 343 \end{array}$$

$$\therefore \sqrt{343} = \sqrt{49}\sqrt{7} = 7\sqrt{7}$$

$$\therefore \sqrt{343} - \sqrt{28} = 7\sqrt{7} - 2\sqrt{7} = 5\sqrt{7}$$

b) REDUCE AND RATIONALIZE

$$\begin{aligned}\sqrt{45} + \frac{20}{\sqrt{5}} &= \sqrt{9}\sqrt{5} + \frac{20}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}} \\ &= 3\sqrt{5} + \frac{20\sqrt{5}}{5} \\ &= 3\sqrt{5} + 4\sqrt{5} \\ &= 7\sqrt{5}\end{aligned}$$

IYGB - SYNOPTIC PAPER Q - PAPER 2

a) THE EQUATION IS GIVEN BY

$$(x-5)^2+(y-4)^2=(3\sqrt{2})^2$$
$$(x-5)^2+(y-4)^2=18$$

b) SOLVING THE EQUATIONS SIMULTANEOUSLY, BY SUBSTITUTION

$\Rightarrow (x-5)^2+(y-4)^2=18$

$\Rightarrow (x-5)^2+((x+1)-4)^2=18$

$\Rightarrow (x-5)^2+(x-3)^2=18$

$\Rightarrow x^2-10x+25+x^2-6x+9=18$

$\Rightarrow 2x^2-16x+34=18$

$\Rightarrow 2x^2-16x+16=0$

$\Rightarrow x^2-8x+8=0$

SOLVING BY COMPLETING THE SQUARE OR QUADRATIC FORMULA

$\Rightarrow (x-4)^2-4^2+8=0$

$\Rightarrow (x-4)^2-16+8=0$

$\Rightarrow (x-4)^2=8$

$\Rightarrow x-4 = \begin{cases}\sqrt{8}\\ -\sqrt{8}\end{cases}$

$\Rightarrow x = \begin{cases}4+2\sqrt{2}\\ 4-2\sqrt{2}\end{cases} \qquad y = \begin{cases}5+2\sqrt{2}\\ 5-2\sqrt{2}\end{cases}$

$\therefore (4+2\sqrt{2},\ 5+2\sqrt{2})$ & $(4-2\sqrt{2},\ 5-2\sqrt{2})$

IYGB - SYNOPTIC PAPER Q - QUESTION 2

c) USING THE DISTANCE FORMULA

$$d = \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2}$$

$$d = \sqrt{[(5+2\sqrt{2})-(5-2\sqrt{2})]^2 + [(4+2\sqrt{2})-(4-2\sqrt{2})]^2}$$

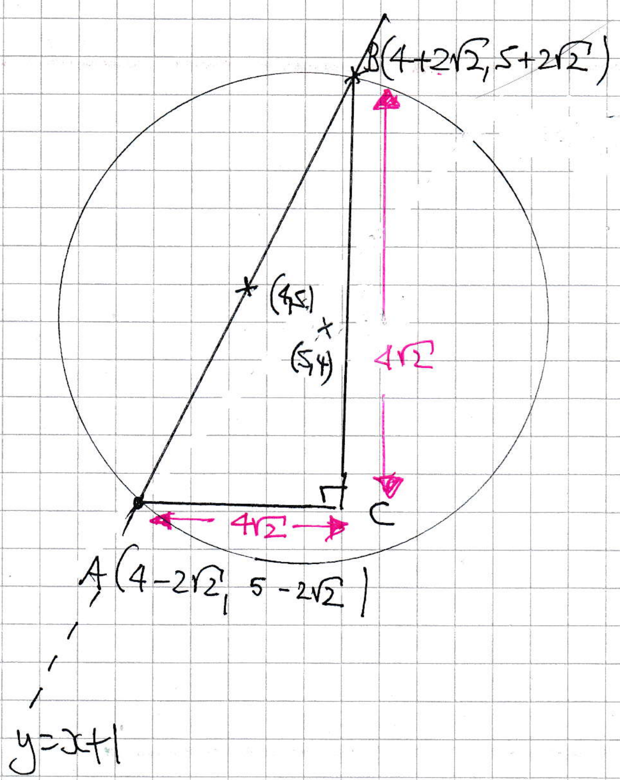
$$d = \sqrt{(4\sqrt{2})^2 + (4\sqrt{2})^2}$$

$$d = \sqrt{32+32}$$

$$d = \sqrt{64}$$

$$d = 8$$

ALTERNATIVE - NOTE $(4,5)$ IS NOT THE CENTRE OF THE CIRCLE, BUT JUST THE MIDPOINT OF AB



BY PYTHAGORAS

$$|AB|^2 = (4\sqrt{2})^2 + (4\sqrt{2})^2$$

$$|AB|^2 = 32+32$$

$$|AB|^2 = 64$$

$$|AB| = 8$$

IYGB - SYNOPTIC PAPER Q - QUESTION 3

METHOD A — REWRITE BACKWARDS

$$1000 + 991 + 982 + 973 + \dots -53$$

$a = 1000$
$d = -9$
$n = 20$

$\Longrightarrow$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$S_{18} = \frac{18}{2}\left[2\times 100 + 17\times(-9)\right]$

$S_{18} = 9\left[2000 - 153\right]$

$S_{18} = 9\times 1847$

$S_{18} = 16623$

METHOD B — BY SUBTRACTION

$a = -53$
$d = 9$

$S_{118} = \frac{118}{2}\left[2(-53) + 117\times 9\right] = 55873$

$S_{100} = \frac{100}{2}\left[2(-53) + 99\times 9\right] = 39250$

$\therefore$ REQUIRED SUM $= 55873 - 39250 = 16623$

IYGB - SYNOPTIC PAPER Q - QUESTION 3

METHOD C - BY WORKING OUT THE FIRST TERM OF THE LAST 18

$$\left.\begin{array}{l} a=-53 \\ d=9 \\ n=101 \end{array}\right\}$$

$$u_n = a + (n-1)d$$
$$u_{101} = -53 + 100 \times 9$$
$$u_{101} = 847$$

THUS FOR THE LAST 18 TERMS

$$\left.\begin{array}{l} a=847 \\ d=9 \\ n=18 \end{array}\right\} \Longrightarrow$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$S_{18} = \frac{18}{2}\left[2 \times 847 + 17 \times 9\right]$$
$$S_{18} = 9\left(1694 + 153\right)$$
$$S_{18} = 9 \times 1847$$
$$S_{18} = 16623$$

