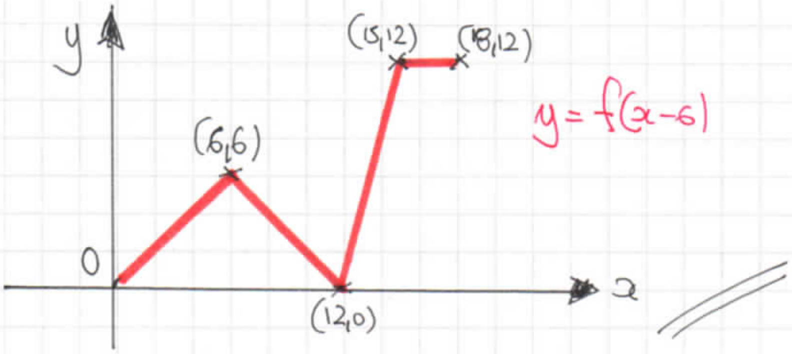
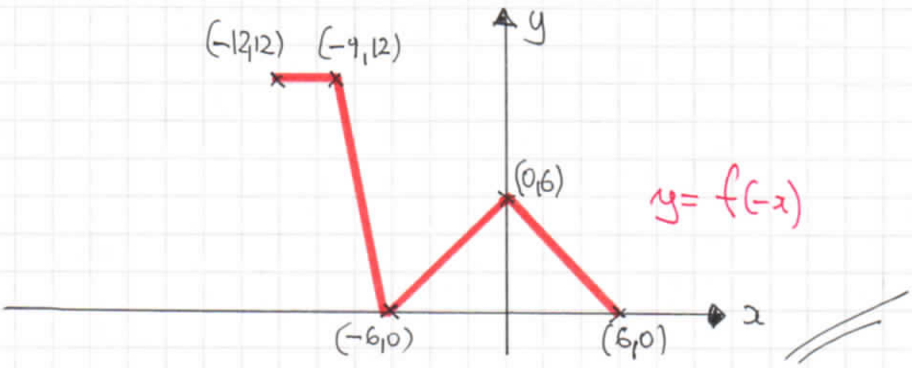
IYGB - SYNOPTIC PAPER E - QUESTION 1

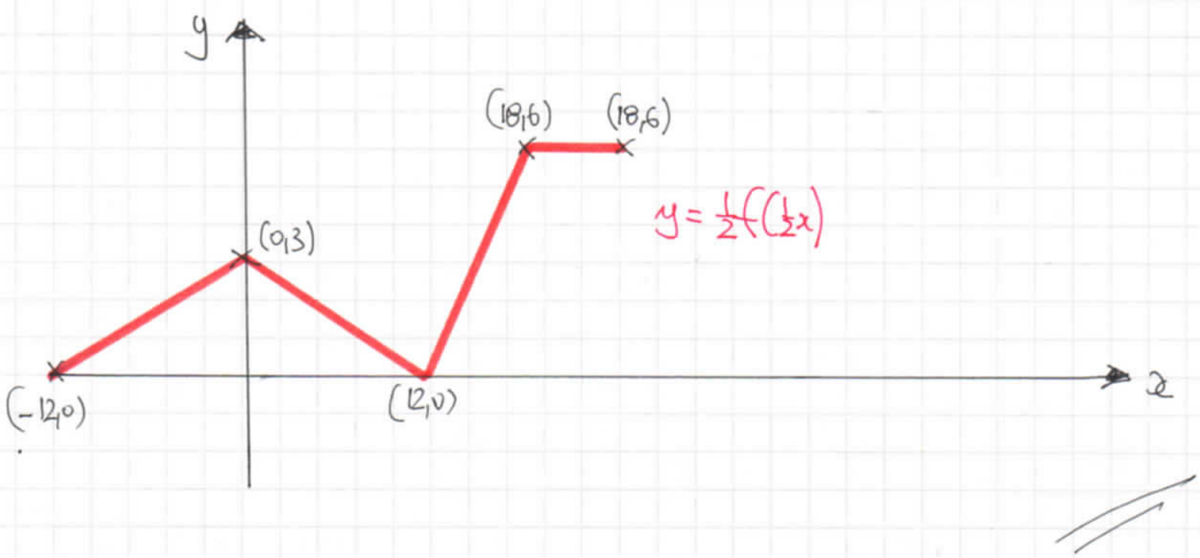
a) $y = f(x-6)$ REPRESENTS A TRANSLATION, 6 UNITS TO THE "RIGHT", IE $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$



b) $y = f(-x)$ REPRESENTS A REFLEXION IN THE y AXIS



c) $y = \frac{1}{2}f(\frac{1}{2}x)$ REPRESENTS A "VERTICAL" STRETCH BY SCALE FACTOR OF $\frac{1}{2}$, FOLLOWED BY A HORIZONTAL STRETCH BY SCALE FACTOR OF 2 (EITHER ORDER)



IYGB - SYNOPTIC PAPER E - QUESTION 2

$$\boxed{f(x) = x^3 + 3x^2 - 24x + 20}$$

a) BY THE FACTOR THEOREM

$$f(1) = 1^3 + 3\times1^2 - 24\times1 + 20$$
$$= 1 + 3 - 24 + 20$$
$$= 0$$

INDEED A FACTOR

b) BY LONG DIVISION, INSPECTION OR MANIPULATION

$$f(x) = x^2(x-1) + 4x(x-1) - 20(x-1)$$
$$f(x) = (x-1)(x^2+4x-20)$$

c) FROM PART (b)

EITHER $x=1$ OR

$$x^2+4x-20=0$$
$$(x+2)^2-4-20=0$$
$$(x+2)^2=24$$
$$x+2=\pm\sqrt{24}$$
$$x = \begin{cases} -2+2\sqrt{6} \\ -2-2\sqrt{6} \end{cases}$$

d) SOLVING THE EQUATION $f(x)=-8$, AND NOTING THAT $x=2$ MUST BE A REPEATED ROOT, IT $(x-2)^2$ MUST BE A FACTOR

$$\Longrightarrow x^3+3x^2-24x+20=-8$$
$$\Longrightarrow x^3+3x^2-24x+28=0$$
$$\Longrightarrow (x-2)^2(x+7)=0$$

$$x = \begin{cases} 2 & \leftarrow Q \\ -7 & \leftarrow P \end{cases}$$

QUICK CHECK

$$(x+7)(x-2)^2 = (x+7)(x^2-4x+4)$$
$$= x^3-4x^2+4x$$
$$+7x^2-28x+28$$
$$= x^3+3x^2-24x+28$$

