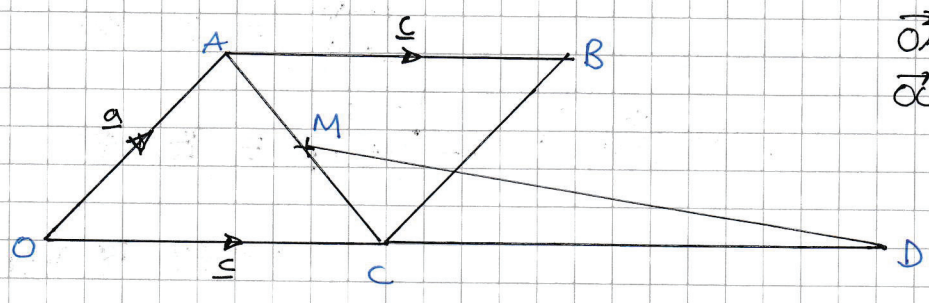


IGCSE - SYNOPTIC PAPER K - QUESTION 1

a) START WITH A DIAGRAM



$$\vec{OA} = \mathbf{a} = (7, 4, 3)$$

$$\vec{OC} = \mathbf{c} = (1, 2, -1)$$

$$\vec{OB} = \vec{OA} + \vec{AB} = \vec{OA} + \vec{OC} = \mathbf{a} + \mathbf{c} = (7, 4, 3) + (1, 2, -1) = (8, 6, 2)$$

B(8, 6, 2)

b) PROCEED AS FOLLOWS

$$\vec{AC} = \vec{AO} + \vec{OC} = -\mathbf{a} + \mathbf{c} = -(7, 4, 3) + (1, 2, -1) = (-6, -2, -4)$$

$$\vec{MC} = \frac{1}{2} \vec{AC} = \frac{1}{2} (-6, -2, -4) = (-3, -1, -2)$$

FINALLY WE HAVE

$$\vec{MD} = \vec{MC} + \vec{CD}$$

$$(1, 7, -6) = (-3, -1, -2) + \vec{CD}$$

$$\vec{CD} = (1, 7, -6) - (-3, -1, -2)$$

$$\vec{CD} = (4, 8, -4)$$

$$\vec{CD} = 4(1, 2, -1)$$

$$\vec{CD} = 4(\vec{OC})$$

∴ RATIO 1:4

1YGB - SYNOPTIC PAPER 1 - QUESTION 2

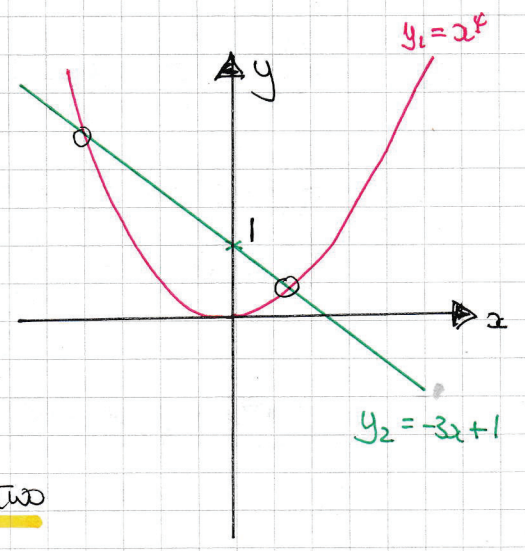
a) SPUT INTO TWO GRAPHS

$$x^4 + 3x - 1 = 0$$

$$x^4 = -3x + 1$$

$$\uparrow \qquad \qquad \uparrow$$

$$y_1 = x^4 \qquad y_2 = -3x + 1$$



∴ TWO REAL ROOTS AS THERE ARE TWO INTERSECTIONS

b)  $f(x) = x^4 + 3x - 1$

$$f(0) = -1 < 0$$

$$f(1) = 3 > 0$$

AS  $f(x)$  IS CONTINUOUS AND THERE IS A CHANGE OF SIGN IN THE INTERVAL  $(0,1)$ , THERE IS AT LEAST ONE ROOT IN THE INTERVAL

c) USING THE RECURRENCE RELATION GIVEN

$$x_1 = 0.3306$$

$$x_2 = 0.3293$$

$$x_3 = 0.3294$$

$$x_4 = 0.3294$$

(4 d.p)