AS BEFORE

IYGB - SYNOPTIC PAPER Q - QUESTION 4

PROCEED BY MANIPULATING THE EXPANSION AS FOLLOWS

$$(1-x)^5(1+x)^6 = (1+x)(1-x)^5(1+x)^5$$

$$= (1+x)\left[(1-x)(1+x)\right]^5$$

$$= (1+x)(1-x^2)^5$$

$$= (1+x)\left[1 + \frac{5}{1}(-x^2)^1 + \frac{5\times4}{1\times2}(-x^2)^2 + \cdots\right]$$

$$= (1+x)(1 - 5x^2 + 10x^4 + \cdots)$$

$10x^5$

I.E 10

IYGB - SYNOPTIC PAPER Q - QUESTION 5

USING THE STANDARD APPROXIMATIONS FOR $\sin x$ & $\cos x$

$\cos x \simeq 1 - \frac{1}{2}x^2$

$\sin x \approx x$

$$\frac{1+\cos x}{1+\sin\left(\frac{1}{2}x\right)} \simeq \frac{1+\left(1-\frac{1}{2}x^2\right)}{1+\frac{1}{2}x} = \frac{2-\frac{1}{2}x^2}{1+\frac{1}{2}x}$$

$$\simeq \frac{4-x^2}{2+x} = \frac{\cancel{(2+x)}(2-x)}{\cancel{2+x}}$$

$$\simeq 2-x$$

AS REQUIRED IE $A=2, B=-1$

IYGB - SYNOPTIC PAPER Q - QUESTION 6

a)

$$y = 22 - 5x - \frac{4}{\sqrt{x}}$$

When $x = 4$, $y = 22 - 5\times4 - \frac{4}{\sqrt{4}}$

$y = 22 - 20 - 2$

$y = 0$

INDEED POINT B, AS B MUST HAVE $x > 1$

b)

$$y = 22 - 5x - 4x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = -5 + 2x^{-\frac{3}{2}}$$

$$\left.\frac{dy}{dx}\right|_{x=1} = -5 + 2\times1^{-\frac{3}{2}} = -3$$

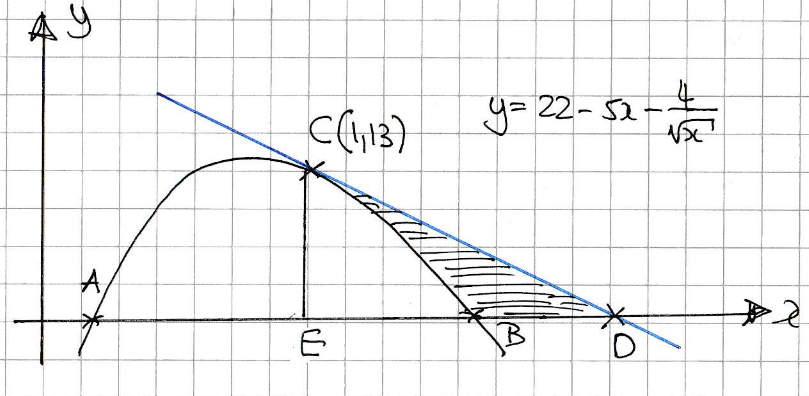
EQUATION OF TANGENT AT $C(1,13)$

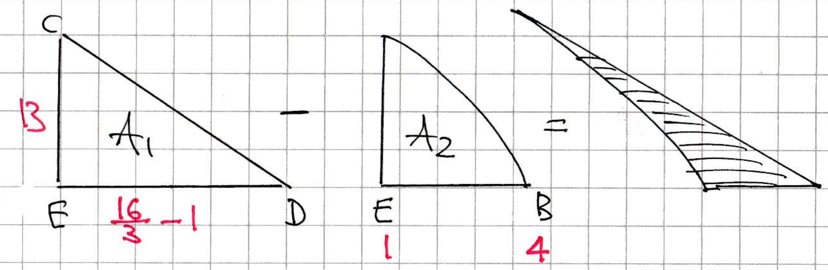
$\Longrightarrow y - y_0 = m(x - x_0)$

$\Longrightarrow y - 13 = -3(x-1)$

$\Longrightarrow y = 16 - 3x$

c)





FIND THE CO-ORDINATES OF D BY SETTING $y=0$

INTO THE EQUATION OF THE TANGENT $\quad y = 0$

$x = \frac{16}{3}$

- $A_1 = \frac{1}{2}\times13\times\left(\frac{16}{3} - 1\right)$

  $A_1 = \frac{169}{6}$

IYGB - SYNOPTIC PAPER Q — QUESTION 6

- $A_2 = \int_1^4 22 - 5x - 4x^{-\frac{1}{2}}\,dx$

$= \left[22x - \frac{5}{2}x^2 - 8x^{\frac{1}{2}}\right]_1^4$

$= (88 - 40 - 16) - (22 - \frac{5}{2} - 8)$

$= 32 - \frac{23}{2}$

$= \frac{41}{2}$

THE REQUIRED AREA IS GIVEN BY $A_1 - A_2$

$= \frac{169}{6} - \frac{41}{2}$

$= \frac{23}{3}$

IYGB - SYNOPTIC PAPER Q - QUESTION 7

a) $y = 2f(x+\alpha)$

TRANSLATION, LEFT OR RIGHT

VERTICAL STRETCH DOUBLING ALL THE y CO-ORDS

EITHER TRANSLATION BY $\begin{pmatrix}2\\0\end{pmatrix}$ OR $\begin{pmatrix}-8\\0\end{pmatrix}$

$\therefore \alpha = \begin{cases} -2 & \leftarrow 2f(x-2) \\ 8 & \leftarrow 2f(x+8) \end{cases}$ //

b) $y = \beta f(x+2)$

TRANSLATION 2 UNITS TO THE LEFT $\therefore \gamma = 1$ //

VERTICAL STRETCH, BUT ALSO A REFLECTION ABOUT THE x AXIS IF NEGATIVE

NEXT TRACE THE x CO-ORDINATE OF THE "VERTEX"