FINALLY $f(-7) = (-7)^3+3(-7)^2-24(-7)+20$ ← JUST AS A CHECK

$$= -343+147+168+20$$
$$= -8$$

$\therefore P(-7,-8)$

IYGB - SYNOPTIC PAPER E - QUESTION 3

$$x = \ln(\sec 3y) \qquad 0 < y < \frac{\pi}{6}$$

PROCEED BY THE INVERSE RULE

$$\frac{dx}{dy} = \frac{1}{\cancel{\sec 3y}} \times \sec^{\cancel{2}} 3y \tan 3y \times 3$$

$$\frac{dx}{dy} = 3\tan 3y$$

$$\frac{dy}{dx} = \frac{1}{3\tan 3y}$$

NOW WE MANIPULATE THE EQUATION AS FOLLOWS

$$\Rightarrow x = \ln(\sec 3y)$$
$$\Rightarrow e^{x} = \sec 3y$$
$$\Rightarrow (e^{x})^2 = (\sec 3y)^2$$
$$\Rightarrow e^{2x} = \sec^2 3y$$
$$\Rightarrow e^{2x} = 1 + \tan^2 3y$$
$$\Rightarrow e^{2x} - 1 = \tan^2 3y$$
$$\Rightarrow \tan 3y = \pm\sqrt{e^{2x}-1}$$

NOW WE OBSERVE THAT

$$0 < y < \frac{\pi}{6}$$
$$0 < 3y < \pi/2$$
$$0 < \tan 3y < +\infty$$
$$\tan 3y > 0$$

$$\Rightarrow \tan 3y = +\sqrt{e^{2x}-1}$$

$$\therefore \frac{dy}{dx} = \frac{1}{3\sqrt{e^{2x}-1}}$$

IYGB - SYNOPTIC PAPER E - QUESTION 4

a) LINEARIZING THE EXPONENTIAL GRAPH AS FOLLOWS

$$\Rightarrow W = ab^t$$
$$\Rightarrow \log W = \log(ab^t)$$
$$\Rightarrow \log W = \log a + \log b^t$$
$$\Rightarrow \log W = \log a + t\log b$$
$$\Rightarrow \log W = (\log b)t + (\log a)$$
$$Y = mX + c$$

b)

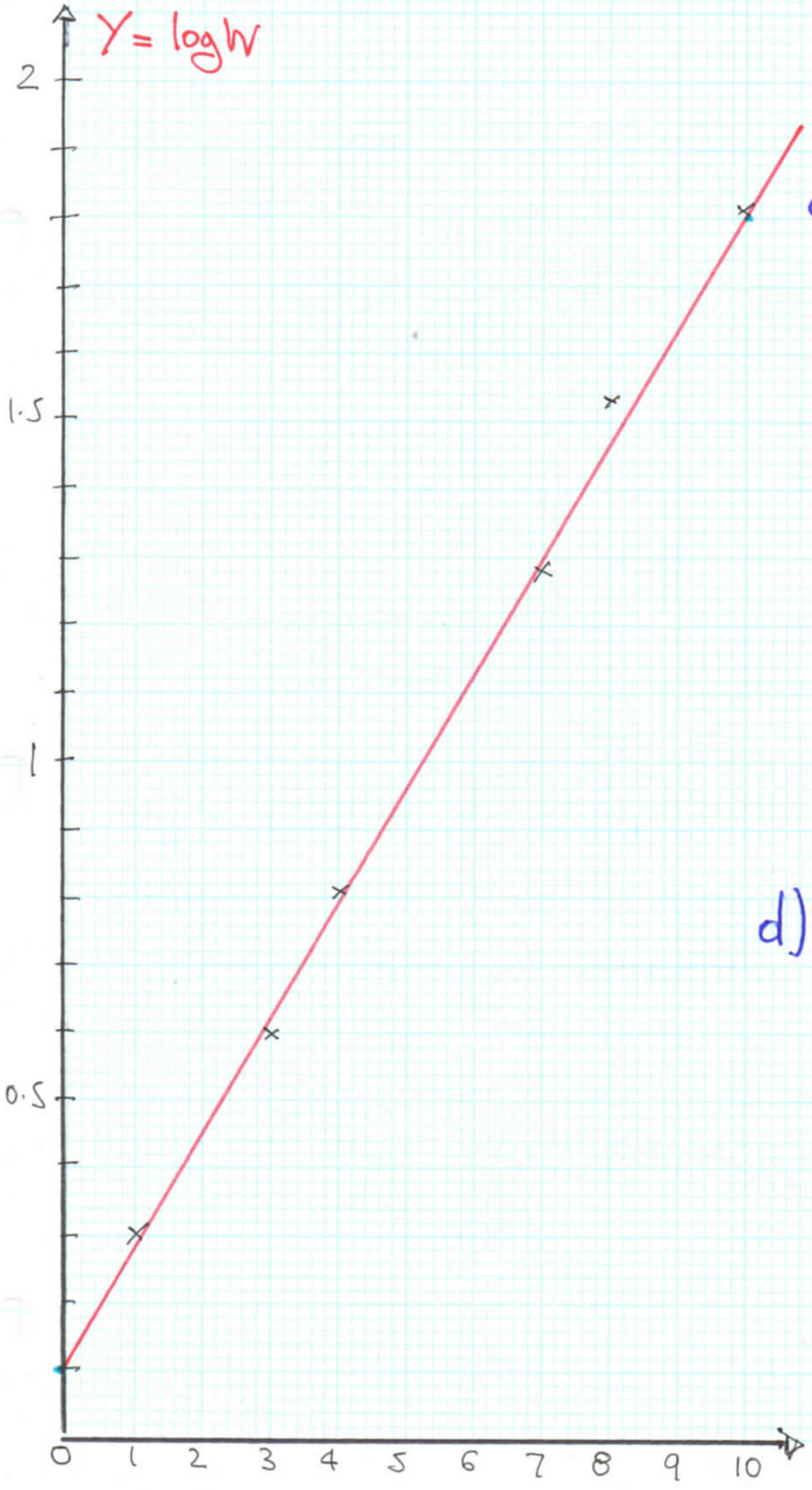
t	1	3	4	7	8	10
W	2	4	6.5	19	34	65

$X = t$	1	3	4	7	8	10
$Y = \log W$	0.30	0.60	0.81	1.28	1.53	1.81

PUTTING THESE VALUES ON AN ACCURATE DIAGRAM

(SEE NEXT PAGE)