d) THIS CAN BE REPRESENTED BY A CORWEB DIAGRAM, AS THE

SUCCESSIVE APPROXIMATION OSCILLATE

1YGB - SYNOPTIC PAPER K - QUESTION K

e)  $f(x) = x^4 + 3x - 1$

$$f(0.329405) = -0.000011... < 0$$

$$f(0.329415) = 0.000020... > 0$$

CONTINUITY & CHANGE OF SIGN IMPLY THE ROOT SATISFIES

$$0.329405 < \alpha < 0.329415$$

$$\therefore \alpha = 0.32941$$

5 d.p

## 17GB - SYNOPSIS PAPER K - QUESTION 3

a) IF THE POINT C(10,0) IS SATISFIED BY BOTH

THE LINE & CURVE

$$y = d - 2x$$

$$0 = d - 2 \times 10$$

$$0 = d - 20$$

$$d = 20$$

$$y = p + 10x - x^2$$

$$0 = p + 10 \times 10 - 10^2$$

$$p = 0$$

b) SOVING SIMULTANEOUSLY

$$y = 20 - 2x$$

$$y = 10x - x^2$$

$$\Rightarrow 20 - 2x = 10x - x^2$$

$$\Rightarrow x^2 - 12x + 20 = 0$$

$$\Rightarrow (x - 10)(x - 2) = 0$$

$$\Rightarrow x = 10 \quad \text{or} \quad x = 2$$

$$\Rightarrow y = 10 \times 2 - 2^2 = 16$$

$$\therefore A(2, 16)$$

FINALLY USING THE QUADRATIC OR SYMMETRY

$$y = 16$$

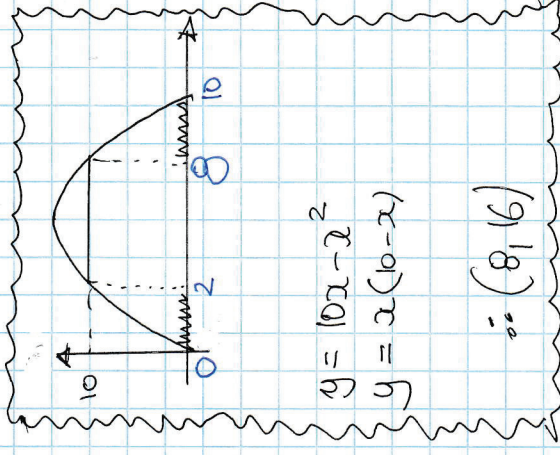
$$16 = 10x - x^2$$

$$x^2 - 10x + 16 = 0$$

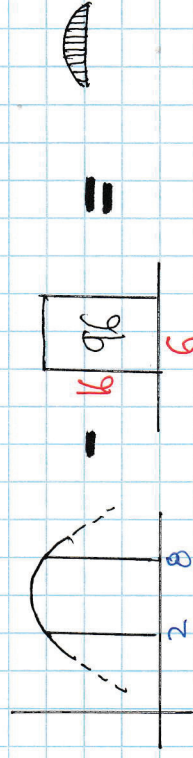
$$(x - 2)(x - 8) = 0$$

$$x = 2 \quad \text{or} \quad x = 8$$

$$\therefore (8, 16)$$



b) LOOKING AT THE DIAGRAM BELOW



$$\int_2^8 10x - x^2 dx = [5x^2 - x^3]_2^8$$

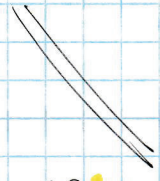
-2-

1XCB - SYNOPSIS PAPER & -QUESTION 3

$$= \left( 320 - \frac{512}{3} \right) - \left( 20 - \frac{8}{3} \right)$$

$$= 132$$

∴ REQUIRED AREA = 132 - 96 = 36



- 1 -

# 1YGB - SYNOPTIC PAPER K - QUESTION 4

$$k = 2^p - 1 \quad N = k^2 - 1 \quad (\text{GIVEN})$$

PROCEED BY DIRECT EVALUATION

$$\begin{aligned} N = k^2 - 1 &= (2^p - 1)^2 - 1 = (2^p)^2 - 2 \times 2^p \times 1 + 1^2 - 1 \\ &= 2^{2p} - 2^{p+1} + 1 - 1 \\ &= 2^{2p} - 2^{p+1} \\ &= 2^{p+1} (2^{p-1} - 1) \end{aligned}$$

INDDED  $2^{p+1}$  IS A FACTOR OF  $N$

[NOTE  $2^{p+1} \times 2^{p-1} = 2^{p+1+p-1} = 2^{2p}$ ]

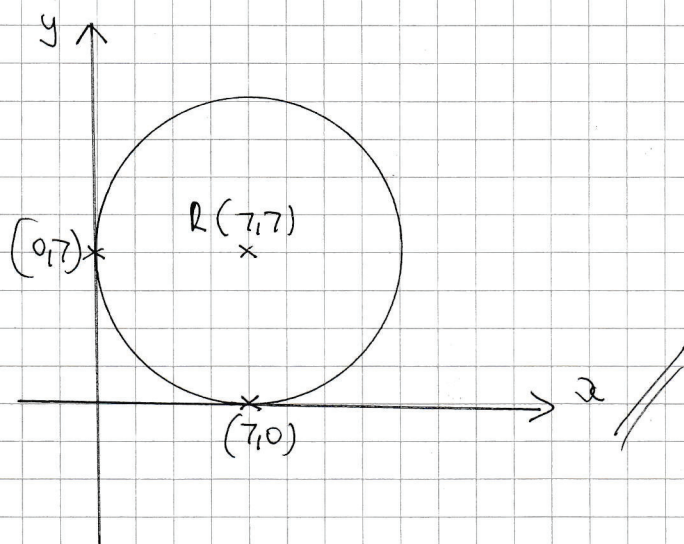
# YGB - SYNOPTIC PAPER K - QUESTION 5

a) COMPLETING THE SQUARE IN x & y

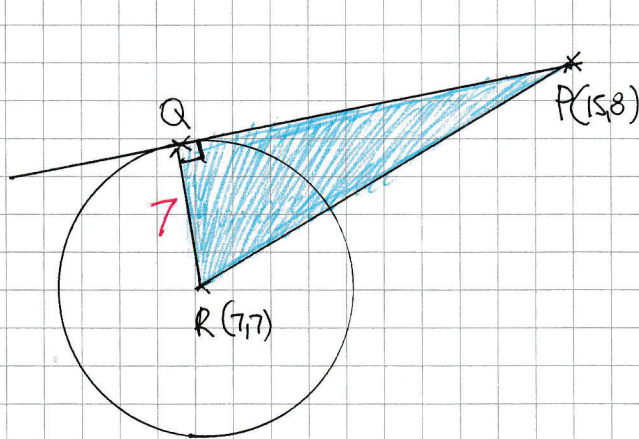
$$\begin{aligned} \Rightarrow x^2 + y^2 - 14x - 14y + 49 &= 0 \\ \Rightarrow x^2 - 14x + y^2 - 14y + 49 &= 0 \\ \Rightarrow (x-7)^2 - 49 + (y-7)^2 - 49 + 49 &= 0 \\ \Rightarrow (x-7)^2 + (y-7)^2 &= 49 \end{aligned}$$

$\therefore$  CENTRE AT  $R(7,7)$  & RADIUS IS 7

b) LOOKING AT THE INFORMATION ABOUT



c) LOOKING AT THE DIAGRAM BELOW - FIND THE LENGTH PR



$$\begin{aligned} \bullet |PR| &= \sqrt{(8-7)^2 + (15-7)^2} \\ |PR| &= \sqrt{1 + 64} \\ |PR| &= \sqrt{65} \end{aligned}$$

• BY PYTHAGORAS

$$\begin{aligned} |QR|^2 + |QP|^2 &= |PR|^2 \\ 7^2 + |QP|^2 &= (\sqrt{65})^2 \\ 49 + |QP|^2 &= 65 \\ |QP|^2 &= 16 \\ |QP| &= 4 \end{aligned}$$

-1-

IYGB - SYNOPTIC PAPER K - QUESTION 6

a) EXPAND BINOMIALLY UP TO  $a^3$

$$(1+ax)^n = 1 + \frac{n}{1}(ax)^1 + \frac{n(n-1)}{1 \times 2}(ax)^2 + \frac{n(n-1)(n-2)}{1 \times 2 \times 3}(ax)^3 + o(x^4)$$

$$(1+ax)^n = 1 + \underbrace{na}_-10x + \underbrace{\frac{1}{2}n(n-1)a^2}_{75}x^2 + \frac{1}{6}n(n-1)(n-2)a^3x^3 + o(x^4)$$

