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IYGB - MMS PAPER II - QUESTION 1

a) USING A CALCULATOR IN STATISTICS MODE, THE P.M.C.C BASED ON 7 OBS

$$r = 0.913$$

b) AGAIN FROM A STATISTICAL CALCULATOR

$$a = -147.6$$

$$b = 50.1$$

$$\Rightarrow y = 50.1x - 147.6$$

$$\Rightarrow y = 50.1 \times 8 - 147.6$$

$$\Rightarrow y \approx 253 \quad \leftarrow "k"$$

AS 8 LIES IN THE RANGE $5 \leq x \leq 17$, WHICH WAS USED TO PRODUCE THE REGRESSION LINE AND r INDICATES STRONG CORRELATION, THE ESTIMATE SHOULD BE VERY RELIABLE

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1YGB - MMS PAPER H - QUESTION 2

a)

$$X = \text{NUMBER OF DEFECTIVE BOOKS}$$
$$X \sim B(60, 0.05)$$

$$H_0 : p = 0.05$$

$H_1 : p < 0.05$, WHERE p REPRESENTS THE PROPORTION OF ALL THE DEFECTIVE BOOKS FROM THE PRODUCTION

TESTING AT 10% LEVEL OF SIGNIFICANCE ON THE BASIS THAT $\alpha = 1$

$$\begin{aligned} P(X \leq 1) &= \text{CALCULATOR OR } P(X=0) + P(X=1) \\ &= 0.1916 \\ &= 19.16\% \\ &> 10\% \end{aligned}$$

THERE IS NO SIGNIFICANT EVIDENCE TO SUPPORT THE CLAIM MADE BY THE MAKERS OF THE NEW MACHINERY

NO SUFFICIENT EVIDENCE TO REJECT H_0

b)

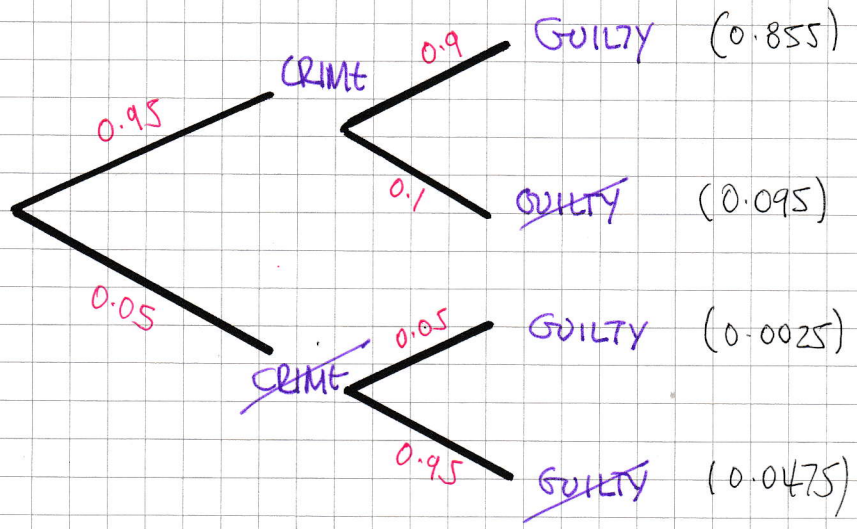
LOOKING AT THE "BOTTOM TAIL" AT 5%

$$\begin{aligned} P(X \leq 0) &= 0.0461 = 4.61\% < 5\% \\ P(X \leq 1) &= 0.1916 = 19.16\% > 5\% \end{aligned}$$

\therefore CRITICAL REGION = $\{0\}$

1YGB - MMS PAPER 1 - QUESTION 3

a) DRAWING A TREE DIAGRAM



FROM THE TREE DIAGRAM, $P(\text{GUILTY}) = 0.855 + 0.0025$
 $= 0.8575$
 $\left(\frac{343}{400}\right)$

b) USING THE "CONDITIONAL" FORMULA

$$P(\text{CRIME} | \text{GUILTY}) = \frac{P(\text{CRIME} \cap \text{GUILTY})}{P(\text{GUILTY})}$$