IYGB - SYNOPTIC PAPER E - QUESTION 4



c) • THE "y INTERCEPT" IS $\log a$

$\log_{10} a \approx 0.1$

$a \approx 10^{0.1}$

$a \approx 1.26$

• THE GRADIENT IS $\log b$

USING $(0, 0.1)$ & $(10, 1.8)$

$\log_{10} b \approx \dfrac{1.8 - 0.1}{10 - 0}$

$\log_{10} b \approx 0.17$

$b \approx 10^{0.17}$

$b \approx 1.48$

d) $W = ab^t$

$W \approx 1.26 \times (1.48)^t$

when $t = 20$

$W \approx 1.26 \times 1.48^{20}$

$W \approx 3200$

IYGB - SYNOPTIC PAPER E - QUESTION 5

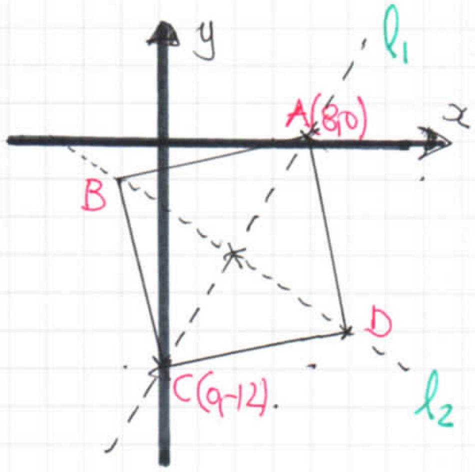
FIND THE CO-ORDINATES OF A & C

$$\Longrightarrow 3x - 2y = 24$$

$$\Longrightarrow x = 0, \quad y = -12 \qquad \therefore C(0,-12)$$

$$\Longrightarrow y = 0, \quad x = 8 \qquad \therefore A(8,0)$$

NEXT LOOKING AT THE DIAGRAM



- $M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = M\left(\frac{0+8}{2}, \frac{-12+0}{2}\right)$

$$= M(4,-6)$$

- $m_{AC} = \frac{y_2-y_1}{x_2-x_1} = \frac{-12-0}{0-8} = \frac{3}{2}$

EQUATION OF l_2 HAS GRADIENT $-\frac{2}{3}$ AND PASSES THROUGH $M(4,-6)$

$$\Longrightarrow y - y_0 = m(x - x_0)$$

$$\Longrightarrow y + 6 = -\frac{2}{3}(x - 4)$$

$$\Longrightarrow 3y + 18 = -2x + 8$$

$$\Longrightarrow 2x + 3x + 10 = 0$$

MGB - SYNOPTIC PAPER E - QUESTION 6

a) USING THE POINT (0,2)

$$y = A\cos(x+60)$$
$$2 = A\cos 60$$
$$2 = \tfrac{1}{2}A$$
$$A = 4$$

b) SOLVING $4\cos(x+60) = 0$

$$\Rightarrow \cos(x+60) = 0$$

$$\Rightarrow \begin{cases} x+60 = 90 \pm 360n \\ x+60 = 270 \pm 360n \end{cases} \quad n = 0,1,2,3,\ldots$$

$$\Rightarrow \begin{cases} x = 30° \pm 360n \\ x = 210° \pm 360n \end{cases}$$

$$\therefore\ Q(30°,0),\ R(210°,0),\ S(390°,0)$$

IYGB - SYNOPTIC PAPER E - QUESTION 7

OBTAIN THE FIRST TWO DERIVATIVES

$$y = e^{-2x} + axe^{-2x}$$

$$\frac{dy}{dx} = -2e^{-2x} + ae^{-2x} + ax\left(-2e^{-2x}\right) = e^{-2x}\left(-2+a-2ax\right)$$

$$\frac{d^2y}{dx^2} = -2e^{-2x}(a-2-2ax) + e^{-2x}(-2a)$$

$$= -2e^{-2x}\left[a-2-2ax+a\right]$$

$$= -2e^{-2x}(2a-2ax-2)$$

$$= 4e^{-2x}(ax-a+1)$$

FOR STATIONARY VALUES $\frac{dy}{dx}=0$

$$\Rightarrow e^{-2x}(-2+a-2ax)=0$$

$$\Rightarrow -2+a-2ax=0 \qquad (e^{-2x}\neq 0)$$

$$\Rightarrow a-2=2ax$$

$$\Rightarrow x = \frac{a-2}{2a}$$

FINALLY WE HAVE

$$\left.\frac{d^2y}{dx^2}\right|_{x=\frac{a-2}{2a}} = 4e^{-2\left(\frac{a-2}{2a}\right)}\left[a \times \frac{a-2}{2a} - a + 1\right]$$

$$= 4e^{-\frac{a-2}{a}}\left[\frac{a-2}{2} - a + 1\right]$$

$$= 4e^{\frac{2-a}{a}}\left[\frac{a}{2} \cancel{-1} - a \cancel{+1}\right]$$

$$= 4e^{\frac{2}{a}-1}\left[-\frac{a}{2}\right]$$

$$= -2ae^{\frac{2}{a}-1}$$

AS REQUIRED

IYGB - SYNOPTIC PAPER E - QUESTION 8

a) PROCEED AS FOLLOWS

$$\sin 3x = \sin(2x+x) = \sin 2x\cos x + \cos 2x \sin x$$
$$= (2\sin x\cos x)\cos x + (1-2\sin^2 x)\sin x$$
$$= 2\sin x\cos^2 x + \sin x - 2\sin^3 x$$
$$= 2\sin x(1-\sin^2 x) + \sin x - 2\sin^3 x$$
$$= 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$$
$$= 3\sin x - 4\sin^3 x$$

AS REQUIRED

b) USING THE RESULT OF PART (a)

$$\int \cos x\,(6\sin x - 2\sin 3x)^{\frac{2}{3}}\,dx$$
$$= \int \cos x\left[6\sin x - 2(3\sin x - 4\sin^3 x)\right]^{\frac{2}{3}}\,dx$$
$$= \int \cos x\left[\cancel{6\sin x} - \cancel{6\sin x} + 8\sin^3 x\right]^{\frac{2}{3}}\,dx$$
$$= \int \cos x\,(8\sin^3 x)^{\frac{2}{3}}\,dx$$
$$= \int \cos x\,(4\sin^2 x)\,dx$$
$$= \int 4\cos x\sin^2 x\,dx$$