SOLVING SIMULTANEOUSLY

$$\left. \begin{matrix} na = -10 \\ \frac{1}{2}n(n-1)a^2 = 75 \end{matrix} \right\} \Rightarrow \left\{ \begin{matrix} na = -10 \\ n(n-1)a^2 = 150 \end{matrix} \right\} \quad \begin{matrix} \text{square} \\ \times n \end{matrix}$$

$$\Rightarrow \left\{ \begin{matrix} n^2a^2 = 100 \\ (n-1)n^2a^2 = 150n \end{matrix} \right\}$$

$$\Rightarrow 100(n-1) = 150n$$

$$\Rightarrow 100n - 100 = 150n$$

$$\Rightarrow -100 = 50n$$

$$\Rightarrow \underline{n = -2} \quad \& \quad \underline{\text{SINCE } na = -10}$$

$$\underline{a = 5}$$

b) SUBSTITUTING  $a=5$ ,  $n=-2$  INTO

$$\frac{1}{6}n(n-1)(n-2)a^3 = \frac{1}{6}(-2)(-3)(-4) \times 5^3 = \underline{-500}$$

c) THE EXPANSION IS VALID FOR  $|ax| < 1$

$$\Rightarrow |5x| < 1$$

$$\Rightarrow \underline{-\frac{1}{5} < x < \frac{1}{5}}$$



# YGB - SYNOPTIC PAPER 2 - QUESTION 7

STARTING WITH THE FORMAL DEFINITION OF THE DERIVATIVE

$$f'(x) = \lim_{h \rightarrow 0} \left[ \frac{f(x+h) - f(x)}{h} \right] \quad \text{with } f(x) = \sin x$$

$$f'(x) = \lim_{h \rightarrow 0} \left[ \frac{\sin(x+h) - \sin x}{h} \right]$$

USING COMPOUND ANGLE IDENTITIES

$$f'(x) = \lim_{h \rightarrow 0} \left[ \frac{\sin x \cosh + \cos x \sinh - \sin x}{h} \right]$$

USING SMALL ANGLE APPROXIMATIONS

$$\begin{aligned} \sinh &= h + O(h^3) \\ \cosh &= 1 + O(h^2) \end{aligned}$$

THUS WE OBTAIN:

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \left[ \frac{\sin x [1 + O(h^2)] + \cos x [h + O(h^3)] - \sin x}{h} \right] \\ &= \lim_{h \rightarrow 0} \left[ \frac{\cancel{\sin x} + O(h^2)\sin x + h \cos x + O(h^3)\cos x - \cancel{\sin x}}{h} \right] \\ &= \lim_{h \rightarrow 0} \left[ \frac{O(h^2)\sin x + h \cos x + O(h^3)\cos x}{h} \right] \\ &= \lim_{h \rightarrow 0} \left[ O(h)\sin x + \cos x + O(h^2)\cos x \right] \\ &= \cos x \end{aligned}$$

~~AS REQUIRED~~

# YGB - SYNOPTIC PAPER K - QUESTION 8

START BY DIFFERENTIATING USING THE PRODUCT RULE

$$\Rightarrow y = x(\ln x)^{\frac{1}{2}}$$

$$\Rightarrow \frac{dy}{dx} = 1 \times (\ln x)^{\frac{1}{2}} + x \times \frac{1}{2}(\ln x)^{-\frac{1}{2}} \times \frac{1}{x}$$

$$\Rightarrow \frac{dy}{dx} = (\ln x)^{\frac{1}{2}} + \frac{1}{2}(\ln x)^{-\frac{1}{2}}$$

$$\Rightarrow \frac{dy}{dx} = \sqrt{\ln x} + \frac{1}{2\sqrt{\ln x}}$$

Now we require  $\frac{dy}{dx} = \frac{3}{2}$

$$\Rightarrow \sqrt{\ln x} + \frac{1}{2\sqrt{\ln x}} = \frac{3}{2}$$

$$\Rightarrow A + \frac{1}{2A} = \frac{3}{2}$$

$$\Rightarrow 2A + \frac{1}{A} = 3$$

$$\Rightarrow 2A^2 + 1 = 3A$$

$$\Rightarrow 2A^2 - 3A + 1 = 0$$

$$\Rightarrow (2A - 1)(A - 1) = 0$$

$$\Rightarrow A = \begin{cases} \frac{1}{2} \\ 1 \end{cases}$$

$$\Rightarrow \sqrt{\ln x} = \begin{cases} \frac{1}{2} \\ 1 \end{cases}$$

$$\Rightarrow \ln x = \begin{cases} \frac{1}{4} \\ 1 \end{cases}$$

$$\Rightarrow x = \begin{cases} e^{\frac{1}{4}} \\ e \end{cases}$$

$A = \sqrt{\ln x}$

$$y = \begin{cases} e^{\frac{1}{4}} \times \frac{1}{2} \\ e \times 1 \end{cases}$$

$\therefore (e^{\frac{1}{4}}, \frac{1}{2}e^{\frac{1}{4}})$  &  $(e, e)$

1YGB - SYNOPTIC PAPER K - QUESTION 9

MODELLING AS FOLLOWS - LET THE "MIDDLE" TERM BE  $x$

$$\begin{array}{cccccc}
 u_{n-2} & u_{n-1} & u_n & u_{n+1} & u_{n+2} \\
 x-2d & x-d & x & x+d & x+2d
 \end{array}$$

THE ARITHMETIC MEAN IS 7

$$\Rightarrow \frac{(x-2d) + (x-d) + x + (x+d) + (x+2d)}{5} = 7$$

$$\Rightarrow \frac{5x}{5} = 7$$

$$\Rightarrow x = 7$$

NEXT THE ARITHMETIC MEAN OF THE SQUARES IS 67

$$\Rightarrow \frac{(x-2d)^2 + (x-d)^2 + x^2 + (x+d)^2 + (x+2d)^2}{5} = 67$$

$$\Rightarrow (7-2d)^2 + (7-d)^2 + 7^2 + (7+d)^2 + (7+2d)^2 = 67 \times 5$$

$$\Rightarrow 49 - 28d + 4d^2 + 49 - 14d + d^2 + 49 + 49 + 14d + d^2 + 49 + 28d + 4d^2 = 335$$

$$\Rightarrow 10d^2 + 49 \times 5 = 335$$

$$\Rightarrow 2d^2 + 49 = 67$$

$$\Rightarrow 2d^2 = 18$$

$$\Rightarrow d^2 = 9$$

$$\Rightarrow d = \pm 3$$

∴ IF  $d = 3$  THE NUMBERS ARE 1, 4, 7, 10, 13

(IF  $d = -3$  THE NUMBERS ARE 13, 10, 7, 4, 1)

-1-

# IYGB - SYNOPTIC PAPER K - QUESTION 10

PROCEED AS FOLLOWS

$$\begin{aligned} \frac{2^n}{2^{\sqrt{n}} \times 2^6} = 1 &\implies \frac{2^n}{2^{\sqrt{n}+6}} = 1 \\ &\implies 2^n = 2^{\sqrt{n}+6} \\ &\implies n = \sqrt{n} + 6 \end{aligned}$$