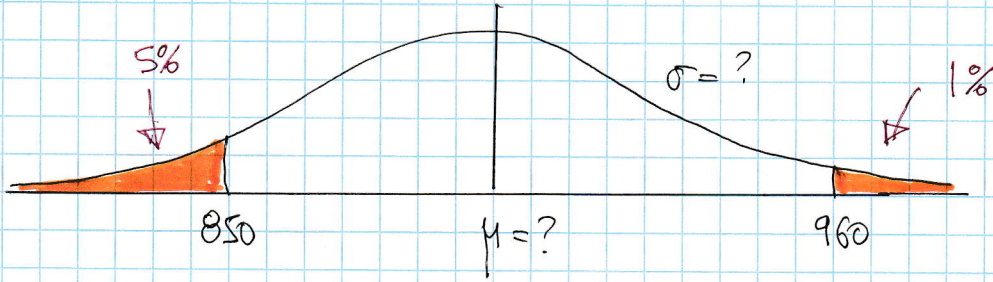
$$= \frac{0.855}{0.8575}$$

$$= 0.9971$$

$$\left(\frac{312}{313}\right)$$

1 YGB - NMS PAPER 11 - QUESTION 4

a) POTTING THE INFORMATION IN A DIAGRAM



$X = \text{weekly mileages}$
 $X \sim N(\mu, \sigma^2)$

$\bullet P(X < 850) = 5\%$
 $\Rightarrow P(X > 850) = 95\%$
 $\Rightarrow P\left(Z > \frac{850 - \mu}{\sigma}\right) = 0.95$
 \downarrow INVERSION
 $\Rightarrow \frac{850 - \mu}{\sigma} = -\Phi^{-1}(0.95)$
 $\Rightarrow \frac{850 - \mu}{\sigma} = -1.6449$
 $\Rightarrow 850 - \mu = -1.6449\sigma$
 $\Rightarrow \boxed{850 + 1.6449\sigma = \mu}$

$\bullet P(X > 960) = 1\%$
 $\Rightarrow P(X < 960) = 99\%$
 $\Rightarrow P\left(Z < \frac{960 - \mu}{\sigma}\right) = 0.99$
 \downarrow INVERSION
 $\Rightarrow \frac{960 - \mu}{\sigma} = +\Phi^{-1}(0.99)$
 $\Rightarrow \frac{960 - \mu}{\sigma} = 2.3263$
 $\Rightarrow 960 - \mu = 2.3263\sigma$
 $\Rightarrow \boxed{960 - 2.3263\sigma = \mu}$

SOLVING SIMULTANEOUSLY

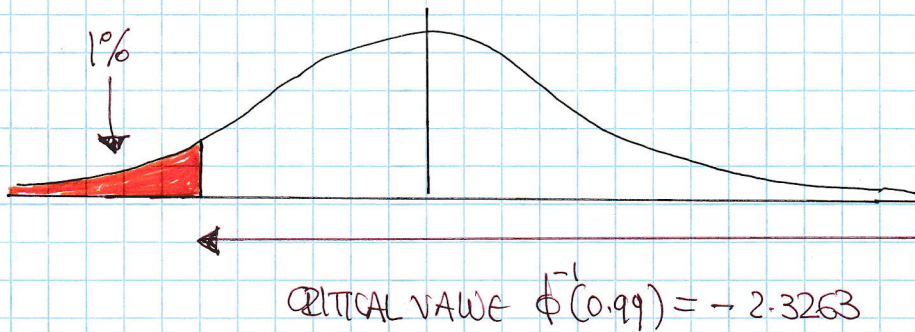
$\Rightarrow 850 + 1.6449\sigma = 960 - 2.3263\sigma$
 $\Rightarrow 3.9712\sigma = 110$
 $\Rightarrow \sigma = 27.69943594\dots$
 $\Rightarrow \sigma \approx 28$
 $\& \mu = 895.5628022\dots$
 $\mu \approx 896$

1YGB - MMS PAPER 1 - QUESTION 4

b) SETTING UP HYPOTHESES

- $H_0 : \mu = 896$
- $H_1 : \mu < 896$, WHERE μ IS THE MEAN OF ALL WEEKLY MILEAGES (POPULATION MEAN)

$n = 4$, $\bar{x}_4 = 863$, $\sigma = 28$, 1% SIGNIFICANCE



• Z-STATISTIC = $\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{863 - 896}{\frac{28}{\sqrt{4}}} = -2.357142\dots$