BY INSPECTION, OR USING THE SUBSTITUTION $u = \sin x$

$$= \frac{4}{3}\sin^3 x + C$$

IYGB - SYNOPTIC PAPER E - QUESTION 7

SOLVING SIMULTANEOUSLY

$$\left.\begin{array}{l} y = x^2+6x+7 \\ y = 2x+c \end{array}\right\} \Longrightarrow x^2+6x+7 = 2x+c$$

$$\Longrightarrow x^2+4x+7-c=0$$

IF A TANGENT THE ABOVE QUADRATIC MUST HAVE REPEATED ROOTS, IE $b^2-4ac=0$

$$\Longrightarrow 4^2-4\times1\times(7-c)=0$$

$$\Longrightarrow 16-4(7-c)=0$$

$$\Longrightarrow 16-28+4c=0$$

$$\Longrightarrow -12+4c=0$$

$$\Longrightarrow 4c=12$$

$$\Longrightarrow c=3$$

FINALLY IF $c=3$ WE HAVE

$$\Longrightarrow x^2+4x+7-c=0$$

$$\Longrightarrow x^2+4x+4=0$$

$$\Longrightarrow (x+2)^2=0$$

$$\Longrightarrow x=-2$$

& USING $y=2x+3$

$$y=2(-2)+3$$

$$y=-1$$

$$\therefore (-2,-1)$$

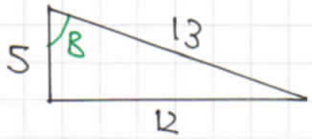
IYGB - SYNOPTIC PAPER E - QUESTION 10

USE THE INFORMATION GIVEN AS FOLLOWS

$\cot A = -\frac{3}{4}$

$\tan A = -\frac{4}{3}$

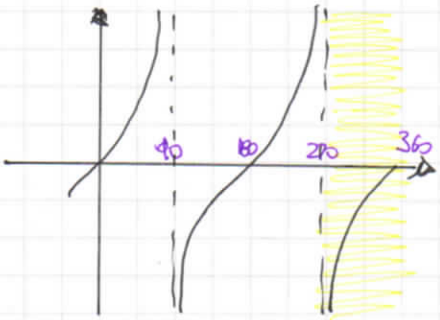
$\cos B = \frac{5}{13}$



BUT "B IS REFLEX" & POSITIVE COSINE

SO $270 < B < 360$ ALSO

SO tangent in this range will be negative



$\therefore \tan B = -\frac{12}{5}$

USING THE COMPOUND TANGENT IDENTITY

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{-\frac{4}{3} - \frac{12}{5}}{1 - \left(-\frac{4}{3}\right)\left(-\frac{12}{5}\right)} = \frac{-\frac{4}{3} - \frac{12}{5}}{1 - \frac{48}{15}}$$

$$= \frac{-20 - 36}{15 - 48} = \frac{-56}{-33} = \frac{56}{33}$$

AS REQUIRED

IYGB - SYNOPTIC PAPER E - QUESTION 11

a) I) SETTING $y=0$ IN EACH OF THE EQUATIONS

- $y = |3x-2|$
  $0 = |3x-2|$
  $0 = 3x-2$
  $3x = 2$
  $x = \frac{2}{3}$
  $\therefore A\left(\frac{2}{3}, 0\right)$

- $y = |x-5|$
  $0 = |x-5|$
  $0 = x-5$
  $x = 5$
  $B(5,0)$

II) SETTING $x=0$ IN EACH OF THE EQUATIONS

- $y = |3x-2|$
  $y = |3\times0-2|$
  $y = |-2|$
  $y =$
  $\therefore C(0,2)$

- $y = |x-5|$
  $y = |0-5|$
  $y = |-5|$
  $y = 5$
  $\therefore D(0,5)$

b) FIND THE CO-ORDINATES OF P & Q

$$\Rightarrow |3x-2| = |x-5|$$

$$\Rightarrow \begin{cases} 3x-2 = x-5 \\ 3x-2 = 5-x \end{cases}$$

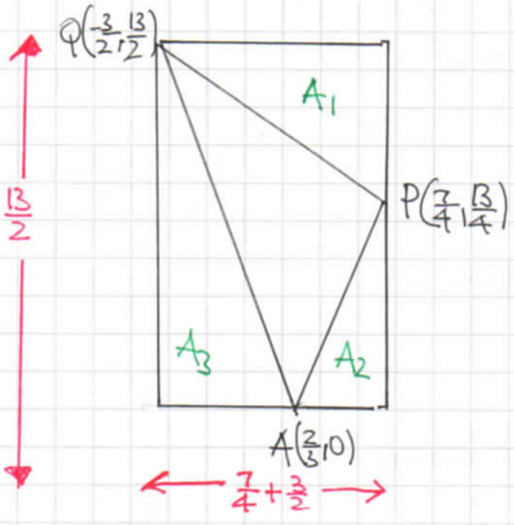
$$\Rightarrow \begin{cases} 2x = -3 \\ 4x = 7 \end{cases}$$

$$\Rightarrow \begin{cases} x = -\frac{3}{2} \\ x = \frac{7}{4} \end{cases} \quad \Rightarrow y \begin{cases} \left|-\frac{3}{2}-5\right| = \left|-\frac{13}{2}\right| = \frac{13}{2} \\ \left|\frac{7}{4}-5\right| = \left|-\frac{13}{4}\right| = \frac{13}{4} \end{cases}$$

IYGB - SYNOPTIC PAPER E - QUESTION 11

$\therefore\ P\left(\frac{7}{4}, \frac{13}{4}\right)$ & $Q\left(-\frac{3}{2}, \frac{13}{2}\right)$

LOOKING AT THE DIAGRAM



AREA OF RECTANGLE

$$\frac{13}{2} \times \left(\frac{7}{4} + \frac{3}{2}\right) = \frac{13}{2} \times \frac{13}{4} = \frac{169}{8}$$

AREA A_1 $= \frac{1}{2} \times \left(\frac{13}{2} - \frac{13}{4}\right)\left(\frac{7}{4} + \frac{3}{2}\right)$

$$= \frac{1}{2} \times \frac{13}{4} \times \frac{13}{4} = \frac{169}{32}$$

AREA A_2 $= \frac{1}{2}\left(\frac{7}{4} - \frac{2}{3}\right)\left(\frac{13}{4}\right)$

$$= \frac{1}{2} \times \frac{13}{12} \times \frac{13}{4} = \frac{169}{96}$$

AREA A_3 $= \frac{1}{2} \times \left(\frac{2}{3} + \frac{3}{2}\right) \times \frac{13}{2}$

$$= \frac{1}{2} \times \frac{13}{6} \times \frac{13}{2}$$

$$= \frac{169}{24}$$

$\therefore$ **REQUIRED AREA** $= \frac{169}{8} - \left(\frac{169}{32} + \frac{169}{96} + \frac{169}{24}\right)$

$$= \frac{169}{24}$$

IYGB – SYNOPTIC PAPER E – QUESTION 12

START BY FINDING THE INTERSECTIONS OF THE TWO CURVES

$$\left.\begin{aligned} y &= \frac{16}{x^2} \\ y &= 17 - x^2 \end{aligned}\right\} \Longrightarrow \frac{16}{x^2} = 17 - x^2$$