$-6 \longmapsto +6 \longmapsto 2$

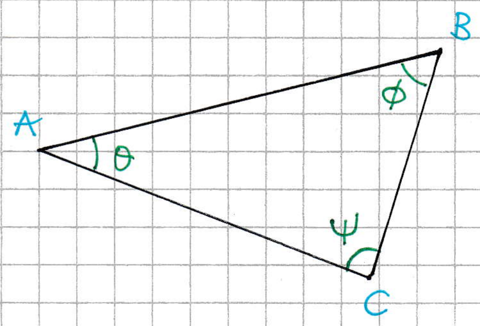
REFLECTION IN x AXIS; VERTICAL STRETCH BY SCALE FACTOR $\frac{1}{3}$

(OR THE OTHER WAY ROUND)

$\therefore \beta = -\frac{1}{3}$ //

IYGB - SYNOPTIC PAPER Q - QUESTION 8

GIVEN THAT $\sin\psi = 0.9703$



$\psi = 180 - \arcsin(0.9703)$

OBTUSE

$\psi = 180 - 76.00...$

$\psi \simeq 104°$

NEXT INFORMATION

$\tan(\theta - \phi) = 0.2493$

$\theta - \phi = \arctan(0.2493)$

$\theta - \phi = 13.9984...$

$\theta - \phi \simeq 14°$

BUT THE ANGLES BELONG TO A TRIANGLE

$\theta + \phi + \psi = 180°$

$\theta + \phi + 104 = 180$

$\theta + \phi = 76°$

FINALLY WE HAVE

$\left.\begin{array}{l}\theta + \phi = 76 \\ \theta - \phi = 14\end{array}\right\} \Rightarrow 2\theta = 90$

$\Rightarrow \theta = 45°$

THUS $\theta = 45°$, $\phi = 31°$, $\psi = 104°$

IYGB - SYNOPTIC PAPER P - QUESTION 9

METHOD A - BY PARTIAL FRACTIONS

$$\frac{3x^2-10x+2}{(x-2)^2(1-2x)} \equiv \frac{A}{(x-2)^2} + \frac{B}{1-2x} + \frac{C}{x-2}$$

$$3x^2-10x+2 \equiv A(1-2x) + B(x-2)^2 + C(x-2)(1-2x)$$

- IF $x=2$

$12-20+2 = -3A$

$3A = 6$

$A = 2$

- IF $x=\frac{1}{2}$

$\frac{3}{4} - 5 + 2 = \frac{9}{4}B$

$3 - 20 + 8 = 9B$

$-9 = 9B$

$B = -1$

- IF $x=0$

$2 = A + 4B - 2C$

$2 = 2 - 4 - 2C$

$2C = -4$

$C = -2$

$$y = \frac{2}{(x-2)^2} - \frac{1}{1-2x} - \frac{2}{x-2}$$

$$\frac{dy}{dx} = \frac{-4}{(x-2)^3} - \frac{2}{(1-2x)^2} + \frac{2}{(x-2)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=3} = -4 - \frac{2}{25} + \frac{2}{1} = -\frac{52}{25}$$

METHOD B - BY LOGARITHMS

$$y = \frac{3x^2-10x+2}{(x-2)^2(1-2x)}$$

$$y\big|_{x=3} = \frac{27-30+2}{1 \times (-5)} = \frac{-1}{-5} = \frac{1}{5}$$

TAKING NATURAL LOGS

$$\ln y = \ln \frac{3x^2-10x+2}{(x-2)^2(1-2x)} = \ln(3x^2-10x+2) - \ln(x-2)^2 - \ln(1-2x)$$

$$\ln y = \ln(3x^2-10x+2) - 2\ln(x-2) - \ln(1-2x)$$

IYGB - SYNOPTIC PAPER Q - QUESTION 9

DIFFERENTIATE IMPLICITLY W.R.T x

$$\frac{1}{y}\frac{dy}{dx} = \frac{6x-10}{3x^2-10x+2} - \frac{2}{x-2} + \frac{2}{1-2x}$$

WHEN $x=3$, $y=\frac{1}{5}$

$$5\left.\frac{dy}{dx}\right|_{x=3} = \frac{8}{27-30+2} - \frac{2}{1} + \frac{2}{-5}$$

$$5\left.\frac{dy}{dx}\right|_{x=3} = -\frac{52}{5}$$

$$\left.\frac{dy}{dx}\right|_{x=3} = -\frac{52}{25}$$

// AS BEFORE

METHOD C - BY TRIPLE PRODUCT RULE

$$y = (3x^2-10x+2)(x-2)^{-2}(1-2x)^{-1}$$

$$\frac{dy}{dx} = (6x-10)(x-2)^{-2}(1-2x)^{-1} + (3x^2-10x+2)(-2)(x-2)^{-3}(1-2x)^{-1} + (3x^2-10x+2)(x-2)^{-2} \times 2(1-2x)^{-2}$$

NO NEED TO TIDY UP

$$\left.\frac{dy}{dx}\right|_{x=3} = 8\times1\times\frac{-1}{5} + (-1)(-2)\times1\times\frac{-1}{5} + (-1)(1)\times\frac{2}{25}$$

$$= -\frac{8}{5} + \frac{2}{5} - \frac{2}{25}$$

$$= -2 - \frac{2}{25}$$

$$= -\frac{52}{25}$$

// AS BEFORE

IYGB – SYNOPTIC PAPER Φ – QUESTION 10

DIFFERENTIATING PARAMETRICALLY

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{12\cos 2t}{-6\sin 2t} = -2\cot 2t$$

NOW SECOND DERIVATIVE

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(-2\cot 2t\right) = 4\operatorname{cosec}^2 2t \times \frac{dt}{dx} = \frac{4\operatorname{cosec}^2 2t}{\frac{dx}{dt}}$$

$$= \frac{4\operatorname{cosec}^2 2t}{-6\sin 2t} = -\frac{2}{3}\operatorname{cosec}^3 2t = -\frac{2}{3\sin^3 2t}$$