• AS $-2.357142\dots < -2.3263$ THERE IS SIGNIFICANT EVIDENCE, AT 1% LEVEL, TO SUPPORT THE SALES REP'S BELIEF

• THERE IS SUFFICIENT EVIDENCE TO REJECT H_0

IYGB - MMS PAPER 11 - QUESTION 5

a) STRATIFIED SAMPLING GUARANTEES REPRESENTATION OF ALL GROUPS WHILE SIMPLE RANDOM SAMPLING DOES NOT

WITH STRATIFIED SAMPLING YOU CAN FURTHER ANALYSE WITHIN A CERTAIN GROUP WHILE WITH SIMPLE RANDOM SAMPLING YOU CANNOT

b) STRATIFIED SAMPLING IS NOT BIASED, WHILE QUOTA SAMPLING IS

YOU CAN ESTIMATE SAMPLING ERRORS IN STRATIFIED SAMPLING WHILE YOU CANNOT WITH QUOTA SAMPLING

1YGB - MMS PAPER 1 - QUESTION 6

a) FORMING A TABLE OF MIDPOINTS

DISTANCE	MIDPOINTS	FREQUENCY
3-5	4	12 (12)
6-7	6.5	14 (26)
8	8	19 (45)
9-11	10	13
12-17	14.5	6

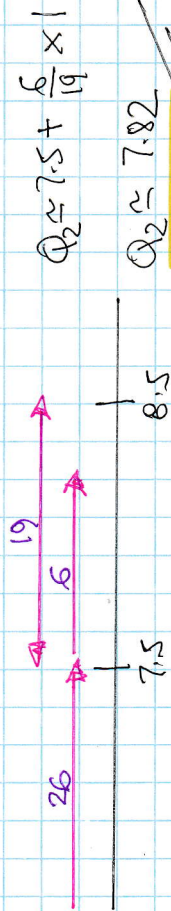
OBTAIN SUMMARY STATISTICS

$$\sum x = 508 \quad \bullet \quad \sum x^2 = 4561 \quad \bullet \quad n = 64$$

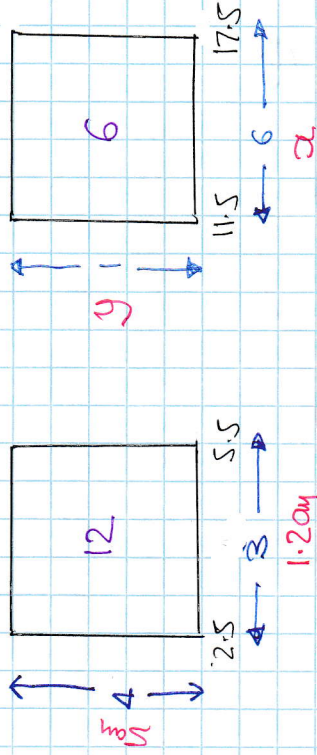
$$\bar{x} = \frac{\sum x}{n} = \frac{508}{64} = 7.9375 = 7.94$$

$$s = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} = 2.87432... \approx 2.87$$

b) MEDIAN (Q_2) IS $\frac{1}{2} \times 64 = 32^{\text{ND}}$ OBS WHICH LIES IN "8"



c) WORKING AT THE DIAGRAM BELOW



$$\frac{3}{1.2} = \frac{6}{x}$$

$$3x = 7.2$$

$$x = 2.4$$

$$\frac{5}{4} = \frac{y}{1}$$

$$4y = 5$$

$$y = 1.25$$

∴ BASE 2.4 cm & HEIGHT 1.25 cm

d) USING THE QUANTILES FOR OBSERVATIONS

$$\begin{aligned} \bullet \text{ LOWER BOUND} &= Q_1 - \frac{2}{3}(Q_3 - Q_1) \\ &= 6.07 - 1.5 \times (9.19 - 6.07) \\ &= 1.39 \end{aligned}$$

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1.68 - IQRS PAPER 1 - QUESTION 6

UPPER BOUND = $Q_3 + 1.5(Q_3 - Q_1) = 9.19 + 1.5(9.19 - 6.07) = 13.87$

$1.39 < 2.5 \Rightarrow$ NO OUTLIERS AT THE BOTTOM END

$11.5 < 13.87 < 17.5 \Rightarrow$ MOST CERTAIN TO HAVE OUTLIERS AT THE TOP END

e)

(MODE) < MEDIAN < MEAN
7.82 7.94

POSITIVE SKEW \Rightarrow NOT APPROPRIATE TO BE MODELLED BY
A NORMAL DISTRIBUTION

1YGB - MINS PAPER - QUESTION 7

a) $P(A) = 0.3 \bullet P(A \cap B') = 0.1 \bullet P(A \cup B') = 0.55$

$\implies P(A \cup B) = P(A) + P(B) - P(A \cap B)$

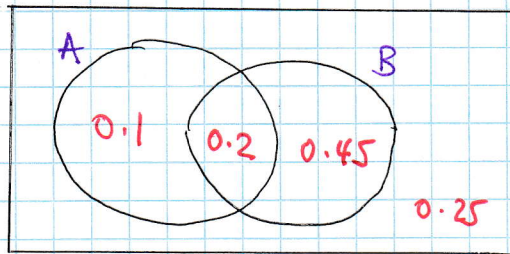
$\implies P(A \cup B') = P(A) + P(B') - P(A \cap B')$