THIS IS A QUADRATIC IN  $\sqrt{n}$

$$\begin{aligned} &\implies (\sqrt{n})^2 - \sqrt{n} - 6 = 0 \\ &\implies (\sqrt{n} - 3)(\sqrt{n} + 2) = 0 \\ &\implies \sqrt{n} = \begin{cases} 3 \\ -2 \end{cases} \\ &\implies \underline{n = 9} \end{aligned}$$

# 1YGB - SYNOPTIC PAPER 2 - QUESTION 11

## METHOD A - BY COMPLETING THE SQUARE

$$\begin{aligned} \Rightarrow 5x^2 - 9x - 1 &= 0 \\ \Rightarrow x^2 - \frac{9}{5}x - \frac{1}{5} &= 0 \\ \Rightarrow x^2 - 1.8x - 0.2 &= 0 \\ \Rightarrow (x - 0.9)^2 - (0.9)^2 - 0.2 &= 0 \\ \Rightarrow (x - 0.9)^2 - 0.81 - 0.2 &= 0 \\ \Rightarrow (x - 0.9)^2 &= 1.01 \\ \Rightarrow x - 0.9 &= \pm \sqrt{1.01} \\ \Rightarrow x - 0.9 &\approx \begin{cases} 1 \\ -1 \end{cases} \\ \Rightarrow x &\approx \begin{cases} 1.9 \\ -0.1 \end{cases} \end{aligned}$$

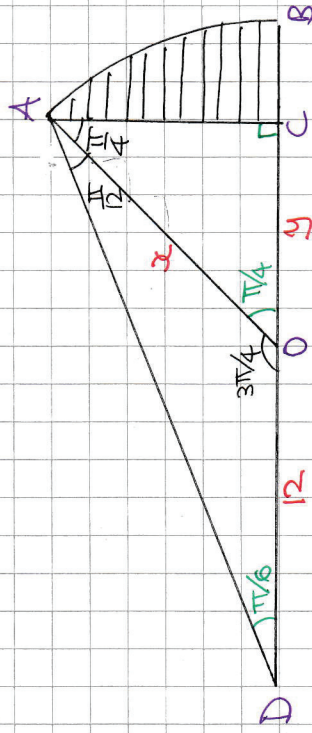
to 1 d.p.

## METHOD B - BY THE QUADRATIC FORMULA

$$\begin{aligned} 5x^2 - 9x - 1 &= 0 \\ x &= \frac{+9 \pm \sqrt{(-9)^2 - 4 \times 5 \times (-1)}}{2 \times 5} \\ x &= \frac{9 \pm \sqrt{81 + 20}}{10} \\ x &= \frac{9 \pm \sqrt{101}}{10} = \begin{cases} \frac{9 + \sqrt{101}}{10} \approx \frac{9 + 10}{10} = \frac{19}{10} = 1.9 \\ \frac{9 - \sqrt{101}}{10} \approx \frac{9 - 10}{10} = \frac{-1}{10} = -0.1 \end{cases} \end{aligned}$$

# 1YGB - SYNOPSIS PAPER K - QUESTION 12

a) STARTING WITH A DIAGRAM & OBTAIN SOME ANGLES



- $\hat{DOA} = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$  (straight line)
- $\hat{DAO} = \pi - (\frac{\pi}{6} + \frac{3\pi}{4}) = \frac{\pi}{12}$  (triangle  $\hat{DAO}$ )
- $\hat{OAC} = \pi - (\frac{\pi}{6} + \frac{\pi}{4}) = \frac{\pi}{4}$  (triangle  $\hat{OAC}$ )

BY THE SINE RULE ON  $\triangle AOD$

$$\frac{10A}{\sin \frac{\pi}{6}} = \frac{10D}{\sin \frac{\pi}{12}} \Rightarrow \frac{x}{\sin \frac{\pi}{6}} = \frac{12}{\sin \frac{\pi}{12}}$$

$$\Rightarrow x = \frac{12 \sin \frac{\pi}{6}}{\sin \frac{\pi}{12}}$$

$$\Rightarrow x = 23.18221983 \dots$$

ALFA OF SECTOR =  $\frac{1}{2} r^2 \theta$  =  $\frac{1}{2} 12^2 \times \frac{\pi}{4}$

$$= \frac{1}{2} (23.1822 \dots)^2 \times \frac{\pi}{4}$$

$$= 211.0$$

b) NEXT WORKING AT  $\triangle AOC$

$$\frac{10C}{10A} = \cos \frac{\pi}{4} \Rightarrow \frac{y}{x} = \cos \frac{\pi}{4}$$

$$\Rightarrow y = x \cos \frac{\pi}{4}$$

$$\Rightarrow y = (23.1822 \dots) \times \frac{\sqrt{2}}{2}$$

$$\Rightarrow y \approx 16.39230480 \dots$$

THE ALFA OF THE TRIANGLE  $\triangle AOC$  IS GIVEN BY

$$\text{ALFA} = \frac{1}{2} |OA| |OC| \sin \frac{\pi}{4}$$

$$= \frac{1}{2} x y \times \frac{\sqrt{2}}{2} = \frac{1}{2} (23.1822 \dots) (16.3923 \dots) \times \frac{\sqrt{2}}{2}$$

$$= 134.3538291 \dots$$

FINALLY THE SHADDED ALFA CAN BE FOUND

ALFA OF SECTOR - ALFA OF TRIANGLE

$$= 211.0 - 134.3538 \dots$$

$$= 76.69 \dots$$

$$\approx 77$$

~~77~~ AS REQUIRED

- 1 -

YGB - SYNOPTIC PAPER K - QUESTION 13

a) IF  $t=1$   
 $P(1,1)$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t}{-\frac{1}{t^2}} = -2t^3$$

$$\left. \frac{dy}{dx} \right|_{t=1} = -2$$

EQUATION OF TANGENT IS

$$y - y_0 = m(x - x_0)$$

$$y - 1 = -2(x - 1)$$

$$y - 1 = -2x + 2$$

$$\underline{y + 2x = 3}$$

AS REQUIRED

b) SOLVING SIMULTANEOUSLY

$$x = \frac{1}{t}, y = t^2 \quad \text{AND} \quad y + 2x = 3$$

$$\Rightarrow t^2 + 2\left(\frac{1}{t}\right) = 3$$

$$\Rightarrow t^2 + \frac{2}{t} = 3$$

$$\Rightarrow t^3 + 2 = 3t$$

$$\Rightarrow t^3 - 3t + 2 = 0$$

$$\Rightarrow (t-1)^2(t+2) = 0$$

$$\Rightarrow t = \begin{cases} 1 & \leftarrow P \\ -2 & \leftarrow Q \end{cases}$$

