

LYGB - FP3 PAPER 2 - QUESTION 1

a) SETTING UP A TABLE OF VALUES WITH GAP OF 0.5

x	0	0.5	1	1.5	2	2.5	3
y	$\ln 4$	$\ln 4.25$	$\ln 5$	$\ln 6.25$	$\ln 8$	$\ln 10.25$	$\ln 13$
	FIRST	ODD	EVEN	ODD	EVEN	ODD	LAST

BY SIMPSON RULE

$$\begin{aligned}
 \text{AREA} &\approx \frac{\text{THICKNESS}}{3} \left[\text{FIRST} + \text{LAST} + (2 \times \text{EVEN}) + (4 \times \text{ODD}) \right] \\
 &\approx \frac{0.5}{3} \left[\ln 4 + \ln 13 + 4[\ln 4.25 + \ln 6.25 + \ln 10.25] + 2[\ln 5 + \ln 8] \right] \\
 &\approx \frac{1}{6} \times 33.75611524 \dots \\
 &\approx \underline{\underline{5.626}}
 \end{aligned}$$

b) USING THE ANSWER FROM PART (a)

$$\begin{aligned}
 \int_0^3 \ln\left(\frac{1}{4}x^2 + 1\right) dx &= \int_0^3 \ln\left[\frac{1}{4}(x^2 + 4)\right] dx = \ln \\
 &= \int_0^3 \ln \frac{1}{4} + \ln(x^2 + 4) dx \\
 &= \int_0^3 \ln(x^2 + 4) dx + \int_0^3 \ln \frac{1}{4} dx \\
 &\approx 5.626 - \ln 4 \int_0^3 1 dx \\
 &\approx 5.626 - (\ln 4) [x]_0^3 \\
 &\approx 5.626 - 3 \ln 4 \\
 &\approx \underline{\underline{1.47}}
 \end{aligned}$$

LYGB - FP3 PAPER 2 - QUESTION 2

SINCE x APPEARS IN THE EXPONENT THE USE OF LOGS MIGHT BE NECESSARY, HOWEVER L'HOSPITAL BUILT WORKS WELL

$$\lim_{x \rightarrow \infty} [x(2^{\frac{1}{x}} - 1)] = \lim_{x \rightarrow \infty} \left[\frac{2^{\frac{1}{x}} - 1}{\frac{1}{x}} \right]$$

THIS IS AN INDETERMINATE FORM OF THE TYPE $\frac{0}{0}$ OR $\frac{\infty}{\infty}$

$$= \lim_{x \rightarrow \infty} \left[\frac{\frac{d}{dx}(2^{\frac{1}{x}} - 1)}{\frac{d}{dx}\left(\frac{1}{x}\right)} \right] = \lim_{x \rightarrow \infty} \left[\frac{2^{\frac{1}{x}} \times \left(-\frac{1}{x^2}\right) \times \ln 2}{-\frac{1}{x^2}} \right]$$

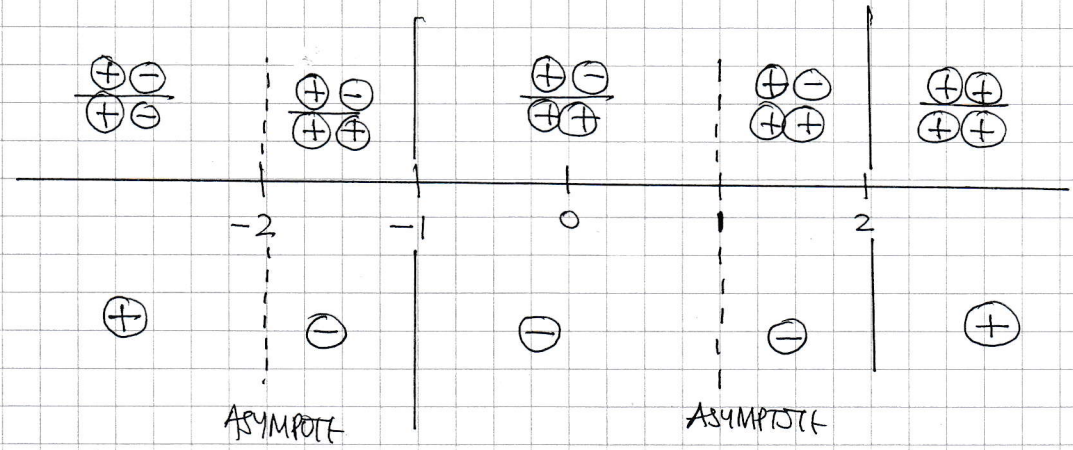
$$= \lim_{x \rightarrow \infty} [2^{\frac{1}{x}} \times \ln 2] = \ln 2$$

