

1YGB - SYNOPTIC PAPER G - QUESTION 1

THE POINT $P(3, k)$ LIES ON THE CURVE

$$y = x^2 + ax - 4$$
$$k = 3^2 + a \times 3 - 4$$
$$k = 9 + 3a - 4$$
$$k = 3a + 5$$

DIFFERENTIATE & USE THE FACT THAT $\frac{dy}{dx} \Big|_{x=3} = 3$

$$\frac{dy}{dx} = 2x + a$$
$$3 = 2 \times 3 + a$$
$$3 = 6 + a$$
$$a = -3$$

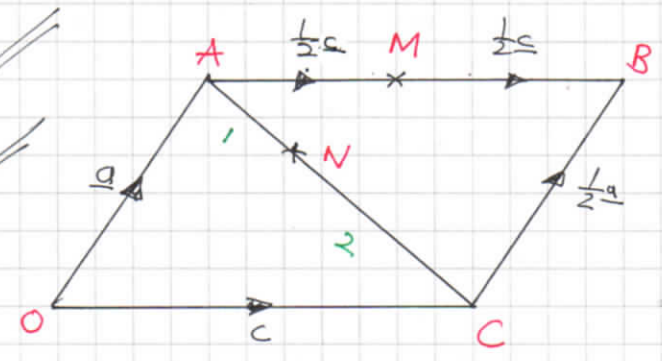
or using $k = 3a + 5$

$$k = 3(-3) + 5$$
$$k = -4$$

IYGB - SYNOPTIC PAPER G - QUESTION 2

a) START BY DRAWING A DIAGRAM, AND LABEL VECTORS

- $\vec{AC} = \vec{AO} + \vec{OC} = -\underline{a} + \underline{c} = \underline{c - a}$
- $\vec{AN} = \frac{1}{3}\vec{AC} = \frac{1}{3}(\underline{c - a}) = \underline{\frac{1}{3}c - \frac{1}{3}a}$
- $\vec{ON} = \vec{OA} + \vec{AN} = \underline{a} + \underline{\frac{1}{3}c - \frac{1}{3}a}$
 $= \underline{\frac{2}{3}a + \frac{1}{3}c}$
- $\vec{NM} = \vec{NA} + \vec{AM} = -\vec{AN} + \vec{AM}$
 $= -(\underline{\frac{1}{3}c - \frac{1}{3}a}) + \underline{\frac{1}{2}c} = \underline{\frac{1}{3}a + \frac{1}{6}c}$



b) PROVE AS FOLLOWS

$$\vec{ON} = \frac{2}{3}a + \frac{1}{3}c = \frac{1}{3}(2a + c)$$

$$\vec{NM} = \frac{1}{3}a + \frac{1}{6}c = \frac{1}{6}(2a + c)$$

AS \vec{ON} & \vec{NM} ARE IN THE SAME DIRECTION & SHARE THE POINT N, O, N & M MUST BE COLLINEAR

1YGB - SYNOPTIC PAPER G - QUESTION 3EXPAND AND COMPARE COEFFICIENTS ON BOTH SIDES

$$\Rightarrow 5x^2 + Ax + 7 \equiv B(x-2)^2 + C$$

$$\Rightarrow 5x^2 + Ax + 7 \equiv B(x^2 - 4x + 4) + C$$

$$\Rightarrow 5x^2 + Ax + 7 \equiv Bx^2 - 4Bx + 4B + C$$

$$\Rightarrow 5x^2 + Ax + 7 \equiv Bx^2 - 4Bx + (4B + C)$$

Hence we have

$$[x^2]: \quad B = 5$$

$$[x]: \quad A = -4B$$

$$A = -4 \times 5$$

$$A = -20$$

$$[x^0] \quad 4B + C = 7$$

$$4 \times 5 + C = 7$$

$$C = -13$$

1 YGB - SYNOPTIC PAPER G - QUESTION 4

a) $x^2 + y^2 - 6x + 14y + 33 = 0$

COMPLETE THE SQUARES IN x & IN y

$$\Rightarrow x^2 - 6x + y^2 + 14y + 33 = 0$$

$$\Rightarrow (x-3)^2 - 9 + (y+7)^2 - 49 + 33 = 0$$

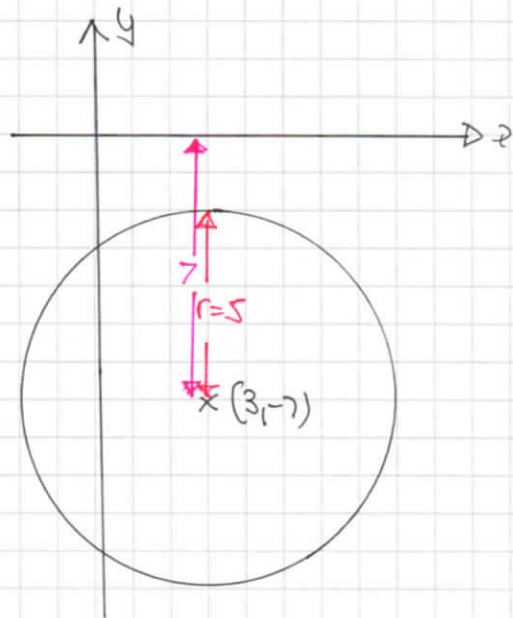
$$\Rightarrow (x-3)^2 + (y+7)^2 = 25$$

\therefore CENTRE AT $(3, -7)$ & RADIUS = $\sqrt{25} = 5$

b) REFERRING TO THE DIAGRAM

AS $r = 5 < 7$ (y COORDINATE OF THE CENTRE IS 7 UNITS BELOW THE x AXIS)