$\therefore Q\left(-\frac{1}{2}, 4\right)$

$t=1$  (REPEATED) MUST BE A SOLUTION FROM THE POINT OF TANGENCY

$$\begin{aligned} & (t+2)(t-1)^2 \\ &= (t+2)(t^2-2t+1) \\ &= t^3 - 2t^2 + t + 2t^2 - 4t + 2 \\ &= t^3 - 3t + 2 \end{aligned} \quad \left. \vphantom{\begin{aligned} & (t+2)(t-1)^2 \\ &= (t+2)(t^2-2t+1) \\ &= t^3 - 2t^2 + t + 2t^2 - 4t + 2 \\ &= t^3 - 3t + 2 \end{aligned}} \right\} \text{CHECK}$$

1YFB - SYNOPTIC PAPER K - QUESTION 14

a) DIFFERENTIATE WITH RESPECT TO x

$$\Rightarrow \frac{d}{dx}(x^2) + \frac{d}{dx}(2xy) - \frac{d}{dx}(3y^2) = \frac{d}{dx}(4x) + \frac{d}{dx}(4y) - \frac{d}{dx}(20)$$

$$\Rightarrow 2x + 2y + 2x \frac{dy}{dx} - 6y \frac{dy}{dx} = 4 + 4 \frac{dy}{dx} - 0$$

$$\Rightarrow 2x + 2y - 4 = 6y \frac{dy}{dx} - 2x \frac{dy}{dx} + 4 \frac{dy}{dx}$$

$$\Rightarrow 2x + 2y - 4 = (6y - 2x + 4) \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x + 2y - 4}{6y - 2x + 4}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x + y - 2}{3y - x + 2} \quad \text{★ REQUIRES$$

b) SOLVING  $\frac{dy}{dx} = 0$

$$\frac{x + y - 2}{3y - x + 2} = 0$$

$$x + y - 2 = 0$$

$$y = 2 - x$$

SUBSTITUTE INTO THE EQUATION OF THE CURVE

$$\Rightarrow x^2 + 2x(2-x) - 3(2-x)^2 = 4x + 4(2-x) - 20$$

$$\Rightarrow x^2 + \cancel{4x} - 2x^2 - \cancel{12} + 12x - 3x^2 = \cancel{4x} + \cancel{8} - 4x - \cancel{20}$$

$$\Rightarrow 0 = 4x^2 - 16x$$

$$\Rightarrow 0 = x^2 - 4x$$

$$\Rightarrow 0 = x(x-4)$$

$$x = \begin{cases} 0 \\ 4 \end{cases}$$

$$y = \begin{cases} 2 \\ -2 \end{cases}$$

$$\therefore \begin{matrix} (0, 2) \\ \& \\ (4, -2) \end{matrix}$$



1YGB-SYNOPTIC PAPER K - QUESTION 14

c) STARTING WITH

$$2x + 2y - 4 = (6y - 2x + 4) \frac{dy}{dx} \quad \Bigg) \div 2$$

$$x + y - 2 = (3y - x + 2) \frac{dy}{dx}$$

← PRODUCT RULE

DIFFERENTIATE AGAIN W.R.T x

$$1 + \frac{dy}{dx} - 0 = \left( 3 \frac{dy}{dx} - 1 + 0 \right) \frac{dy}{dx} + (3y - x + 2) \frac{d^2y}{dx^2}$$

$$1 + \frac{dy}{dx} = 3 \left( \frac{dy}{dx} \right)^2 - \frac{dy}{dx} + (3y - x + 2) \frac{d^2y}{dx^2}$$

$$(2 - 3y - 2) \frac{d^2y}{dx^2} - 3 \left( \frac{dy}{dx} \right)^2 + 2 \frac{dy}{dx} + 1 = 0$$

As Required

d) CHECKING (0,2) & NOTE  $\frac{dy}{dx} = 0$  AT (0,2)

$$(0 - 6 - 2) \frac{d^2y}{dx^2} + 1 = 0$$

$$\frac{d^2y}{dx^2} = \frac{1}{8} > 0$$

∴ (0,2) IS A LOCAL MIN

CHECKING (4,-2) &  $\frac{dy}{dx} = 0$  AT (4,-2)

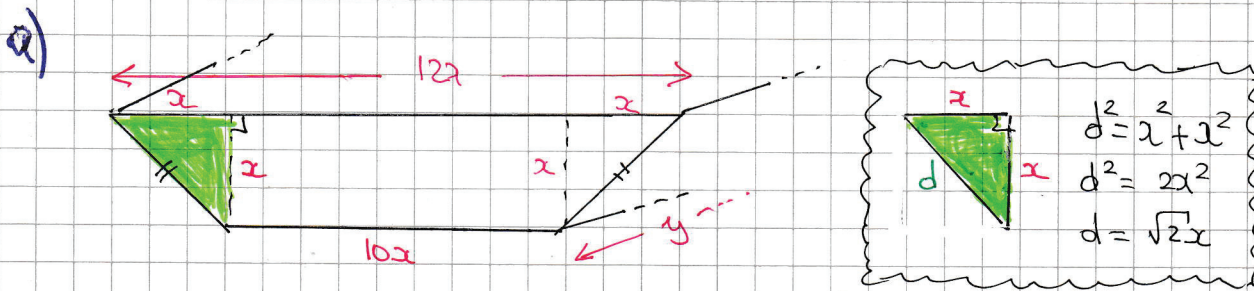
$$(4 + 6 - 2) \frac{d^2y}{dx^2} + 1 = 0$$

$$\frac{d^2y}{dx^2} = -\frac{1}{8} < 0$$

∴ (4,-2) IS A LOCAL MAX

- 1 -

## 1YGB - SYNOPSIS PAPER K - QUESTION 15



### THE TOTAL SURFACE AREA

$$\Rightarrow A = 10xy + 2 \left[ \frac{12x+10x}{2} x \right] + 2dy$$

$\uparrow$  FLOOR                       $\uparrow$  TRAPEZOIDAL SIDES                       $\uparrow$  SLOPING SIDES

$$\Rightarrow A = 10xy + 22x^2 + 2y(\sqrt{2}x)$$

$$\Rightarrow A = 22x^2 + 10xy + 2\sqrt{2}xy$$

$$\Rightarrow A = 22x^2 + (10 + 2\sqrt{2})xy$$

$$\Rightarrow A = 22x^2 + (10 + 2\sqrt{2}) \left( \frac{180}{x} \right)$$

$$\Rightarrow A = 22x^2 + \frac{360}{x} (5 + \sqrt{2})$$

AS REQUIRED

### CONSTRAINT ON CAPACITY

$$V = 1980$$

$$1980 = \left( \frac{12x+10x}{2} \right) x \times y$$

$\underbrace{\hspace{10em}}$  AREA OF TRAPEZIUM ABFE

$$11x^2y = 1980$$

$$x^2y = 180$$

$$x(xy) = 180$$

$$xy = \frac{180}{x}$$

b)