BUT $y = 6\sin 2t$

$$\frac{d^2y}{dx^2} = -\frac{2}{3\left(\frac{y}{6}\right)^3} = -\frac{2}{\frac{y^3}{72}} = -\frac{144}{y^3}$$

$$\therefore \frac{d^2y}{dx^2} = -\frac{144}{y^3}$$

IYGB – SYNOPTIC PAPER Q – QUESTION 11

a)

$$\boxed{f(x) = \ln\left(\frac{e-x}{e+x}\right)}$$

$$f(-x) = \ln\left(\frac{e-(-x)}{e+(-x)}\right) = \ln\left(\frac{e+x}{e-x}\right) = \ln\left(\frac{e-x}{e+x}\right)^{-1}$$

$$= -\ln\left(\frac{e-x}{e+x}\right) = -f(x)$$

$\therefore$ $f(x)$ IS INDEED ODD

b) TO FIND THE LARGEST POSSIBLE DOMAIN

- $e+x \neq 0$

  $x \neq -e$

- THE LOG'S ARGUMENT MUST BE POSITIVE

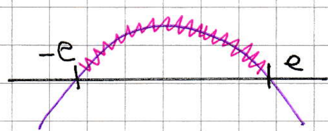
$$\frac{e-x}{e+x} > 0$$

$$\frac{(e-x)(e+x)}{(e+x)(e+x)} > 0$$

$$\frac{(e-x)(e+x)}{(e+x)^2} > 0$$

AS THE DENOMINATOR IS ALWAYS POSITIVE

$$(e-x)(e+x) > 0$$



$$-e < x < e$$

$\therefore$ LARGEST REAL DOMAIN

$$x \in \mathbb{R},\ -e < x < e$$

IYGB - SYNOPTIC PAPER Q - QUESTION 11

FINALLY SOLVING THE EQUATION

$$\Rightarrow f(x) + f(x+1) = 0$$

$$\Rightarrow \ln\left(\frac{e-x}{e+x}\right) + \ln\left[\frac{e-(x+1)}{e+(x+1)}\right] = 0$$

$$\Rightarrow \ln\left(\frac{e-x}{e+x}\right) + \ln\left(\frac{e-x-1}{e+x+1}\right) = 0$$

$$\Rightarrow \ln\left[\frac{e-x}{e+x} \times \frac{e-x-1}{e+x+1}\right] = 0$$

$$\Rightarrow \frac{(e-x)(e-x-1)}{(e+x)(e+x+1)} = 1$$

$$\Rightarrow (e-x)\left[(e-x)-1\right] = (e+x)\left[(e+x)+1\right]$$

$$\Rightarrow (e-x)^2 - (e-x) = (e+x)^2 + (e+x)$$

$$\Rightarrow \cancel{e^2} - 2ex + \cancel{x^2} - e + \cancel{x} = \cancel{e^2} + 2ex + \cancel{x^2} + e + \cancel{x}$$

$$\Rightarrow -2\cancel{e} = 4\cancel{e}x$$

$$\Rightarrow x = -\frac{1}{2}$$

IYGB - SYNOPTIC PAPER P - QUESTION 12

WRITE EACH FORMULA IN y NOTATION & DIFFERENTIATE

- $y = \frac{1}{2}(x^3 - 5)$

  $\frac{dy}{dx} = \frac{3}{2}x^2$

- $y = (2x+5)^{\frac{1}{3}}$

  $\frac{dy}{dx} = \frac{2}{3}(2x+5)^{-\frac{2}{3}}$

- $y = 5(x^2-2)^{-1}$

  $\frac{dy}{dx} = -5(x^2-2)^{-2}(2x)$

  $\frac{dy}{dx} = \frac{-10x}{(x^2-2)^2}$

- $y = \left(2 + \frac{5}{x}\right)^{\frac{1}{2}}$

  $\frac{dy}{dx} = \frac{1}{2}\left(2+\frac{5}{x}\right)^{-\frac{1}{2}} \times \left(-\frac{5}{x^2}\right)$

NEXT EVALUATE THESE DERIVATIVES IN THE NEIGHBOURHOOD OF $x = 2.1$

$\frac{dy}{dx} = 3x^2 \qquad \left.\frac{dy}{dx}\right|_{x=2.1} = +6.615 > 1$

RAPIDLY DIVERGES WITHOUT OSCILLATION (STAIRCASE AWAY FROM α)

$\frac{dy}{dx} = \frac{2}{3}(2x+5)^{-\frac{2}{3}} \qquad \left.\frac{dy}{dx}\right|_{x=2.1} = +0.1518\ldots$

RAPIDLY CONVERGES WITHOUT OSCILLATION AS THIS FIGURE IS BETWEEN 0 & 1 (STAIRCASE TOWARDS α)

$\frac{dy}{dx} = \frac{-10x}{(x^2-2)^2} \qquad \left.\frac{dy}{dx}\right|_{x=2.1} = -3.6156 < -1$

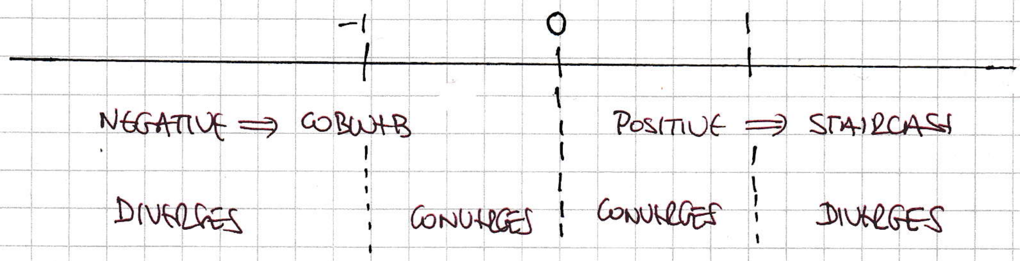
DIVERGES WITH OSCILLATION AS THIS FIGURE IS BELOW -1 (COBWEB AWAY FROM α)

$\frac{dy}{dx} = \frac{1}{2}\left(2+\frac{5}{x}\right)^{-\frac{1}{2}} \times \left(-\frac{5}{x^2}\right) \qquad \left.\frac{dy}{dx}\right|_{x=2.1} = -0.2708$

(RAPIDLY) CONVERGES WITH OSCILLATION AS THIS FIGURE IS VERY CLOSE TO 0 (COBWEB TOWARDS α)