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PICK UP THE CRITICAL VALUES AS L.H.S IS FULLY FACTORIZED

$$\frac{(x+1)^2(x-2)}{(x-1)^2(x+2)} < 0 \implies x = \begin{matrix} -1 & \text{REPEATED} \\ 2 \\ 1 & \text{REPEATED} \\ -2 \end{matrix}$$

USING A NUMBER LINE



$\therefore -2 < x < 2, x \neq 1$

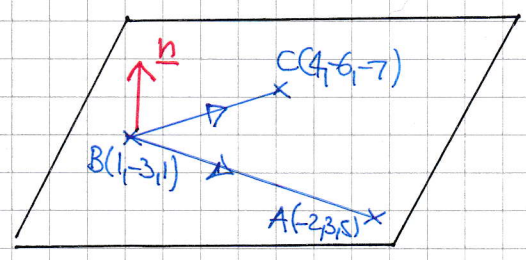
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1YGB - FP3 PAPER R - QUESTION 4

a) START BY FINDING A NORMAL TO THE PLANE

$$\vec{BC} = \underline{c} - \underline{b} = (4, -6, -7) - (1, -3, 1) = (3, -3, -8)$$

$$\vec{BA} = \underline{a} - \underline{b} = (-2, 3, 5) - (1, -3, 1) = (-3, 6, 4)$$



$$\vec{BC} \times \vec{BA} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 3 & -3 & -8 \\ -3 & 6 & 4 \end{vmatrix} = (36, 12, 9)$$

SCALING THE NORMAL VECTOR

$$\underline{n} = (12, 4, 3)$$

EQUATION OF PLANE USING B(1, -3, 1)

$$\Rightarrow 12x + 4y + 3z = \text{CONSTANT}$$

$$\Rightarrow (12 \times 1) + 4(-3) + (3 \times 1) = \text{CONSTANT}$$

$$\Rightarrow \text{CONSTANT} = 3$$

$$\therefore \underline{12x + 4y + 3z = 3}$$

b) EQUATION OF A LINE PASSING THROUGH P(26, 2, 7), PERPENDICULAR TO THE PLANE

$$\underline{r} = (26, 2, 7) + \lambda(12, 4, 3)$$

$$(x, y, z) = (12\lambda + 26, 4\lambda + 2, 3\lambda + 7)$$

SOVING SIMULTANEOUSLY WITH THE EQUATION OF THE PLANE

$$\Rightarrow 12(12\lambda + 26) + 4(4\lambda + 2) + 3(3\lambda + 7) = 3$$

$$\Rightarrow 144\lambda + 312 + 16\lambda + 8 + 9\lambda + 21 = 3$$

$$\Rightarrow 169\lambda = -338$$

$$\Rightarrow \lambda = -2$$

$$\therefore \underline{Q(-2, -6, 1)}$$

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WRITE THE O.D.E AS follows

$$y = y_0, \quad \frac{dy}{dx} = y_1, \quad \frac{d^2y}{dx^2} = y_2, \quad \frac{d^3y}{dx^3} = y_3, \dots, \quad \frac{d^ny}{dx^n} = y_n$$

$$\Rightarrow y_2(x^2+1) + y_1x - 4y_0 = 0$$

DIFFERENTIATE n TIMES BY LEIBNIZ RUL

$$\Rightarrow \frac{d^n}{dx^n} [y_2(x^2+1)] + \frac{d^n}{dx^n} [y_1x] - 4 \frac{d^n}{dx^n} [y_0] = \frac{d^n}{dx^n} [0]$$

$$\begin{aligned} \Rightarrow y_{n+2}(x^2+1) + ny_{n+1}(2x) + \frac{n(n-1)}{2!}y_n(2) + \dots \text{ZERO TERMS} \\ + y_{n+1}x + ny_n(1) + \dots \text{ZERO TERMS} \\ - 4y_n = 0 \end{aligned}$$

$$\Rightarrow y_{n+2}(x^2+1) + (2nx+x)y_{n+1} + (n(n-1)+n-4)y_n = 0$$

$$\Rightarrow y_{n+2}(x^2+1) + (2n+1)xy_{n+1} + (n^2-4)y_n = 0$$

FINALLY SET x=0

$$\Rightarrow y_{n+2} + (n^2-4)y_n = 0$$

$$\Rightarrow \frac{d^{n+2}y}{dx^{n+2}} - (4-n^2)\frac{d^ny}{dx^n} = 0$$

$$\Rightarrow \frac{d^{n+2}y}{dx^{n+2}} = (4-n^2)\frac{d^ny}{dx^n}$$

As required

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$$\frac{dy}{dx} = x + \ln x \quad x=1, y=0$$