$\implies 0.55 = 0.3 + P(B') - 0.1$

$\implies P(B') = 0.35$

$\therefore P(B) = 1 - P(B') = 1 - 0.35 = \underline{0.65}$

b)



c) I) $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$= \frac{0.2}{0.65}$

$= \frac{20}{65}$

$= \underline{\frac{4}{13}}$

II) $P(B'|A') = \frac{P(B' \cap A')}{P(A')}$

$= \frac{0.25}{0.70}$

$= \frac{25}{70}$

$= \underline{\frac{5}{14}}$

IXGB - MMS PAPER 4 - QUESTION 8

a) START WITH A TREE DIAGRAM OR SOME KIND OF SORTER

X = NO OF BLUE DISCS PICKED

$$P(X=1) = P(\text{Red-Red-Blue}) \times 3 \text{ WAYS} = \left(\frac{2}{5} \times \frac{1}{4} \times \frac{3}{3}\right) \times 3 = \frac{3}{10}$$

$$P(X=2) = P(\text{Red-Blue-Blue}) \times 3 \text{ WAYS} = \left(\frac{2}{5} \times \frac{3}{4} \times \frac{2}{3}\right) \times 3 = \frac{6}{10}$$

$$P(X=3) = P(\text{blue-blue-blue}) \times 1 \text{ way} = \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} = \dots = \frac{1}{10}$$

b) ORGANISING OUTCOMES

$$3, 3, 3, 3 \Rightarrow \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = 0.0001$$

$$\left. \begin{array}{l} 3, 3, 3, 2 \\ 3, 3, 2, 3 \\ 3, 2, 3, 3 \\ 2, 3, 3, 3 \end{array} \right\} \Rightarrow \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{6}{10} \times 4 \text{ WAYS} = 0.0024$$

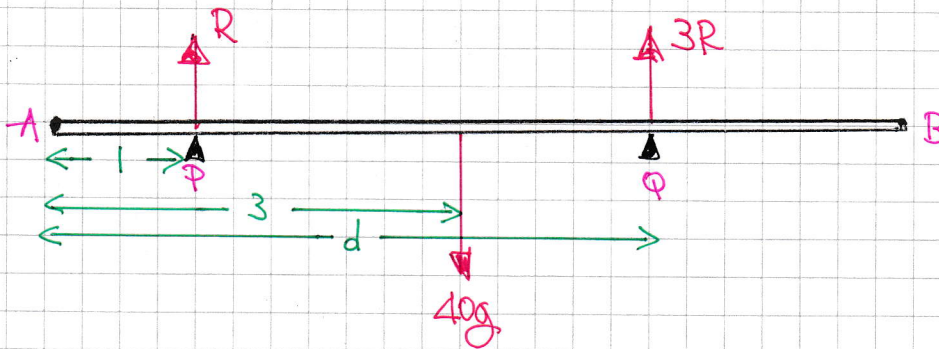
$$\left. \begin{array}{l} 3, 3, 2, 2 \\ 3, 2, 3, 2 \\ 3, 2, 2, 3 \\ 2, 3, 3, 2 \\ 2, 3, 2, 3 \\ 2, 2, 3, 3 \end{array} \right\} \Rightarrow \frac{1}{10} \times \frac{1}{10} \times \frac{6}{10} \times \frac{6}{10} \times 6 \text{ WAYS} = 0.0216$$

$$\left. \begin{array}{l} 3, 3, 3, 1 \\ 3, 1, 3, 3 \\ 3, 1, 3, 3 \\ 1, 3, 3, 3 \end{array} \right\} \Rightarrow \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{3}{10} \times 4 \text{ WAYS} = 0.0012$$