IYGB - SYNOPTIC PAPER Q - QUESTION 12

QUICK SUMMARY FOR THE VALUE OF THE DERIVATIVE CLOSE TO THE ROOT



PICKING THE FORMULA WHICH PRODUCED THE CLOSEST TO ZERO

$$x_{n+1} = (2x_n + 5)^{\frac{1}{3}}$$

$x_1 = 2.1$

$x_2 = 2.095379106$

$x_3 = 2.094677239$

$x_4 = 2.094570591$

$x_5 = 2.094554385$

$x_6 = 2.094551923$

$x_7 = 2.094551549$

$x_8 = 2.094551492$

$\therefore \alpha \approx 2.094551$ // 6 d.p

IYGB – SYNOPTIC PAPER Q – QUESTION 13

MODEL USING THE RESULT SPEED = DISTANCE / TIME

$$\text{TIME} = \frac{\text{DISTANCE}}{\text{SPEED}}$$

LET T_1 BE THE TIME FOR THE FIRST PART OF THE JOURNEY & T_2 THE TIME FOR THE SECOND PART OF THE JOURNEY, SO THAT $T_1 + T_2 = 6$

$$T_1 = \frac{16}{x} \quad \& \quad T_2 = \frac{40-16}{x-2} = \frac{24}{x-2}$$

SOLVING THE RESULTING EQUATION, NOTING THAT $x > 2$

$$\Rightarrow \frac{16}{x} + \frac{24}{x-2} = 6$$

$\div 2$

$$\Rightarrow \frac{8}{x} + \frac{12}{x-2} = 3$$

$\times\ x(x-2)$

$$\Rightarrow 8(x-2) + 12x = 3x(x-2)$$

$$\Rightarrow 8x - 16 + 12x = 3x^2 - 6x$$

$$\Rightarrow 0 = 3x^2 - 26x + 16$$

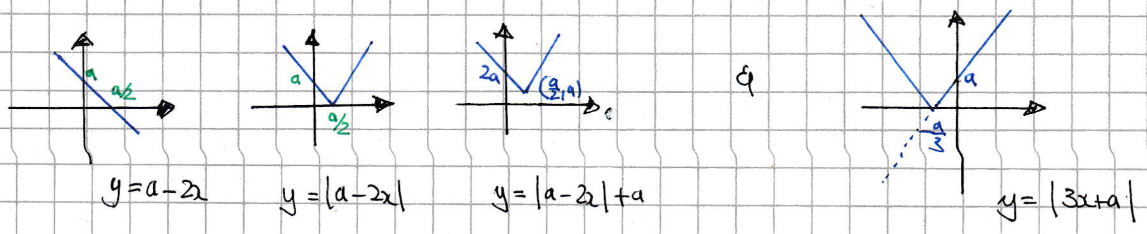
$$\Rightarrow (3x-2)(x-8) = 0$$

$$\Rightarrow x = \begin{cases} 8 \\ \cancel{\frac{2}{3}} \end{cases} \quad (x>2)$$

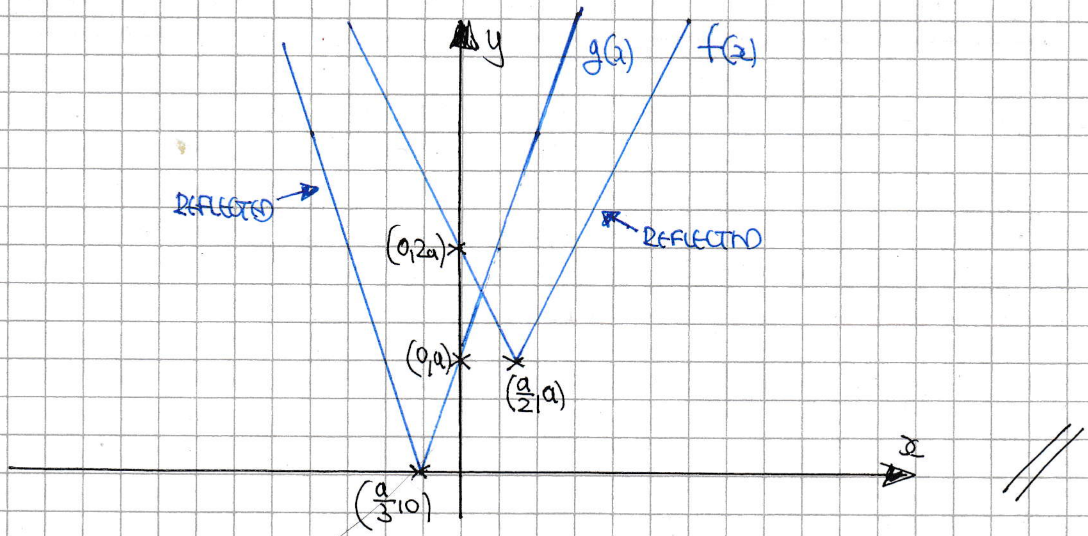
$$\Rightarrow x = 8$$

IYGB – SYNOPTIC PAPER Q – QUESTION 14

a) NOTING THAT a IS POSITIVE



HENCE WE HAVE



b) TO FIND THE "VISIBLE" INTERSECTION WE SOLVE THE NON REFLECTED PARTS

$$\left.\begin{array}{l} (f)\ y = (a-2x)+a \\ (g)\ y = (3x+a) \end{array}\right\} \Longrightarrow \quad 2a - 2x = 3x + a$$

$$a = 5x$$

$$x = \frac{1}{5}a \qquad \& \qquad y = 3\left(\tfrac{1}{5}a\right) + a$$

$$y = \frac{3}{5}a + a$$

$$y = \frac{8}{5}a$$

TO FIND THE "NON VISIBLE IN THE SKETCH" INTERSECTION WE NEED TO SOLVE THE ORIGINAL (NON REFLECTED) $f(x)$ & THE REFLECTED $g(x)$

$$\left.\begin{array}{l} (f)\ y = (a-2x)+a \\ (g)\ y = -(3x+a) \end{array}\right\} \Longrightarrow \quad 2a - 2x = -3x - a$$

$$x = -3a \qquad \& \qquad y = 2a - 2(-3a)$$

$$y = 8a$$

IYGB - SYNOPTIC PAPER Q - QUESTION 14

$\therefore$ INTERSECTIONS ARE $\left(\frac{1}{5}a, \frac{8}{5}a\right)$ & $(-3a, 8a)$ //