\therefore THE CIRCLE LIES ENTIRELY BELOW THE x AXIS



c) USING THE DISTANCE FORMULA FOR THE CENTRE $(3, -7)$ & $P(6, k)$

$$\Rightarrow \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2} > 5$$

$$\Rightarrow \sqrt{[k - (-7)]^2 + (6 - 3)^2} > 5$$

$$\Rightarrow \sqrt{(k+7)^2 + 3^2} > 5$$

$$\Rightarrow \sqrt{k^2 + 14k + 49 + 9} > 5$$

$$\Rightarrow \sqrt{k^2 + 14k + 58} > 5$$

1YGB - SYNOPTIC PAPER G - QUESTION 4

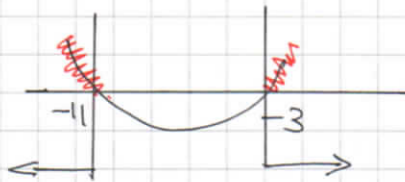
SQUARING BOTH SIDES

$$\Rightarrow k^2 + 14k + 58 > 25$$

$$\Rightarrow k^2 + 14k + 33 > 0$$

$$\Rightarrow (k + 3)(k + 11) > 0$$

$$c.v = \begin{cases} -3 \\ -11 \end{cases}$$



$$\therefore \underline{k < -11 \text{ OR } k > -3}$$

1YGB - SYNOPTIC PAPER G - QUESTION 5

-1-

a) APPROACH AS PER USUAL

$$2\cos\theta + 3\sin\theta \equiv R\cos(\theta - \alpha)$$

$$2\cos\theta + 3\sin\theta \equiv R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$$

$$\underline{2\cos\theta} + \underline{3\sin\theta} \equiv \underline{(R\cos\alpha)\cos\theta} + \underline{(R\sin\alpha)\sin\theta}$$

COMPARING

$$\begin{cases} R\cos\alpha = 2 \\ R\sin\alpha = 3 \end{cases}$$

SQUARE AND ADD

$$R = \sqrt{2^2 + 3^2}$$

$$R = \sqrt{13} \quad (R > 0)$$

DIVIDING SIDE BY SIDE

$$\frac{R\sin\alpha}{R\cos\alpha} = \frac{3}{2}$$

$$\tan\alpha = \frac{3}{2}$$

$$\alpha \approx 0.983^\circ$$

$$\therefore \underline{f(\theta) \approx \sqrt{13} \cos(\theta - 0.983^\circ)}$$

b) $f(\theta)_{\text{MAX}} = \underline{\underline{\sqrt{13}}}$

IT OCCURS WHEN $\cos(\theta - 0.893^\circ) = +1$

$$\theta - 0.893^\circ = 0$$

$$\underline{\underline{\theta = 0.893^\circ}}$$

c) USING PART (a) & PART (b)

$$T = g(t) = 16 + \sqrt{13} \cos\left(\frac{\pi t}{12} + 0.893^\circ\right)$$

$$T_{\text{MAX}} = 16 + \sqrt{13} \approx \underline{\underline{19.6^\circ\text{C}}}$$

& $\theta = 0.893$

$$\frac{\pi t}{12} = 0.893$$

$$\underline{\underline{t \approx 3.75 \text{ (hours)}}}$$

LYGB - SYNOPTIC PAPER G - QUESTION 5

d) finally we have, for T=17

$$\Rightarrow 17 = 16 + 2\cos\frac{\pi t}{12} + 3\sin\frac{\pi t}{12}$$

$$\Rightarrow 1 = \sqrt{13} \cos\left(\frac{\pi t}{12} - 0.893^\circ\right)$$

$$\Rightarrow \cos\left(\frac{\pi t}{12} - 0.893^\circ\right) = \frac{1}{\sqrt{13}}$$

$$\underline{\arccos\left(\frac{1}{\sqrt{13}}\right) = 1.28976\dots}$$

$$\Rightarrow \begin{cases} \frac{\pi t}{12} - 0.893 = 1.28976 \pm 2n\pi \\ \frac{\pi t}{12} - 0.893 = 4.99342 \pm 2n\pi \end{cases} \quad n=0,1,2,3,\dots$$

$$\Rightarrow \begin{cases} \frac{\pi t}{12} = 2.27255\dots \pm 2n\pi \\ \frac{\pi t}{12} = 5.97621\dots \pm 2n\pi \end{cases}$$

$$\Rightarrow \begin{cases} \pi t = 27.2706\dots \pm 24n\pi \\ \pi t = 71.7146\dots \pm 24n\pi \end{cases}$$

$$\Rightarrow \begin{cases} t = 8.6805\dots \pm 24n \\ t = 22.8275\dots \pm 24n \end{cases}$$

$$\therefore t = \begin{cases} 8.681 \approx 08:41 \\ 22.828 \approx 22:50 \end{cases}$$

$\swarrow 0.681 \times 60$
 $\swarrow 0.828 \times 60$

IYGB - SYNOPTIC PAPER G - QUESTION 6

PROCEED AS FOLLOWS

$$\tan 20^\circ = \tan(2 \times 10^\circ) =$$

$$t = \frac{2 \tan 10^\circ}{1 - \tan^2 10^\circ}$$

$$\tan 2\theta \equiv \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$t = \frac{2x}{1 - x^2}$$

where $x = \tan 10^\circ$

REARRANGING

$$\Rightarrow t(1 - x^2) = 2x$$

$$\Rightarrow t - tx^2 = 2x$$

$$\Rightarrow 0 = tx^2 + 2x - t$$

$$\Rightarrow x^2 + \frac{2}{t}x - 1 = 0$$

BY THE QUADRATIC FORMULA OR BY COMPLETING THE SQUARE

$$\Rightarrow \left(x + \frac{1}{t}\right)^2 - \left(\frac{1}{t}\right)^2 - 1 = 0$$

$$\Rightarrow \left(x + \frac{1}{t}\right)^2 = 1 + \frac{1}{t^2}$$

$$\Rightarrow \left(x + \frac{1}{t}\right)^2 = \frac{t^2 + 1}{t^2}$$

$$\Rightarrow x + \frac{1}{t} = \pm \sqrt{\frac{t^2 + 1}{t^2}}$$

$$\Rightarrow x = -\frac{1}{t} \pm \frac{\sqrt{t^2 + 1}}{t}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{t^2 + 1}}{t}$$