ADDING GIVES 0.0253

YGB - NMS PAPER 1 - QUESTION 9

DRAWING A DIAGRAM



RESOLVING VERTICALLY

$$R + 3R = 40g$$

$$4R = 40g$$

$$R = 10g$$

TAKING MOMENTS ABOUT A

$$\curvearrowright A : (R \times 1) + (3R \times d) = 40g \times 3$$

$$10g + 30g \times d = 120g$$

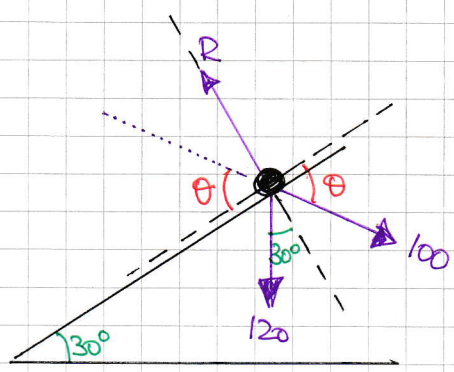
$$10 + 30d = 120$$

$$30d = 110$$

$$d = \frac{11}{3} = \underline{\underline{3\frac{2}{3} \text{ m}}}$$

1YGB - MMS PAPER 1 - QUESTION 10

START WITH A DETAILED DIAGRAM - MODEL THE PUSHING FORCE AS A "PUSHING" FORCE



RESOLVING PARALLEL & PERPENDICULAR TO THE PLANE

$$(1) : 120 \sin 30 = 100 \cos \theta \quad \text{--- (I)}$$

$$(2) : R = 120 \cos 30 + 100 \sin \theta \quad \text{--- (II)}$$

FROM THE FIRST EQUATION (I)

$$\Rightarrow \cos \theta = \frac{120 \sin 30}{100}$$

$$\Rightarrow \cos \theta = 0.6$$

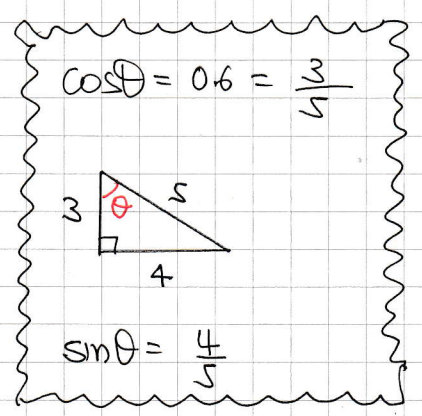
NOW WE HAVE FROM (II)

$$\Rightarrow R = 120 \cos 30 + 100 \sin \theta$$

$$\Rightarrow R = 120 \times \frac{\sqrt{3}}{2} + 100 \times \frac{4}{5}$$

$$\Rightarrow R = 60\sqrt{3} + 80$$

$$\Rightarrow R \approx 184 \text{ N}$$



1YGB - MMS PAPER 1 - QUESTION 11

a) LOOKING AT THE DIAGRAM

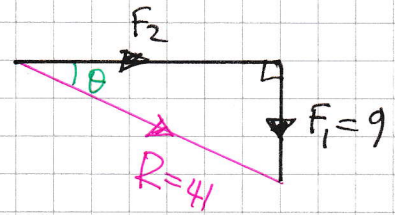
$$\Rightarrow |F_2|^2 + |F_1|^2 = R^2$$

$$\Rightarrow |F_2|^2 + 9^2 = 41^2$$

$$\Rightarrow |F_2|^2 + 81 = 1681$$

$$\Rightarrow |F_2|^2 = 1600$$

$$\Rightarrow |F_2| = 40 \text{ N}$$



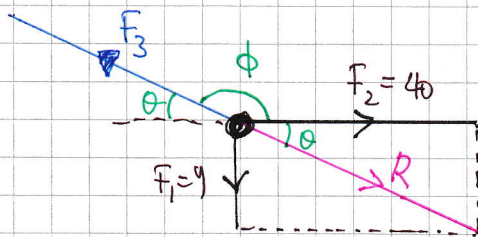
b) BY SIMPLE TRIGONOMETRY

$$\tan \theta = \frac{9}{40} \quad \text{OR} \quad \sin \theta = \frac{9}{41} \quad \text{OR} \quad \cos \theta = \frac{40}{41}$$