$$\Longrightarrow x^2 - 17 + \frac{16}{x^2} = 0$$

$$\Longrightarrow x^4 - 17x^2 + 16 = 0$$

$$\Longrightarrow (x^2 - 1)(x^2 - 16) = 0$$

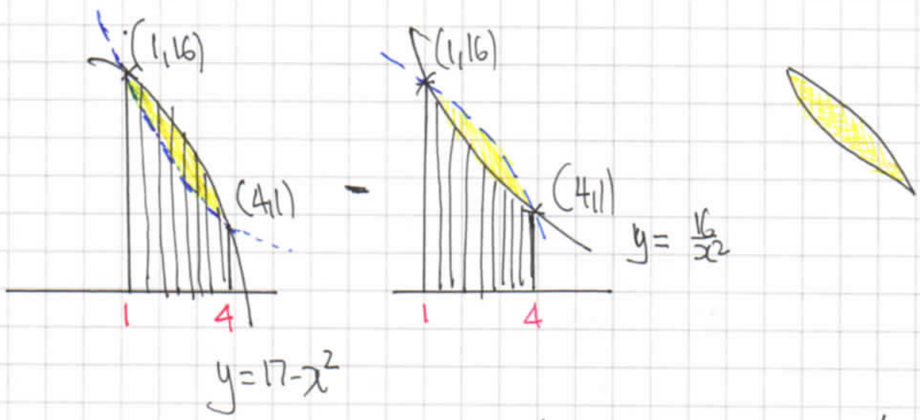
$$\Longrightarrow x^2 = \begin{cases} 1 \\ 16 \end{cases}$$

$$\Longrightarrow x = \begin{cases} +1 \\ +4 \end{cases} \qquad \text{(FIRST QUADRANT)}$$

$$\Longrightarrow y = \begin{cases} 16 \\ 1 \end{cases}$$

$$\therefore \ (1, 16) \ \& \ (4, 1)$$

LOOKING AT THE DIAGRAM BELOW



$$\text{AREA} = \int_1^4 17 - x^2 \, dx - \int_1^4 \frac{16}{x^2} \, dx = \int_1^4 17 - x^2 - \frac{16}{x^2} \, dx$$

IYGB - SYNOPTIC PAPER E - QUESTION 12

$$\text{AREA} = \left[17x - \frac{1}{3}x^3 + \frac{16}{x} \right]_1^4$$

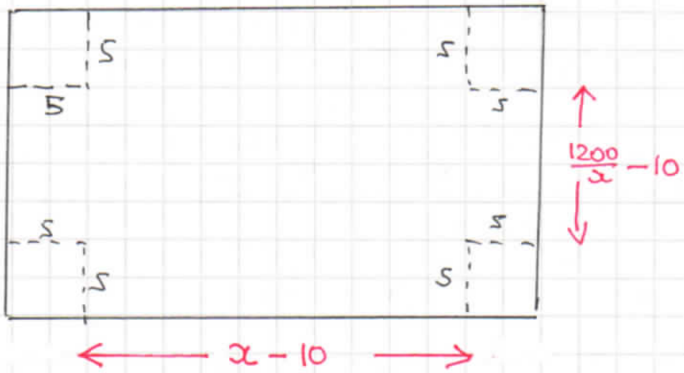
$$= \left(68 - \frac{64}{3} + 4\right) - \left(17 - \frac{1}{3} + 16\right)$$

$$= \frac{152}{3} - \frac{98}{3}$$

$$= \underline{18}$$

IYGB - SYNOPTIC PAPER E - QUESTION 13

a) LOOKING AT THE DIAGRAM



THE MEASUREMENTS OF THE CARD MUST BE

x & $\frac{1200}{x}$

ALSO WE MUST HAVE

$\frac{1200}{x} - 10 > 0$ & $x - 10 > 0$

$\frac{1200}{x} > 10$ $\quad x > 10$

$1200 > 10x$

$x < 120$

FINALLY THE VOLUME MUST EXCEED 2850

$\Rightarrow \left(\frac{1200}{x} - 10\right)(x - 10) \times 5 > 2850$

$\Rightarrow 10\left(\frac{120}{x} - 1\right)(x - 10) \times 5 > 2850$ $\quad \div 50$

$\Rightarrow \left(\frac{120}{x} - 1\right)(x - 10) > 57$

$\Rightarrow 120 - \frac{1200}{x} - x + 10 > 57$ $\quad \times x \quad (x > 0)$

$\Rightarrow 120x - 1200 - x^2 + 10x > 57x$

$\Rightarrow -x^2 + 73x - 1200 > 0$

$\Rightarrow x^2 - 73x + 1200 < 0$

AS REQUIRED

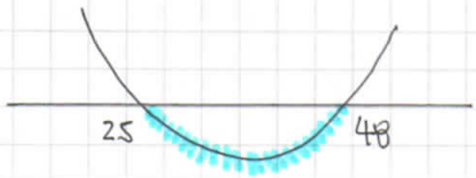
IYGB - SYNOPTIC PAPER E - QUESTION 13

b) BY THE QUADRATIC FORMULA (OR FACTORIZATION)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{+73 \pm \sqrt{(-73)^2 - 4 \times 1 \times 1200}}{2 \times 1}$$

$$x = \begin{cases} 48 \\ 25 \end{cases}$$



$25 < x < 48$

WHICH ALSO SATISFIES

$x > 0$ AND $x < 120$

IYGB – SYNOPTIC PAPER E – QUESTION 14

SOLVE BY SEPARATING VARIABLES

$$\Longrightarrow \frac{dy}{dx} = \frac{y^2-1}{x}$$

$$\Longrightarrow \frac{1}{y^2-1}\,dy = \frac{1}{x}\,dx$$

$$\Longrightarrow \int \frac{1}{(y-1)(y+1)}\,dy = \int \frac{1}{x}\,dx$$

PROCEED BY PARTIAL FRACTIONS

$$\frac{1}{(y-1)(y+1)} \equiv \frac{A}{y-1} + \frac{B}{y+1}$$

$$1 \equiv A(y+1) + B(y-1)$$

- IF $y=1$: $1 = 2A$, $A = \frac{1}{2}$
- IF $y=-1$: $1 = -2B$, $B = -\frac{1}{2}$

RETURNING TO THE O.D.E.