Now $-1 - \sqrt{t^2 + 1} < 0$

AND $x = \tan 10^\circ > 0$

$$\therefore x = \frac{-1 + \sqrt{t^2 + 1}}{t}$$

$$\therefore \tan 10^\circ = \frac{-1 + \sqrt{t^2 + 1}}{t}$$

AS REQUIRED

YGB - SYNOPTIC PAPER G - QUESTION 7

a) SUBSTITUTING INTO THE RECURRENCE RELATION

$$\Rightarrow a_{n+1} = p + q a_n$$

$$\Rightarrow a_2 = p + q a_1$$

$$\Rightarrow a_3 = p + q a_2$$

$$\therefore 220 = p + q \times 250$$

$$\therefore 196 = p + q \times 220$$

SOLVING SIMULTANEOUSLY

$$p + 250q = 220$$

$$p + 220q = 196$$

$$\Rightarrow 30q = 24$$

$$q = \frac{4}{5}$$

$$\Rightarrow p + 250 \times \frac{4}{5} = 220$$

$$p + 200 = 220$$

$$p = 20$$

b) LET THE REQUIRED UNIT BE L

$$\text{As } n \rightarrow \infty \quad a_n \approx a_{n+1} \rightarrow L$$

$$\Rightarrow a_{n+1} = 20 + \frac{4}{5} a_n$$

$$\Rightarrow L = 20 + \frac{4}{5} L$$

$$\Rightarrow 5L = 100 + 4L$$

$$\Rightarrow \underline{L = 100}$$

INDEED IT CONVERGES TO 100

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IVGB - SYNOPSIS PAPER G - QUESTION 8

$$y = \sqrt{x^2 + 16} \quad x \in \mathbb{R}$$

a) LET $y = f(x)$
THEN $y = f(4x) = \sqrt{(4x)^2 + 16} = \sqrt{16x^2 + 16} = \sqrt{16(x^2 + 1)}$
 $= \sqrt{16} \sqrt{x^2 + 1} = 4 \sqrt{x^2 + 1}$

∴ HORIZONTAL STRETCH, BY SCALE FACTOR OF $\frac{1}{4}$

b) LET THE TRANSLATION BY THE VECTOR $\begin{pmatrix} k \\ 0 \end{pmatrix}$ BE $y = f(x - k)$
 (NOTE THAT IF k IS NEGATIVE THEN THE TRANSLATION WILL BE TO THE 'LEFT')

$$y = f(x - k) = \sqrt{(x - k)^2 + 16}$$

NOW THIS CURVE PASSES THROUGH (6, 5)

$$\Rightarrow 5 = \sqrt{(6 - k)^2 + 16}$$

$$\Rightarrow 25 = (6 - k)^2 + 16$$

$$\Rightarrow 9 = (6 - k)^2$$

$$\Rightarrow 9 = (k - 6)^2$$

$$\Rightarrow k - 6 = \begin{matrix} 3 \\ -3 \end{matrix}$$

$$\Rightarrow k = \begin{matrix} 9 \\ 3 \end{matrix}$$

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YGB - SYNOPTIC PAPER G - QUESTION 9

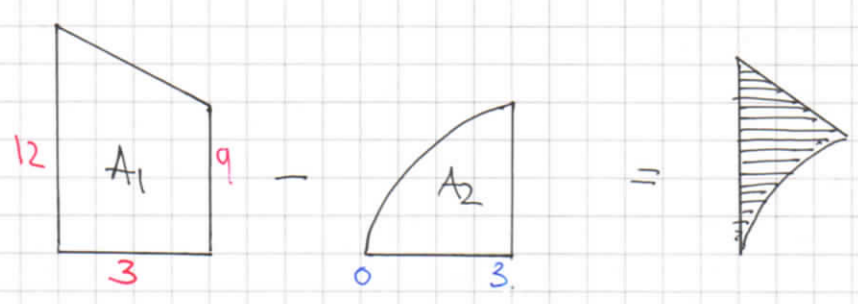
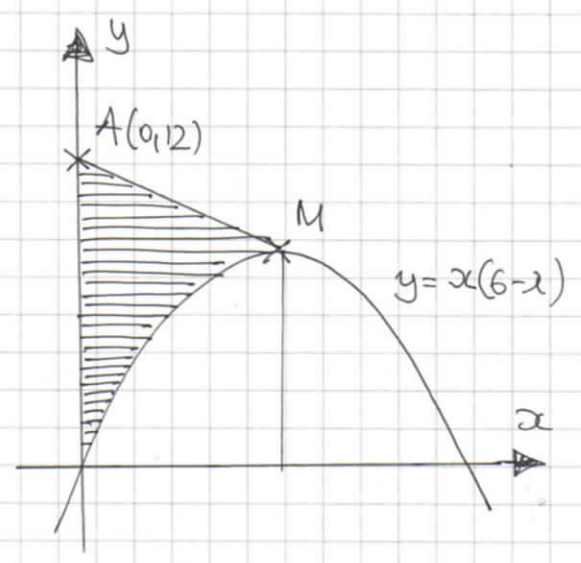
- BY INSPECTION THE CURVE MEETS THE X AXIS AT $x=0$ & $x=6$
- BY SYMMETRY THE MAXIMUM IS LOCATED AT $x=3$ (MIDPOINT)

$$y = x(6-x)$$

$$y = 3(6-3)$$

$$y = 9$$

\therefore $M(3,9)$



$$A_1 = \frac{9+12}{2} \times 3$$

$$A_1 = \frac{63}{2}$$

$$A_2 = \int_0^3 x(6-x) dx = \int_0^3 (6x - x^2) dx$$

$$= \left[3x^2 - \frac{1}{3}x^3 \right]_0^3 = (27 - 9) - (0)$$

$$= \underline{18}$$

- REQUIRED AREA IS $A_1 - A_2$

$$\frac{63}{2} - 18 = \underline{\underline{\frac{27}{2}}}$$

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1YGB - SYNOPTIC PAPER G - QUESTION 10