c) COMPOSING THE FUNCTIONS

$$gf(x) = g(f(x)) = g(|a-2x|+a) = \left|3\left[|a-2x|+a\right]+a\right|$$

NOTE THAT $|a-2x|+a \geqslant 0$ & $a>0$, SO IGNORE OUTER "MOD SIGNS"

$$\Longrightarrow gf(x) = 3\left[|a-2x|+a\right]+a = 3|a-2x|+4a$$ //

d) SOLVING FINALLY $gf(x) = 10a$

$$3|a-2x|+4a = 10a$$
$$3|a-2x| = 6a$$
$$|a-2x| = 2a$$

THIS HAS TWO SOLUTIONS (ALWAYS)

$$a-2x = 2a \qquad\qquad a-2x = -2a$$
$$-2x = a \qquad\qquad 3a = 2x$$
$$x = -\frac{1}{2}a \qquad\qquad x = \frac{3}{2}a$$

$\therefore\ x_1 = -\frac{1}{2}a \cup x_2 = \frac{3}{2}a$ //

IYGB - SYNOPTIC PAPER Q - QUESTION 15

a) FORMING A TABLE

START OF MONTH	£	END OF MONTH
1	200	$200 \times 1.005 = 201$
2	$200 + 201$	$401 \times 1.005 = 403.005$
3	$200 + 403.005$	$603.005 \times 1.005 = 606.020025$

$\therefore$ £ 606.02

AS REQUIRED

b) MONTH END

1	200×1.005
2	$200 \times 1.005^2 + 200 \times 1.005^1$
3	$200 \times 1.005^3 + 200 \times 1.005^2 + 200 \times 1.005^1$
$\vdots$	$\vdots$
60	$200 \times 1.005^{60} + 200 \times 1.005^{59} + 200 \times 1.005^{58} + \dots + 200 \times 1.005$

HENCE THE REQUIRED TOTAL IS

$$\Rightarrow \text{TOTAL} = 200 \times 1.005^1 + 200 \times 1.005^2 + 200 \times 1.005^3 + \dots + 200 \times 1.005^{60}$$

$$\Rightarrow \text{TOTAL} = 200\left[\underbrace{1.005^1 + 1.005^2 + 1.005^3 + \dots + 1.005^{60}}\right]$$

This is a G.P. with $a = 1.005$, $r = 1.005$, $n = 60$

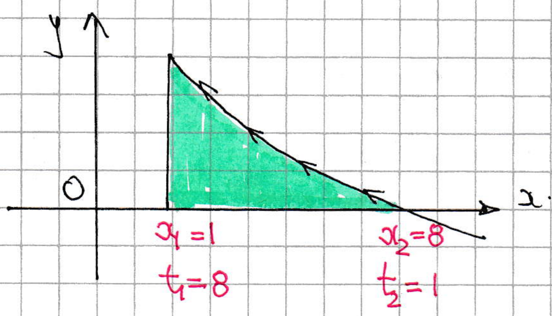
$$\Rightarrow \text{TOTAL} = 200 \times \frac{\overset{a}{1.005}\left(\overset{r^n}{1.005^{60}} - 1\right)}{\underset{r-1}{1.005 - 1}} = £14023.78$$

IYGB - SYNOPTIC PAPER P - QUESTION 16

a) START BY FINDING THE x INTERCEPT OF THE CURVE AND THE VALUE OF t AT 1

- $y=0$ • $\ln t = 0$, $t = e^0$, $t=1$ • $x = \frac{8}{t}$, $x = 8$

- $x=1$
$$1 = \frac{8}{t}$$
$$t = 8$$ I.E THE CURVE IS TRACED "BACKWARDS"



$$\text{AREA} = \int_{x_1}^{x_2} y(x)\,dx = \int_{t_1}^{t_2} y(t)\frac{dx}{dt}\,dt = \int_8^1 (\ln t)\left(-\frac{8}{t^2}\right)dt$$

$$= \int_8^1 -\frac{8\ln t}{t^2}\,dt = \int_1^8 \frac{8\ln t}{t^2}\,dt$$ // AS REQUIRED

b) FOLLOW BY INTEGRATION BY PARTS

$$\int_1^8 \frac{8\ln t}{t^2}\,dt = \left[-\frac{8\ln t}{t}\right]_1^8 - \int_1^8 -\frac{8}{t^2}\,dt$$

$8\ln t$	$\frac{8}{t}$
$-\frac{1}{t}$	$\frac{1}{t^2}$

$$= \left[-\frac{8\ln t}{t}\right]_1^8 + \int_1^8 \frac{8}{t^2}\,dt$$

$$= \left[-\frac{8\ln t}{t} - \frac{8}{t}\right]_1^8$$

IYGB - SYNOPTIC PAPER Q - QUESTION 16

$$= \left[\frac{8}{t} + \frac{8}{t}\ln t \right]_8^1$$

$$= (8 + \cancel{\ln 1}) - (1 + \ln 8)$$

$$= 7 - \ln 8$$

$$= 7 - 3\ln 2$$

c) SWITCHING INTO CARTESIAN

$$\left.\begin{array}{l} x = \frac{8}{t} \\ y = \ln t \end{array}\right\} \Rightarrow t = \frac{8}{x} \Rightarrow y = \ln\frac{8}{x}$$

$$\text{AREA} = \int_{x_1}^{x_2} y(x)\,dx = \int_1^8 \ln\left(\frac{8}{x}\right)dx = \int_1^8 -\ln\left(\frac{x}{8}\right)dx$$

$$= \int_8^1 \ln\left(\tfrac{1}{8}x\right)dx = \int_8^1 1 \times \ln\left(\tfrac{1}{8}x\right)dx$$