$$\theta = 12.7^\circ$$

c) BY INSPECTION, $|F_3| = 41$, SO THE TRIANGLE CLOSES

d) LOOKING AT A NEW DIAGRAM



REQUIRED ANGLE IS ϕ

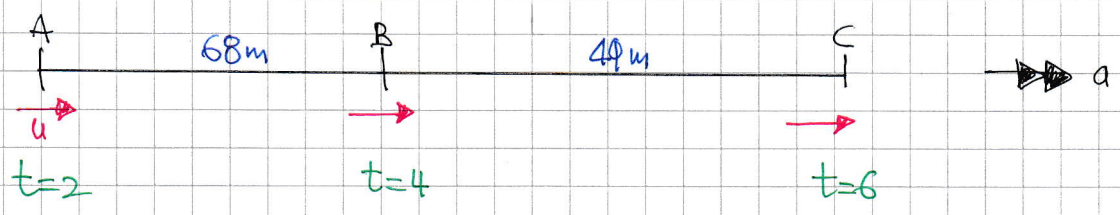
$$\phi = 180 - \theta$$

$$\phi = 180 - 12.7$$

$$\phi \approx 167^\circ$$

1YGB - MMS PAPER 1 - QUESTION 12

PUTTING THE INFORMATION INTO A DIAGRAM



LOOKING AT A TO B

$u = ?$
$a = ?$
$s = 68\text{m}$
$t = 4\text{s}$
$v =$

LOOKING AT A TO C

$u = ?$
$a = ?$
$s = 117\text{m}$
$t = 6$
$v = ?$

$$s = ut + \frac{1}{2}at^2$$

$$68 = 4u + \frac{1}{2}a \times 4^2$$

$$68 = 4u + 8a$$

$$17 = u + 2a$$

$$s = ut + \frac{1}{2}at^2$$

$$117 = 6u + \frac{1}{2}a \times 6^2$$

$$117 = 6u + 18a$$

$$39 = 2u + 6a$$

SOLVING SIMULTANEOUSLY

$$\left. \begin{array}{l} u + 2a = 17 \\ 2u + 6a = 39 \end{array} \right\} \Rightarrow u = 17 - 2a$$

↓

$$2(17 - 2a) + 6a = 39$$

$$34 - 4a + 6a = 39$$

$$2a = 5$$

$$a = 2.5 \text{ ms}^{-2}$$

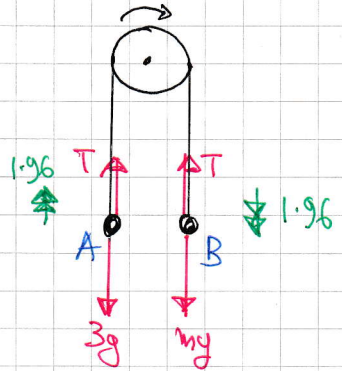
$$\therefore u = 17 - 2 \times 2.5$$

$$u = 12 \text{ ms}^{-1}$$

1YGB - MMS PAPER # - QUESTION 13

a) STARTING WITH A DIAGRAM & CONSIDERING THE EQUATION OF MOTION FOR EACH PARTICLE SEPARATELY

$$\begin{aligned} (A): T - 3g &= 3a \quad [F = ma] \\ T - 3g &= 3 \times 1.96 \\ T &= \underline{35.28 \text{ N}} \end{aligned}$$



b) LOOKING AT THE MOTION OF B

$$\begin{aligned} (B): mg - T &= ma \\ mg - ma &= T \\ m(g - a) &= T \\ m(9.8 - 1.96) &= 35.28 \\ m &= \underline{4.5 \text{ kg}} \end{aligned}$$

c) BY KINEMATICS

$$\begin{cases} u = 0 \\ a = 1.96 \\ s = 1.28 \\ t = \\ v = ? \end{cases}$$

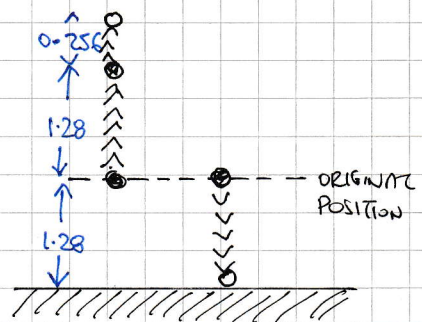
$$\begin{aligned} v^2 &= u^2 + 2as \\ v^2 &= 2 \times 1.96 \times 1.28 \\ v^2 &= 5.0176 \end{aligned}$$

$$\therefore |v| = \underline{2.24 \text{ m/s}}$$

d) ONCE B HITS THE FLOOR - "A" IS MOVING UNDER GRAVITY

$$\begin{cases} u = 2.24 \text{ m/s} \\ a = -9.8 \text{ m/s}^2 \\ s = ? \\ t = \\ v = 0 \end{cases}$$

$$\begin{aligned} v^2 &= u^2 + 2as \\ 0 &= 2.24^2 + 2(-9.8)s \\ 19.6s &= 5.0176 \\ s &= 0.256 \end{aligned}$$