USING THE RESULT $\left(\frac{dy}{dx}\right)_0 = \frac{y_1 - y_0}{h}$ FIRST

$$y'_0 = \frac{y_1 - y_0}{h}$$

$$y_1 = h y'_0 + y_0$$

HERE WE HAVE $x_0 = 1, y_0 = 0, h = 0.1$

$$y_1 = h [x_0 + \ln x_0] + y_0$$

$$y_1 = 0.1 [1 + \ln 1] + 0$$

$$y_1 = 0.1$$

NEXT WE ARE USING $\left(\frac{dy}{dx}\right)_0 = \frac{y_1 - y_{-1}}{2h}$

REWRITE AS

$$y'_0 = \frac{y_1 - y_{-1}}{2h}$$

$$y'_n = \frac{y_{n+1} - y_{n-1}}{2h}$$

$$2h y'_n = y_{n+1} - y_{n-1}$$

$$y_{n+1} = 2h y'_n + y_{n-1}$$

$$y_{n+2} = 2h y'_{n+1} + y_n$$

$$y_{n+2} = 2h [x_{n+1} + \ln x_{n+1}] + y_n$$

WITH $x_0 = 1, y_0 = 0$
 $x_1 = 1.1, y_1 = 0.1$
 $x_2 = 1.2$
 $x_3 = 1.3$

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APPLYING THE FORMULA IN SUCCESSION

• $y_2 \approx 2h [x_1 + \ln x_1] + y_0$

$y_2 \approx 2 \times 0.1 [1.1 + \ln(1.1)] + 0$

$y_2 \approx 0.239062... \approx \underline{0.2391}$

• $y_3 \approx 2h [x_2 + \ln x_2] + y_1$

$y_3 \approx 2(0.1) [1.2 + \ln(1.2)] + 0.1$

$y_3 \approx 0.376464... \approx \underline{0.3765}$

• $y_4 \approx 2h (x_3 + \ln x_3) + y_2$

$y_4 \approx 2(0.1) [1.3 + \ln(1.3)] + 0.239062...$

$y_4 \approx 0.551534... \approx \underline{0.5515}$

IYGB - FP3 PAPER R - QUESTION 7

a) DIFFERENTIATE THE EQUATION WITH RESPECT TO x

$$\Rightarrow \frac{d}{dx} \left[e^{-x} \frac{d^2 y}{dx^2} \right] = \frac{d}{dx} \left[2y \frac{dy}{dx} \right] + \frac{d}{dx} [y^2 + 1]$$

$$\Rightarrow -e^{-x} \frac{d^2 y}{dx^2} + e^{-x} \frac{d^3 y}{dx^3} = 2 \frac{dy}{dx} \frac{dy}{dx} + 2y \frac{d^2 y}{dx^2} + 2y \frac{dy}{dx}$$

$$\Rightarrow e^{-x} \frac{d^3 y}{dx^3} = e^{-x} \frac{d^2 y}{dx^2} + 2y \frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} \frac{dy}{dx} + 2y \frac{dy}{dx}$$

$$\Rightarrow \underline{e^{-x} \frac{d^3 y}{dx^3} = (e^{-x} + 2y) \frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} \left[\frac{dy}{dx} + y \right]}$$

As Required

b) EVALUATE AT $x=0$

$x=0$ $y=1$

$\frac{dy}{dx} = 2$

$\frac{d^2 y}{dx^2} = 6 \rightarrow$

$e^{-0} \frac{d^2 y}{dx^2} \Big|_{x=0} = 2 \times 1 \times 2 + 1^2 + 1$

$\frac{d^3 y}{dx^3} = 30 \rightarrow$

$e^{-0} \frac{d^3 y}{dx^3} = (e^{-0} + 2 \times 1) \times 6 + 2 \times 2 [2 + 1]$

HENCE WE HAVE

$$y = y_0 + xy'_0 + \frac{x^2}{2!} y''_0 + \frac{x^3}{3!} y'''_0 + o(x^4)$$

$$y = 1 + 2x + \frac{x^2}{2} \times 6 + \frac{x^3}{6} \times 30 + o(x^4)$$

$$\underline{y = 1 + 2x + 3x^2 + 5x^3 + o(x^4)}$$

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IF $y = \alpha v(x)$ $\Rightarrow v(x) = \frac{y(x)}{\alpha}$

$$\frac{dy}{dx} = 1 \times v + \alpha \frac{dv}{dx}$$

TRANSFORM THE O.D.E.