INTEGRATION BY PARTS AGAIN

$\ln(\frac{1}{8}x)$	$\frac{1}{x}$
x	1

$$\dots = \left[x\ln\left(\tfrac{1}{8}x\right)\right]_8^1 - \int_8^1 1\,dx$$

$$= \left[x\ln\left(\tfrac{1}{8}x\right) - x\right]_8^1$$

$$= \left(1 \times \ln\left(\tfrac{1}{8}\right) - 1\right) - \left(\cancel{8\ln 1} - 8\right)$$

$$= \ln\left(\tfrac{1}{8}\right) - 1 + 8$$

$$= 7 + \ln\left(\tfrac{1}{8}\right)$$

$$= 7 - \ln 8$$

AS BEFORE

IYGB - SYNOPTIC PAPER Q - QUESTION 17

THIS IS A CUBIC IN e^x

$$\Rightarrow 6e^{3x}+1=7e^{2x}$$

$$\Rightarrow 6e^{3x}-7e^{2x}+1=0$$

$$\Rightarrow 6(e^x)^3-7(e^x)^2+1=0$$

$$\Rightarrow 6A^3-7A^2+1=0 \qquad \boxed{A=e^x}$$

BY INSPECTION $A=1$ IS A SOLUTION - SO LONG DIVIDE BY $A-1$

$$\Rightarrow 6A^2(A-1)-A(A-1)-(A-1)=0$$

$$\Rightarrow (A-1)(6A^2-A-1)=0$$

$$\Rightarrow (A-1)(3A+1)(2A-1)=0$$

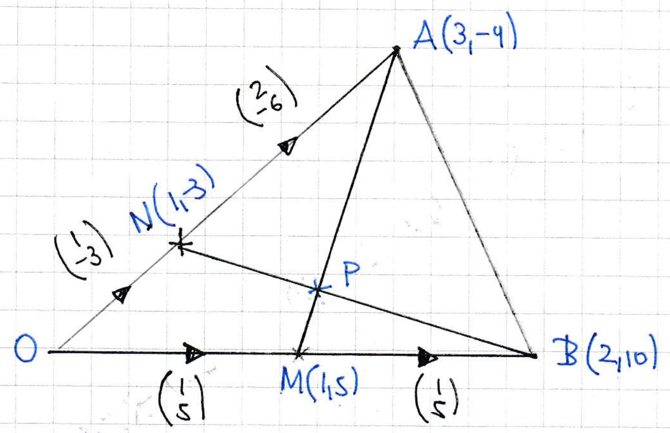
$$\Rightarrow A = \begin{cases} 1 \\ -\frac{1}{3} \\ \frac{1}{2} \end{cases}$$

$$\Rightarrow e^x = \begin{cases} 1 \\ \cancel{-\frac{1}{3}} \\ \frac{1}{2} \end{cases} \qquad e^x>0$$

$$\Rightarrow x = \begin{cases} 0 & (\ln 1) \\ -\ln 2 & (\ln \frac{1}{2}) \end{cases} \qquad \ln\left(\frac{a}{b}\right) \equiv -\ln\left(\frac{b}{a}\right)$$

IYGB - SYNOPTIC PAPER Q - QUESTION 18

START WITH A DIAGRAM (NOT TO SCALE) AND LABEL IT



$$\vec{NB} = \vec{NO} + \vec{OB} = \binom{-1}{3} + \binom{2}{10} = \binom{1}{13}$$

NOW PROCEED AS FOLLOWS

$$\vec{AP} = \vec{AN} + \vec{NP} = \vec{AN} + k(\vec{NB}) = \binom{-2}{6} + k\binom{1}{13} = \binom{k-2}{13k+6}$$

$$\vec{PM} = \vec{PA} + \vec{AO} + \vec{OM} = \binom{2-k}{-13k-6} + \binom{-3}{9} + \binom{1}{5} = \binom{-k}{8-13k}$$

BUT A–P–M IS A STRAIGHT LINE, SO $\vec{AP}$ & $\vec{PM}$ MUST BE IN PROPORTION

$$\Rightarrow \frac{k-2}{13k+6} = \frac{-k}{8-13k}$$

$$\Rightarrow \frac{k-2}{13k+6} = \frac{k}{13k-8}$$

$$\Rightarrow (k-2)(13k-8) = k(13k+6)$$

$$\Rightarrow \cancel{13k^2} - 8k - 26k + 16 = \cancel{13k^2} + 6k$$

$$\Rightarrow 16 = 40k$$

$$\Rightarrow k = \frac{2}{5}$$

$\therefore$ REQUIRED RATIO $\vec{NP} : \vec{PM} = 2 : 3$

IYGB – SYNOPTIC PAPER Q – QUESTION 19

FIND THE x COORDINATE OF THE STATIONARY POINT, IN TERMS OF k

$$y = 2x^3 + \frac{k}{x} - 19$$

$$\frac{dy}{dx} = 6x^2 - \frac{k}{x^2}$$

SOLVING $\frac{dy}{dx} = 0$

$$6x^2 - \frac{k}{x^2} = 0$$

$$6x^2 = \frac{k}{x^2}$$

$$x^4 = \frac{k}{6}$$

$$x = \frac{k^{1/4}}{6^{1/4}}$$

NOW $f\left(\frac{k^{1/4}}{6^{1/4}}\right) = 45$

$$45 = 2\left(\frac{k^{1/4}}{6^{1/4}}\right)^3 + \frac{k}{\frac{k^{1/4}}{6^{1/4}}} - 19$$

$$64 = 2\left(\frac{k^{3/4}}{6^{3/4}}\right) + \frac{6^{1/4} \times k}{k^{1/4}}$$

$$64 = \frac{2}{6^{3/4}}k^{\frac{3}{4}} + 6^{\frac{1}{4}}k^{\frac{3}{4}} \qquad \times 6^{\frac{3}{4}}$$

$$64 \times 6^{\frac{3}{4}} = 2k^{\frac{3}{4}} + 6k^{\frac{3}{4}}$$

$$64 \times 6^{\frac{3}{4}} = 8 \times k^{\frac{3}{4}}$$

$$8 \times 6^{\frac{3}{4}} = k^{\frac{3}{4}}$$

$$\left(k^{\frac{3}{4}}\right)^{\frac{4}{3}} = \left(8 \times 6^{\frac{3}{4}}\right)^{\frac{4}{3}}$$

$$k = 8^{\frac{4}{3}} \times 6$$

$$k = 16 \times 6 \qquad \therefore k = 96$$

IYGB - SYNOPTIC PAPER Q - QUESTION 20

a) AVOID THE TRIPLE ANGLE FORMULAS !!