EXPAND UP TO x^2 IN ASCENDING POWERS OF x

$$f(x) = (2-3x)^2 (1+4x)^7$$

$$f(x) = (4-12x+9x^2) \left[1 + \frac{7}{1}(4x)^1 + \frac{7 \times 6}{1 \times 2}(4x)^2 + \dots \right]$$

$$f(x) = (4-12x+9x^2) (1 + 28x + 336x^2 + \dots)$$

$9x^2$
 $-336x^2$
 $+1344x^2$

REQUIRES COEFFICIENT IS

$$9 - 336 + 1344 = \underline{1017}$$

YGB - SYNOPTIC PAPER G - QUESTION 11

SOLVING THE EQUATIONS SIMULTANEOUSLY

$$\left. \begin{array}{l} y = 7^x \\ y = 2 \times 5^x \end{array} \right\} \Rightarrow 7^x = 2 \times 5^x$$

TAKING LOGARITHMS BASE 2

$$\Rightarrow \log_2 7^x = \log_2 (2 \times 5^x)$$

$$\Rightarrow x \log_2 7 = \log_2 2 + \log_2 5^x$$

$$\Rightarrow x \log_2 7 = 1 + x \log_2 5$$

$$\Rightarrow x \log_2 7 - x \log_2 5 = 1$$

$$\Rightarrow x [\log_2 7 - \log_2 5] = 1$$

$$\Rightarrow x = \frac{1}{\log_2 7 - \log_2 5} \quad \text{AS REQUIRED}$$

1YGB - SYNOPTIC PAPER G - QUESTION 12

a) BY CONSIDERING GRADIENTS

$$\text{GRADIENT AB} = \frac{4-0}{9-1} = \frac{4}{8} = \frac{1}{2}$$

$$\text{GRADIENT BC} = \frac{8-4}{k-9} = \frac{2}{k-9}$$

THESE GRADIENTS MUST BE NEGATIVE
RECIPROCAL OF EACH OTHER

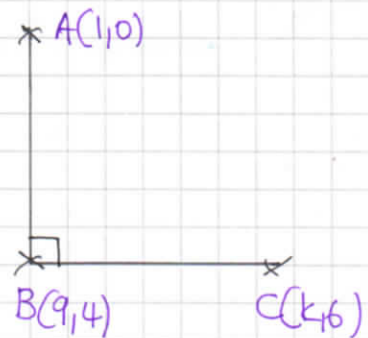
$$\Rightarrow \frac{2}{k-9} = -2 \leftarrow \text{NEGATIVE RECIPROCAL OF } \frac{1}{2}$$

$$\Rightarrow 2 = -2k + 18$$

$$\Rightarrow 2k = 16$$

$$\Rightarrow \underline{k = 8}$$

AS REQUIRED



b) USING PART (a), GRADIENT OF BC MUST BE -2 & PASSING THROUGH B(9,4)

$$y - y_0 = m(x - x_0)$$

$$y - 4 = -2(x - 9)$$

$$y - 4 = -2x + 18$$

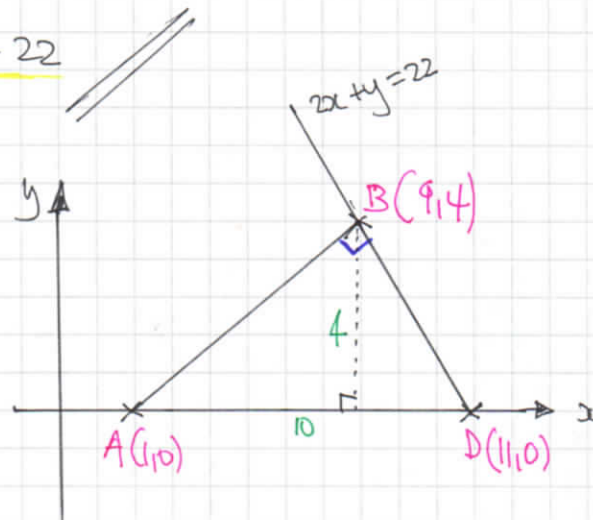
$$\underline{2x + y = 22}$$

c) WHEN $y=0$

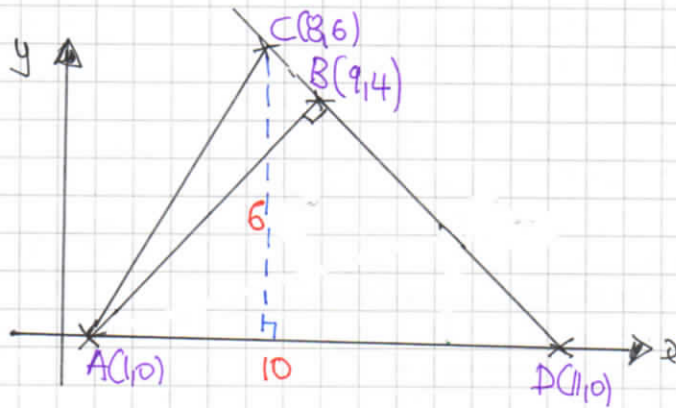
$$2x + 0 = 22$$

$$x = 11$$

$$\therefore D(11,0)$$



$$\text{REQUIRED AREA} = \frac{1}{2} \times 10 \times 4 = \underline{20}$$

1YGB - SYNOPTIC PAPER 6 - QUESTION 12d) LOOKING AT THE DIAGRAM BELOW

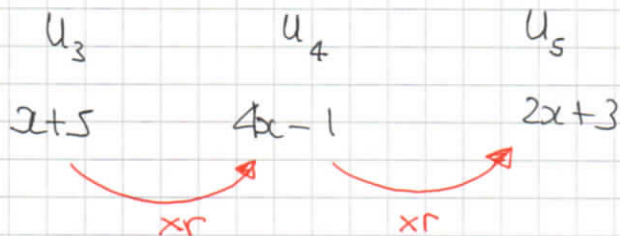
$$\text{AREA OF } \triangle ACD = \frac{1}{2} \times 10 \times 6 = 30$$

$$\text{AREA OF } \triangle ABD = 20 \text{ (PART C)}$$

$$\text{AREA OF } \triangle ABC = 30 - 20 = 10$$

1YGB - SYNOPTIC PAPER G - QUESTION 13

LOOKING AT THE "GEOMETRIC PATTERN"