$$\begin{aligned} \Rightarrow 2 \frac{dy}{dx} &= 1 + \frac{y^2}{\alpha^2} \\ \Rightarrow 2 \left[v + \alpha \frac{dv}{dx} \right] &= 1 + v^2 \\ \Rightarrow 2v + 2\alpha \frac{dv}{dx} &= 1 + v^2 \\ \Rightarrow 2\alpha \frac{dv}{dx} &= v^2 - 2v + 1 \\ \Rightarrow 2\alpha \frac{dv}{dx} &= (v-1)^2 \\ \Rightarrow \frac{2}{(v-1)^2} dv &= \frac{1}{\alpha} dx \end{aligned}$$

$$\begin{aligned} \Rightarrow \frac{2}{-2} - \frac{2}{v-1} &= \ln \alpha - \ln e \\ \Rightarrow -1 - \frac{2}{v-1} &= \ln \alpha - 1 \\ \Rightarrow -\frac{2}{v-1} &= \ln \alpha \\ \Rightarrow -\frac{2}{\ln \alpha} &= v-1 \\ \Rightarrow v &= 1 - \frac{2}{\ln \alpha} \\ \Rightarrow \frac{y}{\alpha} &= 1 - \frac{2}{\ln \alpha} \\ \Rightarrow y &= \alpha - \frac{2\alpha}{\ln \alpha} \end{aligned}$$

INTEGRATING BOTH SIDES SUBJECT

$x=e \quad y=-e \quad v=-1$

$$\begin{aligned} \Rightarrow \int_{v=-1}^v \frac{2}{(v-1)^2} dv &= \int_{x=e}^x \frac{1}{\alpha} dx \\ \Rightarrow \left[-\frac{2}{v-1} \right]_{-1}^v &= \left[\ln \alpha \right]_e^x \\ \Rightarrow \left[\frac{2}{v-1} \right]_v^{-1} &= \left[\ln \alpha \right]_e^x \end{aligned}$$

IYGB - FP3 PAPER 2 - QUESTION 9

a) DIFFERENTIATE W.R.T x

$$\frac{d}{dx} \left(\frac{x^2}{a^2} - \frac{y^2}{b^2} \right) = \frac{d}{dx} (1)$$

$$\frac{2x}{a^2} - \frac{2y}{b^2} \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{b^2 x}{a^2 y}$$

EVALUATE AT P

$$\begin{aligned} \left. \frac{dy}{dx} \right|_P &= \frac{b^2 (a \sec \theta)}{a^2 (b \tan \theta)} = \frac{b}{a} \sec \theta \cot \theta = \frac{b}{a} \frac{1}{\cancel{\cos \theta}} \frac{\cancel{\cos \theta}}{\sin \theta} \\ &= \frac{b}{a \sin \theta} \end{aligned}$$

NORMAL EQUATION IS GIVEN BY

$$y - b \tan \theta = - \frac{a \sin \theta}{b} (x - a \sec \theta)$$

$$by - b^2 \tan \theta = -a x \sin \theta + a^2 \sin \theta \sec \theta$$

$$by + a x \sin \theta = b^2 \tan \theta + a^2 \sin \theta \times \frac{1}{\cos \theta}$$

$$by + a x \sin \theta = b^2 \tan \theta + a^2 \tan \theta$$

$$\underline{by + a x \sin \theta = (b^2 + a^2) \tan \theta}$$

AS REQUIRED

b) FIRSTLY FIND THE CO-ORDINATES OF X $\Rightarrow y=0$

$$a x \sin \theta = (a^2 + b^2) \tan \theta$$

$$x = \frac{a^2 + b^2}{a} \frac{\tan \theta}{\sin \theta}$$

$$x = \frac{a^2 + b^2}{a} \sec \theta$$

$$\therefore X \left(\frac{a^2 + b^2}{a} \sec \theta, 0 \right)$$

1YGB - FP3 PAPER 2 - QUESTION 9

USING THE ECCENTRICITY RELATION

$$b^2 = a^2(e^2 - 1)$$

$$b^2 = a^2\left(\frac{9}{4} - 1\right)$$

$$b^2 = \frac{5}{4}a^2$$

$$\Rightarrow X\left(\frac{a^2 + \frac{5}{4}a^2}{a} \sec\theta, 0\right)$$

$$X\left(\frac{9}{4}a \sec\theta, 0\right)$$

NEXT THE FOCI S' WITH POSITIVE x CO-ORDINATE

$$S'(ae, 0) \Rightarrow S'\left(\frac{3}{2}a, 0\right)$$

FINALLY WE HAVE

$$\Rightarrow |OX| = 3|OS|$$

$$\Rightarrow \frac{9}{4}a \sec\theta = 3 \times \frac{3}{2}a$$

$$\Rightarrow \frac{9}{4} \sec\theta = \frac{9}{2}$$

$$\Rightarrow \sec\theta = 2$$

$$\Rightarrow \cos\theta = \frac{1}{2}$$

$$\therefore \theta = \frac{\pi}{3} \text{ and } \frac{5\pi}{3}$$