$$\Longrightarrow \int \frac{\frac{1}{2}}{y-1} - \frac{\frac{1}{2}}{y+1}\,dy = \int \frac{1}{x}$$

$$\Longrightarrow \int \frac{1}{y-1} - \frac{1}{y+1}\,dy = \int \frac{2}{x}$$

$$\Longrightarrow \ln|y-1| - \ln|y+1| = 2\ln|x| + \ln A$$

$$\Longrightarrow \ln\left|\frac{y-1}{y+1}\right| = \ln|Ax^2|$$

$$\Longrightarrow \frac{y-1}{y+1} = Ax^2$$

APPLY CONDITION $(1,2)$

$$\frac{2-1}{2+1} = A \times 1^2$$

$$\frac{1}{3} = A$$

$$\therefore \frac{y-1}{y+1} = \frac{1}{3}x^2$$

REARRANGING

$$\Longrightarrow \frac{y-1}{y+1} = \frac{x^2}{3}$$

$$\Longrightarrow 3y - 3 = x^2y + x^2$$

$$\Longrightarrow 3y - yx^2 = 3 + x^2$$

$$\Longrightarrow y(3-x^2) = 3 + x^2$$

$$\Longrightarrow y = \frac{3+x^2}{3-x^2}$$

AS REQUIRED

IYGB - SYNOPTIC PAPER E - QUESTION 15

a) FORMING AN EQUATION

$$\Rightarrow u_4 - u_1 = 5(u_3 - u_2)$$

$$u_n = ar^{n-1}$$

$$\Rightarrow ar^3 - a = 5(ar^2 - ar)$$

$$\Rightarrow ar^3 - a = 5ar^2 - 5ar$$

$$\Rightarrow ar^3 - 5ar^2 + 5ar - a = 0$$

$$\Rightarrow r^3 - 5r^2 + 5r - 1 = 0$$

// AS REQUIRED

b) BY INSPECTION $r=1$, IS A SOLUTION

$$\Rightarrow (r-1)(r^2 + Ar + 1) = 0$$

$-Ar$

r

$$r - Ar = 5r$$
$$-4r = Ar$$
$$A = -4$$

$$\Rightarrow (r-1)(r^2 - 4r + 1) = 0$$

$$\Rightarrow r^2 - 4r + 1 = 0$$

$$\Rightarrow (r-2)^2 - 2^2 + 1 = 0$$

$$\Rightarrow (r-2)^2 = 3$$

$$\Rightarrow r - 2 = \pm\sqrt{3}$$

$$\Rightarrow r = 2 \pm \sqrt{3}$$

$$\therefore r = \begin{cases} 1 \\ 2+\sqrt{3} \\ 2-\sqrt{3} \end{cases}$$

IYGB - SYNOPTIC PAPER E - QUESTION 15

c) THE COMMON RATIO IS $2-\sqrt{3}$, AS THIS IS THE ONLY VALUE OF r WHICH PRODUCES A SUM TO INFINITY $(-1<r<1)$

$$\Rightarrow S_{\infty} = \frac{a}{1-r}$$

$$\Rightarrow \sqrt{6}+\sqrt{2} = \frac{a}{1-(2-\sqrt{3})}$$

$$\Rightarrow \sqrt{6}+\sqrt{2} = \frac{a}{-1+\sqrt{3}}$$

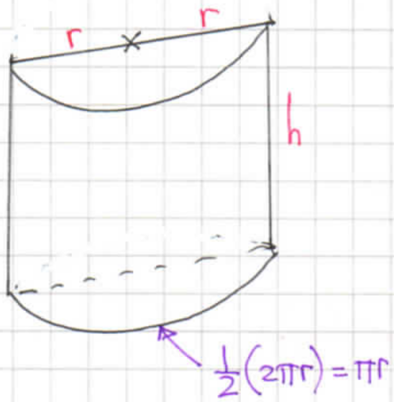
$$\Rightarrow a = (\sqrt{6}+\sqrt{2})(-1+\sqrt{3})$$

$$\Rightarrow a = \cancel{-\sqrt{6}} + \sqrt{18} - \sqrt{2} \cancel{+\sqrt{6}}$$

$$\Rightarrow a = 3\sqrt{2} - \sqrt{2}$$

$$\Rightarrow a = 2\sqrt{2}$$

IYGB - SYNOPTIC PAPER E - QUESTION 16



- LET THE RADIUS BE r & THE VERTICAL HEIGHT h
- CONSTRAINT ON SURFACE AREA

$$\Rightarrow \pi r^2 + \pi r h + 2rh = \sqrt[3]{27\pi}$$

$$\Rightarrow \pi r h + 2rh = \sqrt[3]{27\pi} - \pi r^2$$

$$\Rightarrow rh(\pi + 2) = 3\pi^{\frac{1}{3}} - \pi r^2$$

$$\Rightarrow \boxed{rh = \frac{3\pi^{\frac{1}{3}} - \pi r^2}{\pi + 2}}$$

- NOW LOOKING AT THE VOLUME

$$\Rightarrow V = \frac{1}{2}(\pi r^2 h) = \frac{1}{2}\pi r(rh) = \frac{1}{2}\pi r\left(\frac{3\pi^{\frac{1}{3}} - \pi r^2}{\pi + 2}\right)$$

$$\Rightarrow V = \frac{\pi r(3\pi^{\frac{1}{3}} - \pi r^2)}{2(\pi + 2)}$$

$$\Rightarrow \boxed{V = \frac{\pi}{2(\pi+2)}\left[3\pi^{\frac{1}{3}} r - \pi r^3\right]}$$

- DIFFERENTIATE & SOLVE FOR ZERO

$$\Rightarrow \frac{dV}{dr} = \frac{\pi}{2(\pi+2)}\left[3\pi^{\frac{1}{3}} - 3\pi r^2\right]$$

$$\Rightarrow 0 = \cancel{\frac{\pi}{2(\pi+2)}}\left[3\pi^{\frac{1}{3}} - 3\pi r^2\right]$$

$$\Rightarrow \cancel{3}\pi^{\frac{1}{3}} - \cancel{3}\pi r^2 = 0$$

$$\Rightarrow \pi^{\frac{1}{3}} = \pi r^2$$

IYGB - SYNOPTIC PAPER E - QUESTION 16

$$\Rightarrow r^2 = \pi^{-\frac{2}{3}}$$

$$\Rightarrow r^2 = \frac{1}{\pi^{2/3}}$$

$$\Rightarrow r = +\frac{1}{\pi^{1/3}}$$

FINALLY TO OBTAIN THE MAXIMUM VOLUME

$$\Rightarrow V = \frac{\pi}{2(\pi+2)}\left[3\pi^{\frac{1}{3}}r - \pi r^3\right]$$

$$\Rightarrow V = \frac{\pi r}{2(\pi+2)}\left[3\pi^{\frac{1}{3}} - \pi r^2\right]$$

$$\Rightarrow V_{MAX} = \frac{\pi}{2(\pi+2)}\left(\frac{1}{\pi^{1/3}}\right)\left[3\pi^{\frac{1}{3}} - \pi \times \frac{1}{\pi^{2/3}}\right]$$

$$\Rightarrow V_{MAX} = \frac{\pi^{\frac{2}{3}}}{2(\pi+2)}\left[3\pi^{\frac{1}{3}} - \pi^{\frac{1}{3}}\right]$$

$$\Rightarrow V_{MAX} = \frac{\pi^{\frac{2}{3}}}{2(\pi+2)} \times 2\pi^{\frac{1}{3}}$$