FORMING TWO EQUATIONS

$$(2x+5)r = 4x-1$$

$$(4x-1)r = 2x+3$$

DIVIDE EQUATIONS TO ELIMINATE r

$$\frac{(2x+5)r}{(4x-1)r} = \frac{4x-1}{2x+3} \Rightarrow (4x-1)^2 = (2x+3)(2x+5)$$

$$\Rightarrow 16x^2 - 8x + 1 = 2x^2 + 13x + 15$$

$$\Rightarrow 14x^2 - 21x - 14 = 0$$

$$\Rightarrow 2x^2 - 3x - 2 = 0$$

$$\Rightarrow (2x+1)(x-2)$$

$$\Rightarrow x = < \frac{-1}{2}$$

USING EACH OF THE VALUES OF x FOUND

• IF $x = -\frac{1}{2}$

$$\begin{matrix} u_3 = \frac{9}{2} \\ u_4 = -3 \\ u_5 = 2 \end{matrix} \left. \begin{matrix} \downarrow \\ \downarrow \\ \downarrow \end{matrix} \right\} \begin{matrix} \times -\frac{2}{3} \\ \times -\frac{2}{3} \\ \times -\frac{2}{3} \end{matrix}$$

• IF $x = 2$

~~$$\begin{matrix} u_3 = 7 \\ u_4 = 7 \\ u_5 = 7 \end{matrix}$$~~

NO G.P.

1XGB - SYNOPTIC PAPER G - QUESTION 13

If $\lambda = -\frac{1}{2}$ we get $r = -\frac{2}{3}$, so using $u_n = ar^{n-1}$

$$\Rightarrow u_3 = \frac{9}{2}$$

$$\Rightarrow ar^2 = \frac{9}{2}$$

$$\Rightarrow a\left(-\frac{2}{3}\right)^2 = \frac{9}{2}$$

$$\Rightarrow \frac{4}{9}a = \frac{9}{2}$$

$$\Rightarrow a = \frac{81}{8}$$

FINALLY THE SUM TO INFINITY CAN BE FOUND

$$\Rightarrow \sum_{n=0}^{\infty} u_n = \frac{a}{1-r}$$

$$\Rightarrow \sum_{n=0}^{\infty} u_n = \frac{81/8}{1 - (-\frac{2}{3})}$$

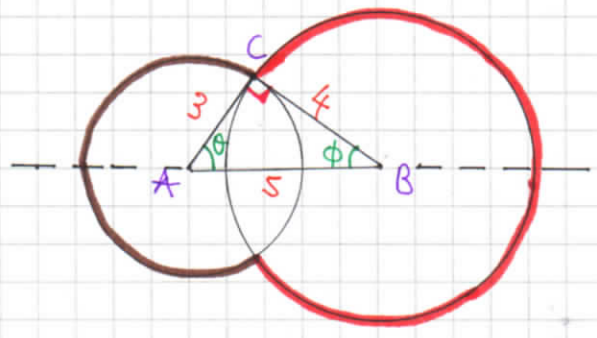
$$\Rightarrow \sum_{n=0}^{\infty} u_n = \frac{81/8}{5/3}$$

$$\Rightarrow \sum_{n=0}^{\infty} u_n = \frac{243}{40}$$

1YGB - SYNOPTIC PAPER Q - QUESTION 14

LOOKING AT THE DIAGRAM WE OBSERVE THAT $\triangle ABC$ IS RIGHT ANGLED AS A "3-4-5" TRIANGLE

$\bullet \sin \phi = \frac{3}{5} \quad \bullet \sin \theta = \frac{4}{5}$
 $\phi = 0.6435^\circ \quad \theta = 0.9273$



$\bullet 2\pi - 2\phi \quad \leftarrow \quad \text{Diagram of a circle with angle } 2\phi$
 $= 2\pi - 2 \times 0.6435$
 $= 4.9962^\circ$

$\bullet 2\pi - 2\theta \quad \leftarrow \quad \text{Diagram of a circle with angle } 2\theta$
 $= 2\pi - 2 \times 0.9273$
 $= 4.4286^\circ$

\bullet "Brown" ARC-LENGTH = " $r\theta$ " = $3 \times 4.4286 = 13.2857\dots$

\bullet "Red" ARC-LENGTH = " $r\theta$ " = $4 \times 4.962 = 19.9848\dots$

\therefore REQUIRES LENGTH IS $13.2857\dots + 19.9848\dots = 33.2705\dots$

\approx 33.3 cm

1YGB - SYNOPTIC PAPER G - QUESTION 15

USING THE SUBSTITUTION GIVEN

$$\Rightarrow u = 1 + \alpha e^{\sin x}$$

$$\Rightarrow \frac{du}{dx} = 1 \cdot e^{\sin x} + \alpha \cdot e^{\sin x} (\cos x)$$

$$\Rightarrow \frac{du}{dx} = e^{\sin x} + \alpha e^{\sin x} \cos x$$

$$\Rightarrow \frac{du}{dx} = e^{\sin x} (1 + \alpha \cos x)$$

$$\Rightarrow dx = \frac{du}{e^{\sin x} (1 + \alpha \cos x)}$$

CHANGING THE LIMITS

$$x=0 \quad \mapsto \quad u=1$$

$$x=\pi \quad \mapsto \quad u=1+\pi$$

TRANSFORMING THE INTEGRAL

$$\int_0^{\pi} \frac{1 + \alpha \cos x}{x + e^{-\sin x}} dx = \int_1^{1+\pi} \frac{\cancel{1 + \alpha \cos x}}{x + e^{-\sin x}} \times \frac{du}{e^{\sin x} \cancel{(1 + \alpha \cos x)}}$$