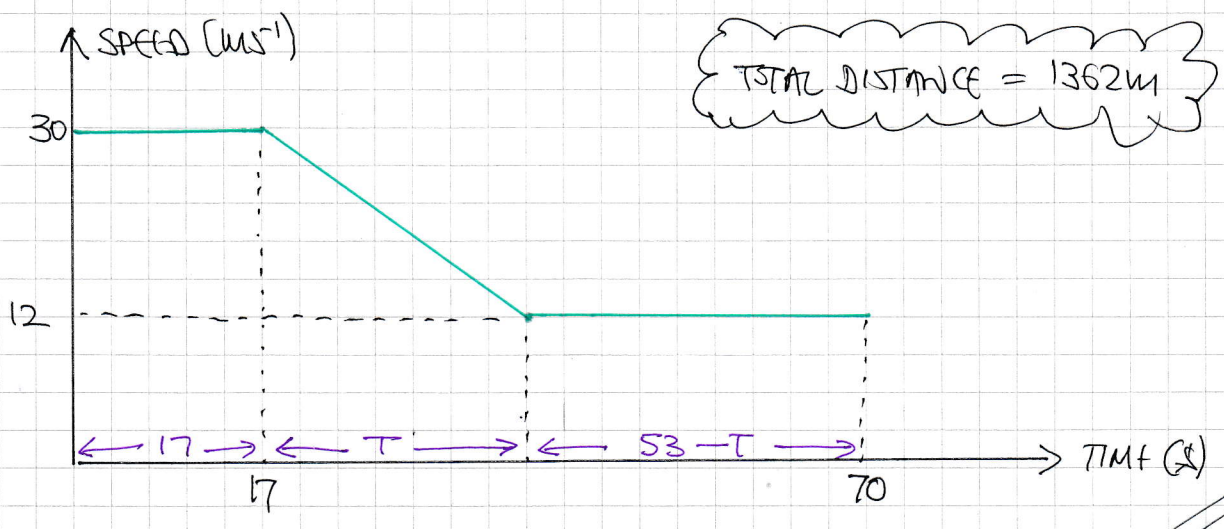


$$\therefore \text{MAX HEIGHT} = 1.28 + 1.28 + 0.256 = \underline{2.816 \text{ m}}$$

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1968 MMS PAPER 1 - QUESTION 14

a) DRAWING A SPEED TIME GRAPH



b) "DISTANCE = AREA"

$$(17 \times 30) + \frac{1}{2} (30 + 12) \times T + (53 - T) \times 12 = 1362$$

$$510 + 21T + 636 - 12T = 1362$$

$$9T = 216$$

$$T = 24$$

FINALLY GRADIENT = DECELERATION

$$a = \frac{\Delta v}{\Delta t} = \frac{12 - 30}{T} = \frac{-18}{24} = -0.75 \text{ m/s}^2$$

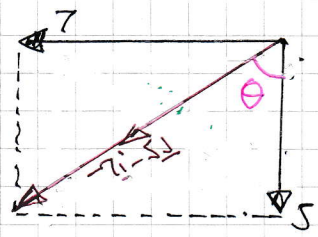
(+ DECELERATION) 0.75 m/s²

YGB-MMS PAPER 4 - QUESTION 15

a) SPEED = MAGNITUDE OF VELOCITY VECTOR

$$\Rightarrow \text{SPEED} = |\underline{v}| = | -7\underline{i} - 5\underline{j} | = \sqrt{49 + 25} = \sqrt{74} \approx \underline{8.60 \text{ kmh}^{-1}}$$

b) DRAWING A DIAGRAM



$$\tan \theta = \frac{7}{5}$$

$$\theta = 54.46\dots^\circ$$

$$\therefore \text{BEARING} = 180 + 54.46\dots \approx \underline{234^\circ}$$

c) OBTAIN A GENERAL EXPRESSION FOR THE POSITION VECTOR OF THE SHIP AS IT WILL BE NEEDED IN PART (d) ALSO

$$\underline{r} = \underline{r}_0 + \underline{v}t$$

$$\underline{r} = (40\underline{i} + 28\underline{j}) + (-7\underline{i} - 5\underline{j})t$$

$$\underline{r} = (40 - 7t)\underline{i} + (28 - 5t)\underline{j}$$

$$\underline{r}_4 = (40 - 7 \times 4)\underline{i} + (28 - 5 \times 4)\underline{j}$$

$$\underline{r}_4 = 12\underline{i} + 8\underline{j}$$

DISTANCE BETWEEN $(-12\underline{i} + \underline{j})$ & $(12\underline{i} + 8\underline{j})$

$$d = \sqrt{(-12 - 12)^2 + (1 - 8)^2} = \sqrt{576 + 49} = \underline{25 \text{ km}}$$

d) EAST OF THE LIGHT-HOUSE \Rightarrow SAME \underline{j} & \underline{i} GREATER THAN 12