$$\Rightarrow V_{MAX} = \frac{2\pi}{2(\pi+2)}$$

$$\Rightarrow V_{MAX} = \frac{\pi}{\pi+2}$$

IYGB - SYNOPTIC PAPER E - QUESTION 17

a) SUBSTITUTE, EXPAND & TIDY

$$4x^2+4x+17 = 4\left[\tfrac{1}{2}(-1+4\tan\theta)\right]^2+4\left[\tfrac{1}{2}(-1+4\tan\theta)\right]+17$$

$$= \cancel{4}\times\tfrac{1}{\cancel{4}}(-1+4\tan\theta)^2+2(-1+4\tan\theta)+17$$

$$= 1-\cancel{8\tan\theta}+16\tan^2\theta-2+\cancel{8\tan\theta}+17$$

$$= 16+16\tan^2\theta$$

$$= 16(1+\tan^2\theta)$$

$$= 16\sec^2\theta$$

// AS REQUIRED

b) BY SUBSTITUTION FROM PART (a)

$$\Rightarrow x=\tfrac{1}{2}(-1+4\tan\theta) = -\tfrac{1}{2}+2\tan\theta$$

$$\Rightarrow \frac{dx}{d\theta}=2\sec^2\theta$$

$$\Rightarrow dx = 2\sec^2\theta\,d\theta$$

- when $x=-\frac{1}{2}$

  $-\frac{1}{2}=-\frac{1}{2}+2\tan\theta$

  $0=2\tan\theta$

  $\theta=0$

- when $x=\frac{3}{2}$

  $\frac{3}{2}=-\frac{1}{2}+2\tan\theta$

  $2=2\tan\theta$

  $\tan\theta=1$

  $\theta=\frac{\pi}{4}$

IYGB - SYNOPTIC PAPER E - QUESTION 17

TRANSFORMING THE INTEGRAL YIELDS

$$\int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{1}{4x^2+4x+17}\,dx = \int_0^{\frac{\pi}{4}} \frac{1}{16\cancel{\sec^2\theta}}\left(2\cancel{\sec^2\theta}\,d\theta\right)$$

$$= \int_0^{\frac{\pi}{4}} \frac{1}{8}\,d\theta$$

$$= \left[\frac{1}{8}\theta\right]_0^{\frac{\pi}{4}}$$

$$= \frac{\pi}{32} - 0$$

$$= \frac{\pi}{32}$$

IYGB - SYNOPTIC PAPER E - QUESTION 18

a) COMPLETING THE SQUARE

$\Rightarrow x^2+y^2-14x+33=0$

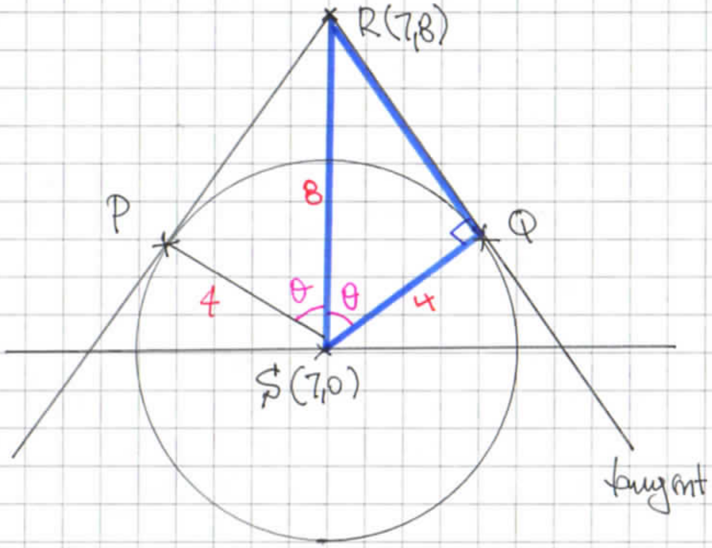
$\Rightarrow x^2-14x+y^2+33=0$

$\Rightarrow (x-7)^2-49+y^2+33=0$

$\Rightarrow (x-7)^2+y^2=16$

CENTRE $(7,0)$ & RADIUS $=\sqrt{16}=4$

b) LOOKING AT THE DIAGRAM



USING THE RIGHT ANGLED TRIANGLE $\hat{SQR}$

$\cos\theta = \frac{4}{8} = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

AREA OF SECTOR

"$A = \frac{1}{2}r^2\theta^c$"

"θ" IS 2θ IN THE PICTURE

$A = \frac{1}{2}\times 4^2 \times \left(2\times\frac{\pi}{3}\right) = \frac{16}{3}\pi$

AREA OF TRIANGLE $\hat{SQR}$

$A = \frac{1}{2}|RS||SQ|\sin\frac{\pi}{3}$

$A = \frac{1}{2}\times 4\times 8\times\frac{\sqrt{3}}{2}$

$A = 8\sqrt{3}$

REQUIRED AREA

$2\times$ TRIANGLE SQR $-$ SECTOR

$= 2\times 8\sqrt{3} - \frac{16}{3}\pi$

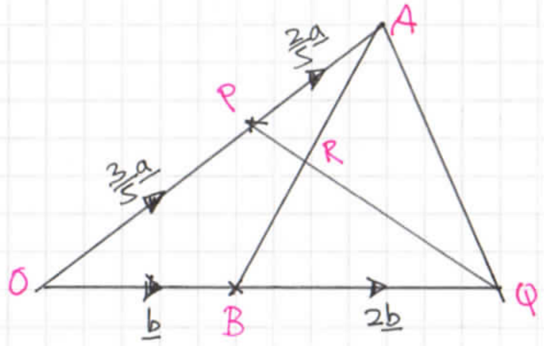
$= 16\sqrt{3} - \frac{16}{3}\pi$

$= \frac{16}{3}\left[3\sqrt{3}-\pi\right]$

AS REQUIRED

IYGB - SYNOPTIC PAPER E - QUESTION 19

a) LOOKING AT THE DIAGRAM



$\vec{OR} = \vec{OA} + \vec{AR}$

$\vec{OR} = \underline{a} + h\vec{AB}$

$\vec{OR} = \underline{a} + h(\vec{AO} + \vec{OB})$

$\vec{OR} = \underline{a} + h(-\underline{a} + \underline{b})$

$\vec{OR} = \underline{a} - h\underline{a} + h\underline{b}$

$\vec{OR} = (1-h)\underline{a} + h\underline{b}$

b) SIMILARLY WE HAVE

$\vec{OR} = \vec{OP} + \vec{PR}$

$\vec{OR} = \vec{OP} + k\vec{PQ}$

$\vec{OR} = \vec{OP} + k(\vec{PO} + \vec{OQ})$

$\vec{OR} = \frac{3}{5}\underline{a} + k(-\frac{3}{5}\underline{a} + 3\underline{b})$

$\vec{OR} = \frac{3}{5}\underline{a} - \frac{3}{5}k\underline{a} + 3k\underline{b}$