$$= \int_1^{1+\pi} \frac{1}{x e^{\sin x} + 1} du$$

$$= \int_1^{1+\pi} \frac{1}{u} du$$

$$= \left[\ln |u| \right]_1^{1+\pi}$$

$$= \ln(1+\pi) - \cancel{\ln 1}$$

$$= \underline{\ln(1+\pi)}$$

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1YGB - SYNOPTIC PAPER G - QUESTION 16

$$y = x(x-2)^3, x \in \mathbb{R}$$

● START BY DIFFERENTIATION (PRODUCT RULE)

$$\Rightarrow \frac{dy}{dx} = 1 \times (x-2)^3 + x \times 3(x-2)^2 \times 1$$

$$\Rightarrow \frac{dy}{dx} = (x-2)^3 + 3x(x-2)^2$$

$$\Rightarrow \frac{dy}{dx} = (x-2)^2 [(x-2) + 3x]$$

$$\Rightarrow \frac{dy}{dx} = (4x-2)(x-2)^2$$

$$\Rightarrow \frac{dy}{dx} = 2(2x-1)(x-2)^2$$

● NOW BY INSPECTION

$$\text{IF } x=3 \quad \frac{dy}{dx} = 2 \times 5 \times 1^2 = 10$$

● EXPAND THE GRADIENT FUNCTION

$$\Rightarrow \frac{dy}{dx} = 2(2x-1)(x^2-4x+4)$$

$$\Rightarrow \frac{dy}{dx} = 2(2x^3 - 8x^2 + 8x - x^2 + 4x - 4)$$

$$\Rightarrow \frac{dy}{dx} = 2(2x^3 - 9x^2 + 12x - 4)$$

● SETTING EQUAL TO 10, NOTING THAT (x-3) WILL BE A FACTOR OF THE RESULTING POLYNOMIAL

$$\Rightarrow 2(2x^3 - 9x^2 + 12x - 4) = 10$$

$$\Rightarrow 2x^3 - 9x^2 + 12x - 4 = 5$$

1Y0B - SYNOPTIC PAPER A - QUESTION 16

$$\Rightarrow 2x^3 - 9x^2 + 12x - 9 = 0$$

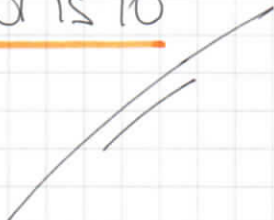
$$\Rightarrow 2x^2(x-3) - 3x(x-3) + 3(x-3) = 0$$

(OR LONG DIVISION INSTEAD)

$$\Rightarrow (x-3)(2x^2 - 3x + 3) = 0$$

$$\begin{aligned} b^2 - 4ac &= (-3)^2 - 4 \times 2 \times 3 \\ &= 9 - 24 \\ &= -15 < 0 \end{aligned}$$

- ONLY SOLUTION IS $x=3$, THENCE THERE IS ONLY ONE POINT ON THE CURVE, WHERE THE GRADIENT IS 10



TYGB - SYNOPTIC PAPER G - QUESTION 17

FORM A DIFFERENTIAL EQUATION

$$\frac{dx}{dt} = k \frac{1}{x}$$

↑ RATE
↑ PROPORTIONALITY CONSTANT
↑ INVERSELY PROPORTIONAL TO x

x = DISTANCE FROM 0
 t = TIME

 $t=0, x=50$
 $t=4, x=30$

SOLVING BY SEPARATING VARIABLES

$$\begin{aligned} \Rightarrow dx &= \frac{k}{x} dt \\ \Rightarrow x dx &= k dt \\ \Rightarrow \int x dx &= \int k dt \\ \Rightarrow \frac{1}{2}x^2 &= kt + C \\ \Rightarrow x^2 &= At + B \end{aligned}$$

APPLY CONDITIONS TO FIND THE CONSTANTS

- $t=0, x=50 \Rightarrow 50^2 = B$
 $\Rightarrow B = 2500$
 $\Rightarrow \underline{\underline{x^2 = At + 2500}}$

1YGB - SYNOPTIC PAPER G - QUESTION 17

● $t=4, x=30 \Rightarrow 30^2 = A \times 4 + 2500$
 $\Rightarrow 900 = 4A + 2500$
 $\Rightarrow 4A = -1600$
 $\Rightarrow A = -400$
 $\Rightarrow \underline{\underline{x^2 = -400t + 2500}}$

● $\text{min } x=0 \Rightarrow 0^2 = -400t + 2500$
 $\Rightarrow 400t = 2500$
 $\Rightarrow 4t = 25$
 $\Rightarrow \underline{\underline{t = 6.25}}$

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IGB-SYNOPTIC PAPER 6 - QUESTION 18

a) DIFFERENTIATE THE EQUATION WITH RESPECT TO x

$$\Rightarrow 2x^2 + xy - y^2 - 4x - y + 20 = 0$$

$$\Rightarrow \frac{d}{dx}(2x^2) + \frac{d}{dx}(xy) - \frac{d}{dx}(y^2) - \frac{d}{dx}(4x) - \frac{d}{dx}(y) + \frac{d}{dx}(20) = \frac{d}{dx}(0)$$

$$\Rightarrow 4x + y + x \frac{dy}{dx} - 2y \frac{dy}{dx} - 4 - \frac{dy}{dx} + 0 = 0$$

$$\Rightarrow (x - 2y - 1) \frac{dy}{dx} = -4x - y + 4$$

$$\Rightarrow \frac{dy}{dx} = \frac{-4x - y + 4}{x - 2y - 1} \quad \text{MULTIPLY "TOP & BOTTOM" BY -1}$$

$$\Rightarrow \frac{dy}{dx} = \frac{4x + y - 4}{2y - x + 1} \quad \text{AS EQUIVANT}$$

b) SOWING $\frac{dy}{dx} = 0 \Rightarrow 4x + y - 4 = 0$
 $\Rightarrow y = 4 - 4x$