$$f(x) = \frac{\sin 3x}{\cos x} + \frac{\cos 3x}{\sin x} = \frac{\sin 3x \sin x + \cos 3x \cos x}{\cos x \sin x}$$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$= \frac{\cos(3x - x)}{\cos x \sin x} = \frac{\cos 2x}{\cos x \sin x} = \frac{2\cos 2x}{2\cos x \sin x} = \frac{2\cos 2x}{\sin 2x} = 2\cot 2x$$

AS REQUIRED

b) FINALLY SOLVING THE EQUATION IN $0 \leq x < 2\pi$

$$\Rightarrow \frac{1}{4}f(x) + 1 = \tan x$$

$$\Rightarrow \frac{1}{4}(2\cot 2x) + 1 = \tan x$$

$$\Rightarrow \frac{1}{2}\cot 2x + 1 = \tan x$$

$$\Rightarrow \cot 2x + 2 = 2\tan x$$

$$\Rightarrow \frac{1}{\tan 2x} + 2 = 2\tan x$$

$$\tan 2\theta \equiv \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$\Rightarrow \frac{1 - \tan^2 x}{2\tan x} + 2 = 2\tan x$$

$$\Rightarrow \frac{1 - T^2}{2T} + 2 = 2T$$

$$\Rightarrow 1 - T^2 + 4T = 4T^2$$

$$\Rightarrow 0 = 5T^2 - 4T - 1$$

$$\Rightarrow (5T + 1)(T - 1) = 0$$

$$\Rightarrow T = \begin{cases} 1 \\ -\frac{1}{5} \end{cases}$$

$$\Rightarrow \tan\theta = \begin{cases} 1 \\ -\frac{1}{5} \end{cases}$$

$$\begin{cases} \theta = \arctan(1) \pm n\pi \\ \theta = \arctan\left(-\frac{1}{5}\right) \pm n\pi \end{cases} \qquad n = 0, 1, 2, 3, \ldots$$

$\theta_1 = 0.785^c \quad (\pi/4)$

$\theta_2 = 3.927^c \quad (3\pi/4)$

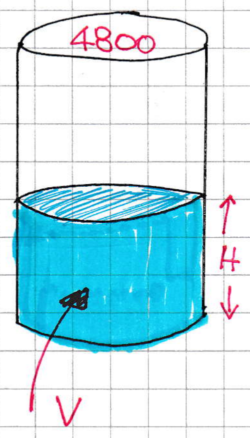
$\theta_3 = 2.944^c$

$\theta_4 = 6.086^c$

IYGB - SYNOPTIC PAPER Q - QUESTION 21

a) SETTING UP A MODEL

- IN FLOW $\frac{dV}{dt} = 2400$
- OUT FLOW $\frac{dV}{dt} = -kH^{\frac{1}{2}}$
- NET FLOW $\frac{dV}{dt} = 2400 - kH^{\frac{1}{2}}$



$V = 4800H$

$\frac{dV}{dH} = 4800$

RELATING VARIABLES H & V

$$\Rightarrow \frac{dV}{dH} \times \frac{dH}{dt} = 2400 - kH^{\frac{1}{2}}$$

$$\Rightarrow 4800\frac{dH}{dt} = 2400 - kH^{\frac{1}{2}}$$

$$\Rightarrow \frac{dH}{dt} = \frac{1}{2} - \frac{k}{4800}H^{\frac{1}{2}}$$

$$\Rightarrow \frac{dH}{dt} = \frac{1}{2} - BH^{\frac{1}{2}} \qquad \left(B = \frac{k}{4800} = \text{CONSTANT}\right)$$

AS REQUIRED

b) USING THE CONDITION GIVEN: $H = 16$, $\frac{dH}{dt} = -120$

$$\Rightarrow -120 = -k \times 16^{\frac{1}{2}} \quad \text{(OUTFLOW ONLY)}$$

$$\Rightarrow -120 = -4k$$

$$\Rightarrow k = 30$$

$$\Rightarrow B = \frac{k}{4800} = \frac{30}{4800} = \frac{1}{160}$$

$$\therefore \frac{dH}{dt} = \frac{1}{2} - \frac{1}{160}H^{\frac{1}{2}}$$

$$\frac{dH}{dt} = \frac{80 - H^{\frac{1}{2}}}{160}$$

AS REQUIRED

IYGB - SYNOPTIC PAPER Q - QUESTION 21

c) USING THE SUBSTITUTION GIVEN

$$u = 80 - \sqrt{H}$$
$$\sqrt{H} = 80 - u$$
$$H = (80-u)^2$$
$$\frac{dH}{du} = 2(80-u)^1(-1)$$
$$\frac{dH}{du} = -2(80-u)$$
$$\frac{dH}{du} = 2(u-80)$$
$$dH = 2(u-80)\,du$$

$$\int \frac{1}{80-\sqrt{H}}\,dH = \int \frac{1}{u} \times 2(u-80)\,du$$

$$= \int \frac{2u-160}{u}\,du = \int 2 - \frac{160}{u}\,du$$

$$= 2u - 160\ln|u| + C$$

$$= 2(80-\sqrt{H}) - 160\ln|80-\sqrt{H}| + C$$

$$= 160 - 2\sqrt{H} - 160\ln|80-\sqrt{H}| + C$$

$$= -2\sqrt{H} - 160\ln|80-\sqrt{H}| + C$$

d) SEPARATING VARIABLES

$$\Rightarrow \frac{dH}{dt} = \frac{80-\sqrt{H}}{160}$$

$$\Rightarrow \int \frac{1}{80-\sqrt{H}}\,dH = \int \frac{1}{160}\,dt$$

PART (c)

$$\Rightarrow -2\sqrt{H} - 160\ln|80-\sqrt{H}| = \frac{1}{160}t + C$$

APPLY CONDITION $t=0 \quad H=0 \Rightarrow C = -160\ln 80$

$$\Rightarrow -2\sqrt{H} - 160\ln|80-\sqrt{H}| = \frac{1}{160}t - 160\ln 80$$

FINALLY WHEN $H = 4\text{m} = 400\text{cm}$

$$\Rightarrow -2\times 20 - 160\ln(80-20) = \frac{1}{160}t - 160\ln 80$$

$$\Rightarrow \frac{1}{160}t = 160\ln 80 - 160\ln 60 - 40$$

$$\Rightarrow t \approx 964.66105\ldots \text{ seconds} \approx 16 \text{ minutes}$$