$\vec{OR} = \frac{3}{5}(1-k)\underline{a} + 3k\underline{b}$

b) SOLVING SIMULTANEOUSLY

I) $\Longrightarrow (1-h)\underline{a} + h\underline{b} = \frac{3}{5}(1-k)\underline{a} + 3k\underline{b}$

$$\left\{\begin{array}{l} 1-h = \frac{3}{5}(1-k) \\ h = 3k \end{array}\right\} \Longrightarrow$$

$1 - 3k = \frac{3}{5}(1-k)$

$5 - 15k = 3 - 3k$

$2 = 12k$

$k = \frac{1}{6}$

& $h = \frac{1}{2}$

II) BY INSPECTION

$\vec{PR} = k\vec{PQ}$

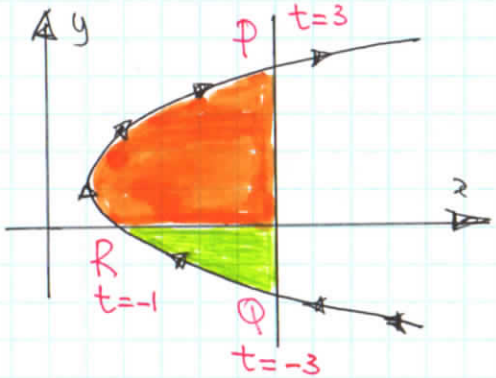
$\vec{PR} = \frac{1}{6}\vec{PQ}$

$\frac{|\vec{PR}|}{|\vec{PQ}|} = \frac{1}{6}$

$\therefore\ 1 : 6$

IYGB - SYNOPTIC PAPER E - QUESTION 20

START BY DETERMINING THE VALUE OF t, AT P, Q & R (DIAGRAM)



- $y=0$
  $2t+2=0$
  $t=-1$
  ↑ $R(2,0)$

- $x=10$
  $t^2+1=10$
  $t^2=9$
  $t = \begin{cases} -3 & \leftarrow Q(10,-4) \\ 3 & \leftarrow P(10,8) \end{cases}$

INTEGRATING IN PARAMETRIC IN "ONE GO"

$$\text{AREA} = \int_{x_1}^{x_2} y(x)\,dx = \int_{t_1}^{t_2} y(t)\frac{dx}{dt}\,dt$$

$$= \int_{-3}^{3}(2t+2)(2t)\,dt = \int_{-3}^{3} \underbrace{4t^2}_{\text{EVEN}} + \underbrace{\cancel{4t}}_{\text{ODD}}\,dt \quad (\times 2)$$

$$= 2\int_0^3 4t^2\,dt = 8\int_0^3 t^2\,dt$$

// AS REQUIRED

$$= \left[\frac{8}{3}t^3\right]_0^3 = \left(\frac{8}{3}\times 27\right) - 0$$

$$= 72$$

ALTERNATIVE BY SPLITTING THE AREA IN 2

$$\text{"ORANGE AREA"} = \int_{-1}^{3} 4t^2+4t\,dt$$

$$\text{"GREEN AREA"} = -\int_{-1}^{-3} 4t^2+4t\,dt$$

↑ AS THIS AREA IS BELOW THE x-AXIS THE MINUS WILL MAKE IT POSITIVE

IYGB - SYNOPTIC PAPER E - QUESTION 20

HENCE THE TOTAL AREA CAN BE FOUND

$$\text{TOTAL AREA} = \int_{-1}^{3} 4t^2+4t \, dt - \int_{-1}^{-3} 4t^2+4t \, dt$$

$$= \int_{-1}^{3} 4t^2+4t \, dt + \int_{-3}^{-1} 4t^2+4t \, dt$$

$$= \int_{-3}^{3} 4t^2+\cancel{4t} \, dt$$

ODD IN A SYMMETRICAL DOMAIN

EVEN IN A SYMMETRICAL DOMAIN $\Rightarrow \times 2$

$$= 2\int_{-3}^{3} 4t^2 \, dt$$

$= \dots$ AS BEFORE.

IYGB – SYNOPTIC PAPER E – QUESTION 21

a) PROCEED TO ELIMINATE THE LOGARITHMS

$$\Rightarrow \log_2(256x^2) = 1 + 2\log_2\left(\tfrac{1}{2}x^4\right)$$

$$\Rightarrow \log_2(256x^2) = \log_2 2 + \log_2\left(\tfrac{1}{2}x^4\right)^2$$

$$\Rightarrow \log_2(256x^2) = \log_2 2 + \log_2\left(\tfrac{1}{4}x^8\right)$$

$$\Rightarrow \log_2(256x^2) = \log_2\left[2 \times \tfrac{1}{4}x^8\right]$$

$$\Rightarrow \log_2\left[256x^2\right] = \log_2\left[\tfrac{1}{2}x^8\right]$$

EXTRACTING LOGS

$$\Rightarrow 256x^2 = \tfrac{1}{2}x^8$$

$$\Rightarrow \tfrac{1}{2}x^8 - 256x^2 = 0$$

$$\Rightarrow x^8 - 512x^2 = 0$$

$$\Rightarrow x^2(x^6 - 512) = 0$$

$$\Rightarrow x^6 - 512 = 0 \qquad (x^2 \neq 0 \text{ BECAUSE OF THE LOGS})$$

$$\Rightarrow (x^2)^3 = 512 \qquad (512 \text{ CUBE ROOTS, BUT IT DOES NOT SQUARE ROOT})$$

$$\Rightarrow x^2 = 8$$

$$\Rightarrow x = \pm\sqrt{8} = \pm 2\sqrt{2}$$

BOTH ARE FINE

IYGB - SYNOPTIC PAPER E - QUESTION 21

b) WITH A SIMILAR METHOD TO PART (a)

$$\Rightarrow 2\log_2\left(\frac{y}{2}\right) + \log_2\sqrt{y} = 8$$

$$\Rightarrow \log_2\left(\frac{y}{2}\right)^2 + \log_2 y^{\frac{1}{2}} = 8\log_2 2$$

$$\Rightarrow \log_2\left(\frac{y^2}{4}\right) + \log_2 y^{\frac{1}{2}} = \log_2 256$$

$$\Rightarrow \log_2\left(\frac{y^2}{4} \times y^{\frac{1}{2}}\right) = \log_2 256$$

EXTRACTING FROM THE LOGS

$$\Rightarrow \frac{y^{\frac{5}{2}}}{4} = 256$$

$$\Rightarrow y^{\frac{5}{2}} = 1024$$

$$\Rightarrow \left(y^{\frac{5}{2}}\right)^{\frac{2}{5}} = 1024^{\frac{2}{5}}$$

$$\Rightarrow y^1 = \left(\sqrt[5]{1024}\right)^2$$

$$\Rightarrow y = 4^2$$

$$\Rightarrow y = 16$$