SUBSTITUTE INTO THE EQUATION OF THE CURVE

$$\Rightarrow 2x^2 + x(4 - 4x) - (4 - 4x)^2 - 4x - (4 - 4x) + 20 = 0$$

$$\Rightarrow 2x^2 + 4x - 4x^2 - (16 - 32x + 16x^2) - 4x - 4 + 4x + 20 = 0$$

$$\Rightarrow 2x^2 + 4x - 4x^2 - 16 + 32x - 16x^2 - 4 + 20 = 0$$

$$\Rightarrow -18x^2 + 36x = 0$$

$$\Rightarrow -18x(x - 2) = 0$$

$$x = \begin{cases} 0 \\ 2 \end{cases} \quad y = \begin{cases} 4 - 4 \times 0 = 4 \\ 4 - 4 \times 2 = -4 \end{cases}$$

\therefore $(0, 4)$ & $(2, -4)$

1YGB - SYNOPTIC PAPER G - QUESTION 18

c) STARTING FROM

$$\Rightarrow 4x + y + x \frac{dy}{dx} - 2y \frac{dy}{dx} - 4 - \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{d}{dx}(4x) + \frac{d}{dx}(y) + \frac{d}{dx}\left(x \frac{dy}{dx}\right) - 2 \frac{d}{dx}\left(y \frac{dy}{dx}\right) - \frac{d}{dx}(4) - \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(0)$$

$$\Rightarrow 4 + \frac{dy}{dx} + \left[1 \frac{dy}{dx} + x \frac{d^2y}{dx^2}\right] - 2 \left[\frac{dy}{dx} \frac{dy}{dx} + y \frac{d^2y}{dx^2}\right] - 0 - \frac{d^2y}{dx^2} = 0$$

$$\Rightarrow 4 + \frac{dy}{dx} + \frac{dy}{dx} + x \frac{d^2y}{dx^2} - 2 \left(\frac{dy}{dx}\right)^2 - 2y \frac{d^2y}{dx^2} - \frac{d^2y}{dx^2} = 0$$

$$\Rightarrow \underline{4 + 2 \frac{dy}{dx} - 2 \left(\frac{dy}{dx}\right)^2 + (x - 2y - 1) \frac{d^2y}{dx^2} = 0}$$

As required

d) Checking (0, 4), $\frac{dy}{dx} = 0$

$$4 + 0 - 0 + (0 - 8 - 1) \frac{d^2y}{dx^2} = 0$$

$$4 = -9 \frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2} = \frac{4}{9} > 0$$

\therefore (0, 4) is a LOCAL MIN

Checking (2, -4) $\frac{dy}{dx} = 0$

$$4 + 0 - 0 + (2 + 8 - 1) \frac{d^2y}{dx^2} = 0$$

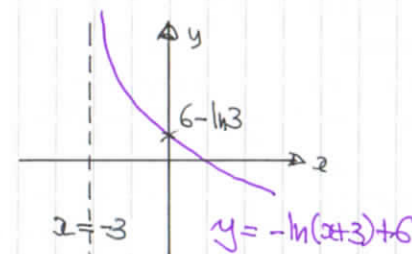
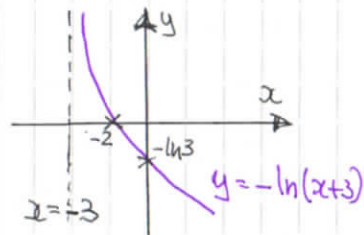
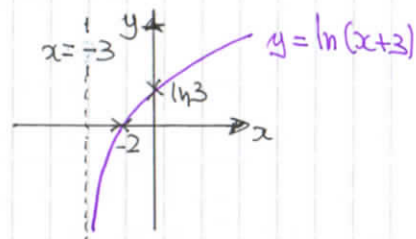
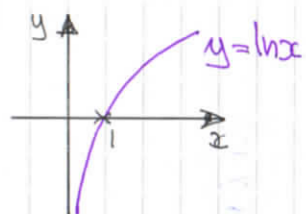
$$9 \frac{d^2y}{dx^2} = -4$$

$$\frac{d^2y}{dx^2} = -\frac{4}{9} < 0$$

\therefore (2, -4) is a LOCAL MAX

1YGB - SYNOPTIC PAPER G - QUESTION 19

a) SKETCHING & DESCRIBING, STAGE BY STAGE



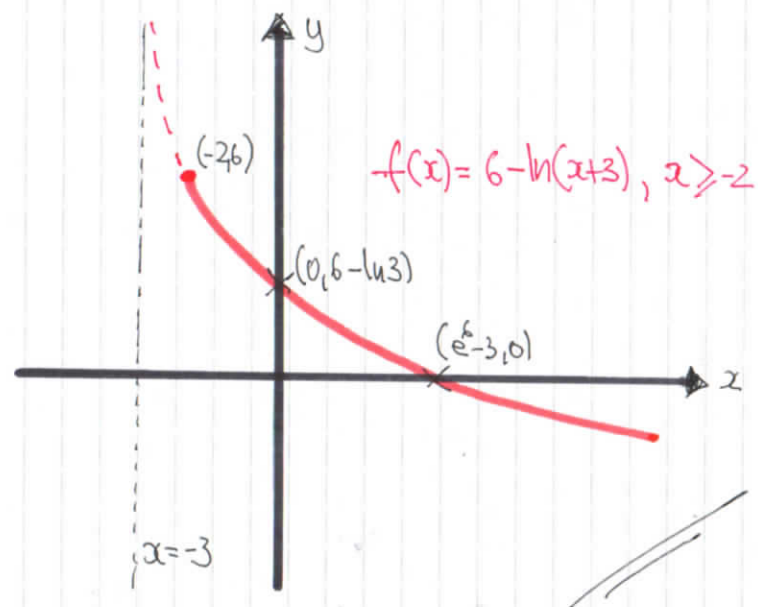
T_1 : TRANSLATION, "LEFT", 3 UNITS

T_2 : REFLECTION, ABOUT THE x AXIS

T_3 : TRANSLATION, "UP", 6 UNITS

SKETCHING THE GRAPH OF $f: x \mapsto 6 - \ln(x+3)$ FOR ITS GIVEN DOMAIN

- $x = -2$ $y = 6 - \ln t$ $t \in (-2, 6)$
- $y = 0$ $0 = 6 - \ln(x+3)$
 $\ln(x+3) = 6$
 $x+3 = e^6$
 $x = e^6 - 3$ $t \in (e^6 - 3, 0)$