### DIFFERENTIATE THE ABOVE EQUATION & SETTING FOR ZERO

$$\Rightarrow A = 22x^2 + 360(5 + \sqrt{2})x^{-1}$$

$$\Rightarrow \frac{dA}{dx} = 44x - 360(5 + \sqrt{2})x^{-2}$$

IYGB - SYNOPTIC PAPER K - QUESTION 15

$$\Rightarrow \frac{dA}{dx} = 44x - \frac{360(5+\sqrt{2})}{x^2}$$

SOLVING FOR ZERO

$$\Rightarrow 0 = 44x - \frac{360(5+\sqrt{2})}{x^2}$$

$$\Rightarrow 0 = 44x^3 - 360(5+\sqrt{2})$$

$$\Rightarrow x^3 = \frac{90(5+\sqrt{2})}{11}$$

$$\Rightarrow x \approx 3.744$$

USING THE SECOND DERIVATIVE

$$\Rightarrow \frac{d^2A}{dx^2} = 44 + 720(5+\sqrt{2})x^{-3}$$

$$\Rightarrow \frac{d^2A}{dx^2} = 132 > 0$$

INDICED  $x \approx 3.744$  MINIMIZES A

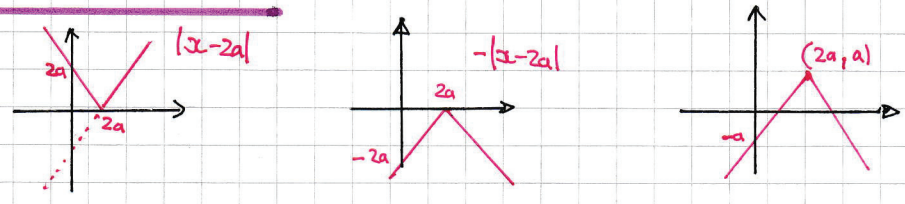
c)  $A = 22x^2 + \frac{360}{x}(5+\sqrt{2})$

$$\Rightarrow A_{\min} = 22(3.744)^2 + \frac{360}{3.744}(5+\sqrt{2})$$

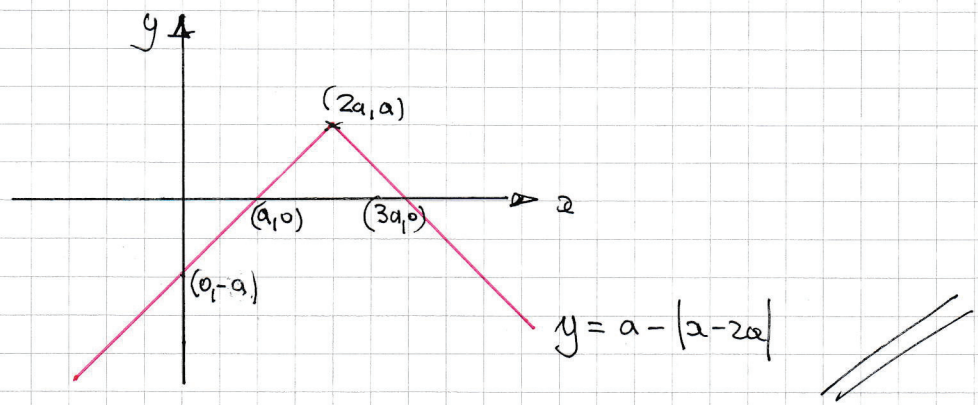
$$\Rightarrow A_{\min} \approx 925 \text{ m}^3$$

NYGB - SYNOPTIC PAPER K - QUESTION 16

a) BY TRANSFORMATIONS



HENCE WE CAN SKETCH (x CO-ORDINATES BY INSPECTION)



b) WORKING AT THE GRAPH ABOVE

$$\begin{aligned}
 \int_0^{3a} f(x) dx &= \text{area of triangle below x-axis} + \text{area of triangle above x-axis} \\
 &= -\frac{1}{2}a^2 + a^2 \\
 &= \frac{1}{2}a^2
 \end{aligned}$$

1YGB - SYNOPTIC PAPER K - QUESTION 17

a)  $f(x) = x^2 - 4x + 1 \quad x > 4$

$y = x^2 - 4x + 1$

$y = (x-2)^2 - 3$

$y+3 = (x-2)^2$

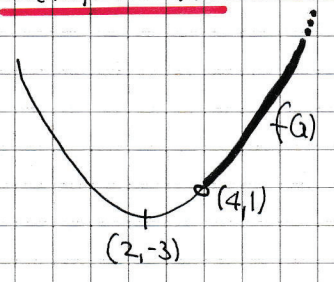
$x-2 = \pm \sqrt{y+3}$

$x-2 = + \sqrt{y+3} \quad (x > 4)$

$x = 2 + \sqrt{y+3}$

$f^{-1}(x) = 2 + \sqrt{x+3}$

b) Sketching  $f(x)$



	$f$	$f^{-1}$
D	$x > 4$	$x > 1$
R	$f(x) > 1$	$f^{-1}(x) > 4$

$\therefore$  DOMAIN of  $f^{-1}$ :  $x > 1$

RANGE of  $f^{-1}$ :  $f^{-1}(x) > 4$

c) USING THE FACT THAT  $f^{-1}(x) = f(x)$  CAN BE SOLVED AS  $f(x) = x$  OR  $f^{-1}(x) = x$  (IF IT IS EASIER) WE HAVE:

$\Rightarrow x^2 - 4x + 1 = x$   $f(x) = x$

$\Rightarrow x^2 - 5x + 1 = 0$

$\Rightarrow (x - \frac{5}{2})^2 - \frac{25}{4} + 1 = 0$

$\Rightarrow (x - \frac{5}{2})^2 = \frac{21}{4}$

$\Rightarrow (x - \frac{5}{2}) = \pm \frac{\sqrt{21}}{2}$

$\Rightarrow x = \frac{5}{2} \pm \frac{\sqrt{21}}{2}$

BUT AS  $x > 4$

$\Rightarrow x = \frac{5 + \sqrt{21}}{2}$

# MGB - SYNOPTIC PAPER K - QUESTION 18

## a) FORMING AN O.D.E

- IN:  $\frac{dv}{dt} = 50$
- OUT:  $\frac{dv}{dt} = -10h$
- NET:  $\frac{dv}{dt} = 50 - 10h$