$$\underline{r} = (40 - 7t)\underline{i} + (28 - 5t)\underline{j}$$

$$\therefore 28 - 5t = 1$$

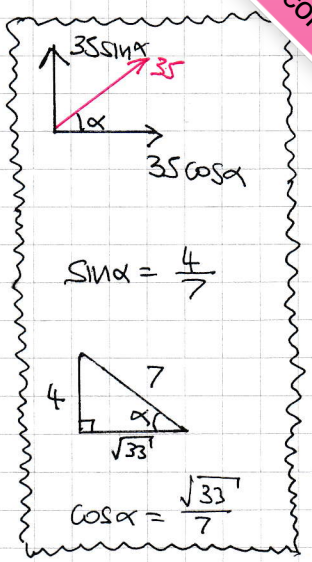
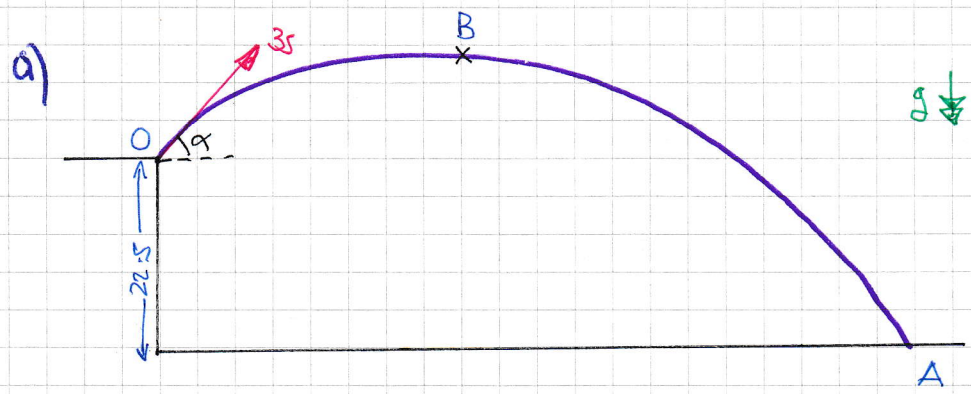
$$5t = 27$$

$$t = 5.4$$

$$\text{NOW } 0.4 \times 60 = 24$$

$$\therefore \underline{17:24}$$

IYGB - MMS PAPER 1 - QUESTION 16



LOOKING AT THE VERTICAL MOTION (O TO B)

$u = 35 \sin \alpha$
$a = -9.8$
$s = ?$
$t =$
$v = 0$

$$v^2 = u^2 + 2as$$

$$0 = (35 \sin \alpha)^2 + 2(-9.8)s$$

$$0 = (35 \times \frac{4}{7})^2 - 19.6s$$

$$19.6s = 400$$

$$s = \frac{1000}{49} \approx 20.408...$$

\therefore MAX HEIGHT = $22.5 + 20.408... = 42.91...m$

b) LOOKING AT THE VERTICAL MOTION (O TO A)

$u = 35 \sin \alpha$
$a = -9.8$
$s = -22.5$
$t = ?$
$v =$

$$s = ut + \frac{1}{2}at^2$$

$$-22.5 = (35 \sin \alpha)t + \frac{1}{2}(-9.8)t^2$$

$$-22.5 = 20t - 4.9t^2$$

$$4.9t^2 - 20t - 22.5 = 0$$

$$49t^2 - 200t - 225 = 0$$

FACTORIZE USING "SUM FACT" THAT $t = 5$

$$(t - 5)(49t + 45) = 0$$

$$t = \frac{5}{-\frac{45}{49}}$$

1YGB - MMS PAPER # - QUESTION 16

c) LOOKING AT THE VERTICAL MOTION FROM O TO A

$$\left| \begin{array}{l} u = 35 \sin \alpha \\ a = -9.8 \\ s = -22.5 \\ t = 5 \\ v \end{array} \right|$$

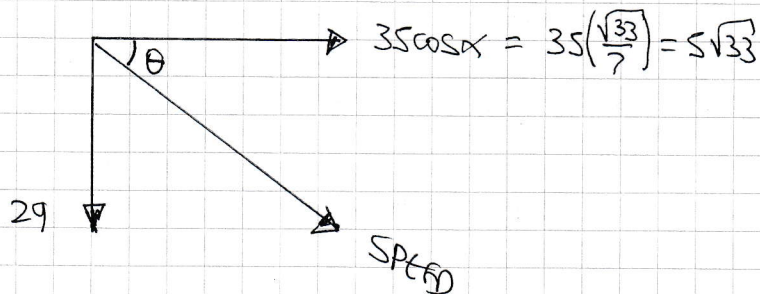
$$v = u + at$$

$$v = 35 \sin \alpha + (-9.8) \times 5$$

$$v = 35 \times \frac{4}{7} - 9.8 \times 5$$

$$v = -29$$

LOOKING AT "SPEEDS" AT A



• $SPEED = \sqrt{29^2 + (5\sqrt{33})^2} = \sqrt{1666} \approx \underline{40.82 \text{ ms}^{-1}}$

• $\tan \theta = \frac{29}{5\sqrt{33}}$

$\theta = \underline{45.3^\circ}$ TO THE HORIZONTAL