-2-

1YGB - SYNOPTIC PAPER G - QUESTION 19

b) Let $y = 6 - \ln(x+3)$, for simplicity

$$\Rightarrow y = 6 - \ln(x+3)$$

$$\Rightarrow \ln(x+3) = 6 - y$$

$$\Rightarrow x+3 = e^{6-y}$$

$$\Rightarrow x = e^{6-y} - 3$$

$$\therefore f^{-1}(x) = e^{6-x} - 3$$

	f	f^{-1}
D	$x \geq -2$	$x \leq 6$
R	$f(x) \leq 6$	$f^{-1}(x) \geq -2$

\therefore DOMAIN: $x \leq 6$

RANGE: $f^{-1}(x) \geq -2$

c) FINALLY THE COMPOSITION

$$f(g(x)) = f(e^{x^2} - 3)$$

$$= 6 - \ln[(e^{x^2} - 3) + 3]$$

$$= 6 - \ln(e^{x^2})$$

$$= \underline{6 - x^2}$$

-2-

LYGB - SYNOPSIS PAPER 5 - QUESTION 19

b) Let $y = 6 - \ln(x+3)$, for simplicity

$$\Rightarrow y = 6 - \ln(x+3)$$

$$\Rightarrow \ln(x+3) = 6 - y$$

$$\Rightarrow x+3 = e^{6-y}$$

$$\Rightarrow x = e^{6-y} - 3$$

$$\therefore f^{-1}(x) = e^{6-x} - 3$$

c) Find the composition

$$\begin{aligned} f(g(x)) &= f(e^{x^2} - 3) \\ &= 6 - \ln[(e^{x^2} - 3) + 3] \\ &= 6 - \ln(e^{x^2}) \\ &= 6 - x^2 \end{aligned}$$

D	f	f
R	$x \geq -2$	$x \leq 6$
	$f(x) \leq 6$	$f(x) \geq 2$

\therefore DOMAIN: $x \leq 6$
RANGE: $f(x) \geq -2$

Y6B - SYNOPTIC PAPER G - QUESTION 20

a) OBTAIN THE GRADIENT FUNCTION

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2 - 4}{2}$$

AT THE POINT A(2,4) THE VALUE OF t = -1 SINCE $2t + 4 = 2$
 $2t = -2$
 $t = -1$

GRADIENT AT A(2,4)

$$\frac{dy}{dx} \Big|_{t=-1} = \frac{3(-1)^2 - 4}{2} = -\frac{1}{2}$$

EQUATION OF TANGENT IS GIVEN BY

$$y - y_0 = m(x - x_0)$$

$$y - 4 = -\frac{1}{2}(x - 2)$$

$$2y - 8 = -x + 2$$

$$x + 2y = 10$$

AS REQUIRED

b) SOLVING SIMULTANEOUSLY THE EQUATION OF THE TANGENT AND THE EQUATION OF THE CURVE IN PARAMETRIC

$$\Rightarrow x + 2y = 10$$

$$\Rightarrow (2t + 4) + 2(t^3 - 4t + 1) = 10$$

$$\Rightarrow 2t + 4 + 2t^3 - 8t + 2 = 10$$

$$\Rightarrow 2t^3 - 6t - 4 = 0$$

$$\Rightarrow t^3 - 3t - 2 = 0$$

$$\Rightarrow (t + 1)^2(t - 2) = 0$$

POINT OF TANGENCY MUST BE A REPEATED SOLUTION

$$\therefore t = 2 \quad y = 1$$

$$B(8, 1)$$

← QUICK CHECK $(t - 2)(t^2 + 2t + 1)$
 $= t^3 + 2t^2 + t - 2t^2 - 4t - 2$
 $= t^3 - 3t - 2$

IYGB - SYNOPTIC PAPER G - QUESTION 21

STARTING WITH $\frac{dv}{dt} = 2t$, NOTING $8.1\ell = 8100\text{ cm}^3$ & 2 MINUTES = 120S

$$\Rightarrow dv = 2t dt$$

$$\Rightarrow \int_{v=8100}^v 1 dv = \int_{t=0}^{t=120} 2t dt$$

$$\Rightarrow [v]_{8100}^v = [t^2]_0^{120}$$

$$\Rightarrow v - 8100 = 120^2 - 0$$

$$\Rightarrow v = 14400 + 8100$$

$$\Rightarrow \underline{v = 22500}$$

NEXT WE HAVE

$$\Rightarrow \frac{dh}{dt} = \frac{dh}{dv} \times \frac{dv}{dt}$$

$$\Rightarrow \frac{dh}{dt} = \frac{1}{72h} \times 2t$$

$$\Rightarrow \boxed{\frac{dh}{dt} = \frac{t}{36h}}$$

$$\begin{aligned} V &= 36h^2 \\ \frac{dV}{dh} &= 72h \\ \frac{dh}{dV} &= \frac{1}{72h} \end{aligned}$$

WHEN $V = 22500$, $V = 36h^2$
 $22500 = 36h^2$
 $h^2 = 625$
 $\underline{h = 25}$

FINALLY WE HAVE

$$\left. \frac{dh}{dt} \right|_{\substack{t=120 \\ h=25}} = \frac{120}{36 \times 25} = \frac{2}{15} \approx \underline{0.133 \text{ cm s}^{-1}}$$