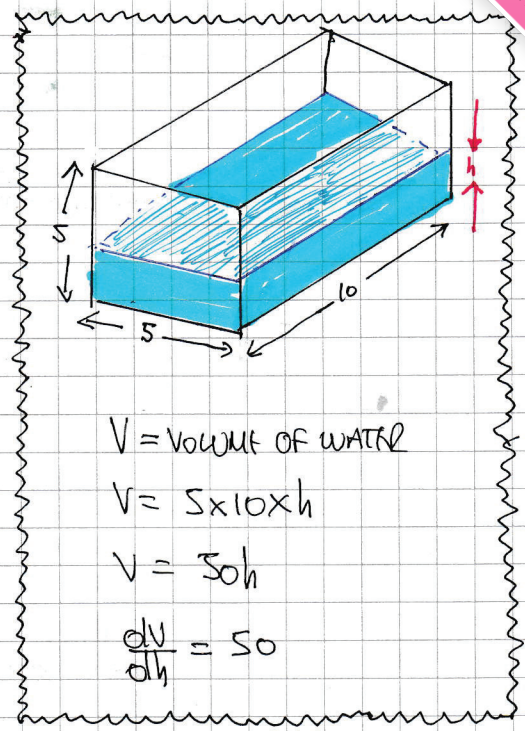
## RELATING V & h

$$\frac{dv}{dh} \times \frac{dh}{dt} = 50 - 10h$$

$$50 \times \frac{dh}{dt} = 50 - 10h$$

$$5 \frac{dh}{dt} = 5 - h$$

AS REQUIRED



## b) SOLVING BY SEPARATION OF VARIABLES

$$\Rightarrow 5 dh = (5 - h) dt$$

$$\Rightarrow \int \frac{1}{5-h} dh = \int \frac{1}{5} dt$$

$$\Rightarrow -\ln|5-h| = \frac{1}{5}t + C$$

$$\Rightarrow \ln|5-h| = -\frac{1}{5}t + C$$

$$\Rightarrow 5-h = e^{-\frac{1}{5}t + C}$$

$$\Rightarrow 5-h = e^{-\frac{1}{5}t} \times e^C$$

$$\Rightarrow 5-h = Ae^{-\frac{1}{5}t} \quad (A = e^C)$$

$$\Rightarrow 5 + Ae^{-\frac{1}{5}t} = h$$

1YCB - SYNOPSIS PAPER K - QUESTION 18

$$\Rightarrow h = 5 + Ae^{-\frac{1}{5}t}$$

When  $t=0$   $h=2$  (ARBITRARY START OF CLOCK)

$$2 = 5 + Ae^0$$

$$2 = 5 + A$$

$$A = -3$$

$$\therefore h = 5 - 3e^{-\frac{1}{5}t}$$

When  $h=4$

$$4 = 5 - 3e^{-\frac{1}{5}t}$$

$$3e^{-\frac{1}{5}t} = 1$$

$$e^{-\frac{1}{5}t} = \frac{1}{3}$$

$$e^{\frac{1}{5}t} = 3$$

$$\frac{1}{5}t = \ln 3$$

$$t = 5 \ln 3$$

YOB - SYNOPTIC PAPER K - QUESTION 19

Let  $\theta = \arctan\left(\frac{3}{x}\right)$  &  $\phi = \arctan\left(\frac{2x}{9}\right)$

$\Rightarrow 2\arctan\left(\frac{3}{x}\right) = \arctan\left(\frac{2x}{9}\right)$

$\Rightarrow 2\theta = \phi$

$\Rightarrow \tan 2\theta = \tan \phi$

$\Rightarrow \frac{2\tan\theta}{1-\tan^2\theta} = \tan\phi$

BUT IF  $\theta = \arctan\left(\frac{3}{x}\right) \Rightarrow \tan\theta = \frac{3}{x}$

$\phi = \arctan\left(\frac{2x}{9}\right) \Rightarrow \tan\phi = \frac{2x}{9}$

$\Rightarrow \frac{2\left(\frac{3}{x}\right)}{1-\left(\frac{3}{x}\right)^2} = \frac{2x}{9}$

$\Rightarrow \frac{\frac{6}{x}}{1-\frac{9}{x^2}} = \frac{2x}{9}$

$\Rightarrow \frac{6x}{x^2-9} = \frac{2x}{9}$

$\Rightarrow \frac{6}{x^2-9} = \frac{2}{9}$

$\Rightarrow 2(x^2-9) = 54$

$\Rightarrow x^2-9 = 27$

$\Rightarrow x^2 = 36$

$\Rightarrow x = \pm 6$

MULTIPLY "TOP" & "BOTTOM" OF THE DOUBLE FRACTION BY  $x^2$

AS  $x \neq 0$ , WE MAY DIVIDE BOTH SIDES BY 2



IYGB - SYNOPTIC PAPER K - QUESTION 20

TIDY USING RULES OF INDICES

$$\Rightarrow 4^{x+1} \times 3^{1-2x} = 24$$

$$\Rightarrow 4 \times 4^x \times 3 \times 3^{-2x} = 24$$

$$\Rightarrow 12 \times 4^x \times 3^{-2x} = 24$$

$$\Rightarrow 4^x \times 3^{-2x} = 2$$

$$\Rightarrow 4^x \times (3^2)^{-x} = 2$$

$$\Rightarrow 4^x \times 9^{-x} = 2$$

$$\Rightarrow \left(\frac{4}{9}\right)^x = 2$$

$$\Rightarrow \log\left(\frac{4}{9}\right)^x = \log 2$$

$$\Rightarrow x \log\left(\frac{4}{9}\right) = \log 2$$

$$\Rightarrow x = \frac{\log 2}{\log \frac{4}{9}} \approx -0.854$$

ALTERNATIVE, BY TAKING LOGS STRAIGHT AWAY

$$\Rightarrow 4^{x+1} \times 3^{1-2x} = 24$$

$$\Rightarrow \log(4^{x+1} \times 3^{1-2x}) = \log 24$$

$$\Rightarrow \log 4^{x+1} + \log 3^{1-2x} = \log 24$$

$$\Rightarrow (x+1)\log 4 + (1-2x)\log 3 = \log 24$$

$$\Rightarrow 2\log 4 + \log 4 = \log 3 - 2\log 3 = \log 24$$

$$\Rightarrow x \log 4 - 2\log 3 = \log 24 - \log 3 - \log 4$$

$$\Rightarrow x [\log 4 - 2\log 3] = \log\left(\frac{24}{3 \times 4}\right)$$

1YGB - SYNOPTIC PAPER K - QUESTION 20

$$\Rightarrow a(\log 4 - \log 9) = \log 2$$

$$\Rightarrow a = \frac{\log 2}{\log 4 - \log 9}$$

$$\Rightarrow \underline{a \approx -0.854}$$

As BWS/2f

-1-

IXGB - SYNOPTIC PART 2 - QUESTION 21

$A: \sqrt{x} + \sqrt{y} = 1$  (GIVEN)

● B IS A REFLECTION OF A ABOUT THE y AXIS

⇒ REPLACE  $x$  FOR  $-x$ .

⇒  $\sqrt{-x} + \sqrt{y} = 1$

● C IS A REFLECTION OF B ABOUT THE x AXIS

⇒ REPLACE  $y$  FOR  $-y$  IN THE ABOVE EQUATION

⇒  $\sqrt{-x} + \sqrt{-y} = 1$

● D IS A REFLECTION OF A IN THE x AXIS

⇒ REPLACE  $y$  FOR  $-y$  IN THE "A" EQUATION

⇒  $\sqrt{x} + \sqrt{-y} = 1$

1YGB - SYNOPTIC PAPER K - QUESTION 22

PROCEED BY SPLITTING THE FRACTION

$$\int_{\pi/6}^{\pi/4} \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x} dx = \int_{\pi/6}^{\pi/4} \frac{\sin^6 x}{\sin^2 x \cos^2 x} + \frac{\cos^6 x}{\sin^2 x \cos^2 x} dx$$

$$= \int_{\pi/6}^{\pi/4} \frac{\sin^4 x}{\cos^2 x} + \frac{\cos^4 x}{\sin^2 x} dx = \int_{\pi/6}^{\pi/4} \frac{(1 - \cos^2 x)^2}{\cos^2 x} + \frac{(1 - \sin^2 x)^2}{\sin^2 x} dx$$

EXPANDING & SPLIT THE FRACTIONS AGAIN

$$= \int_{\pi/6}^{\pi/4} \frac{1 - 2\cos^2 x + \cos^4 x}{\cos^2 x} + \frac{1 - 2\sin^2 x + \sin^4 x}{\sin^2 x} dx$$

$$= \int_{\pi/6}^{\pi/4} \sec^2 x - 2 + \cos^2 x + \operatorname{cosec}^2 x - 2 + \sin^2 x dx$$

$$= \int_{\pi/6}^{\pi/4} \sec^2 x + \operatorname{cosec}^2 x + (\cos^2 x + \sin^2 x) - 4 dx$$

$$= \int_{\pi/6}^{\pi/4} \sec^2 x + \operatorname{cosec}^2 x - 3 dx$$

$$= \left[ \tan x - \cot x - 3x \right]_{\pi/6}^{\pi/4}$$

- 2 -

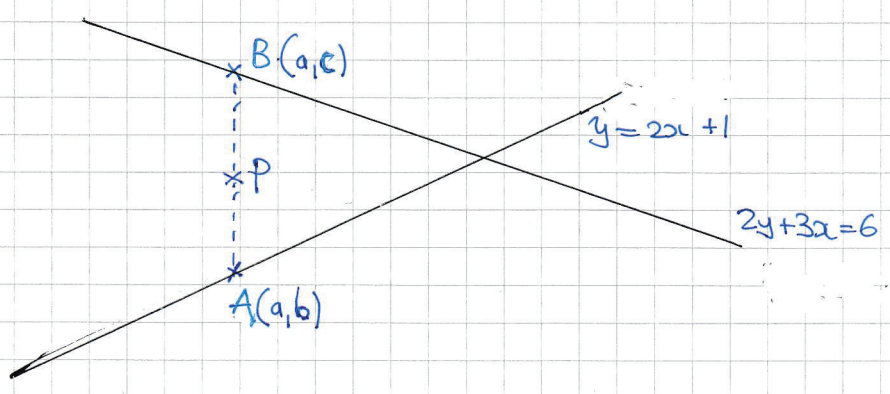
1YGB - SYNOPTIC PAPER K - QUESTION 22

TIDY BEFORE GAWATING

$$\begin{aligned}
 &= \left[ \tan x - \frac{1}{\tan x} - 3x \right]_{\frac{\pi}{8}}^{\frac{3\pi}{4}} \\
 &= \left[ \frac{\tan^2 x - 1}{\tan x} - 3x \right]_{\frac{\pi}{8}}^{\frac{3\pi}{4}} \\
 &= - \left[ 3x + \frac{1 - \tan^2 x}{\tan x} \right]_{\frac{\pi}{8}}^{\frac{3\pi}{4}} \\
 &= \left[ 3x + 2 \left( \frac{1 - \tan^2 x}{2 \tan x} \right) \right]_{\frac{\pi}{8}}^{\frac{3\pi}{4}} \\
 &= \left[ 3x + 2 \cot 2x \right]_{\frac{\pi}{8}}^{\frac{3\pi}{4}} \\
 &= \left[ 3x + \frac{2}{\tan 2x} \right]_{\frac{\pi}{8}}^{\frac{3\pi}{4}} \\
 &= \left( \frac{3\pi}{8} + \frac{2}{\tan \frac{\pi}{2}} \right) - \left( \frac{3\pi}{4} + \frac{2}{\tan \frac{\pi}{2}} \right) \\
 &= -\frac{3\pi}{8} + 2 \\
 &= \frac{1}{8} (16 - 3\pi)
 \end{aligned}$$

1YGB - SYNOPTIC PAPER 4 - QUESTION 23

STARTING WITH A DIAGRAM



EVIDENTLY P IS THE MIDPOINT OF AB

•  $A(a, b)$  LIES ON  $y = 2x + 1$   
 $\Rightarrow \underline{b = 2a + 1}$

•  $B(a, c)$  LIES ON  $2y + 3x = 6$   
 $\Rightarrow 2c + 3a = 6$   
 $\Rightarrow c + \frac{3}{2}a = 3$   
 $\Rightarrow \underline{c = 3 - \frac{3}{2}a}$

THUS P HAS y CO-ORDINATE

$$"y" = \frac{1}{2}(b+c) = \frac{1}{2} \left[ 2a+1 + 3 - \frac{3}{2}a \right] = \frac{1}{2} \left( \frac{1}{2}a + 4 \right) = \frac{1}{4}a + 2$$

THUS THE POINT P HAS GENERAL CO-ORDINATES  $(a, \frac{1}{4}a + 2)$

$$\Rightarrow \begin{cases} x = a \\ y = \frac{1}{4}a + 2 \end{cases}$$

$$\Rightarrow \underline{y = \frac{1}{4}x + 2}$$