

# YGB - FP2 PAPER M - QUESTION 1

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 12(x + e^x)$$

START WITH THE AUXILIARY EQUATION

$$\lambda^2 + 5\lambda + 6 = 0$$

$$(\lambda + 2)(\lambda + 3) = 0$$

$$\lambda = \begin{cases} -2 \\ -3 \end{cases}$$

$\therefore$  COMPLEMENTARY FUNCTION:  $y = Ae^{-2x} + Be^{-3x}$

FOR PARTICULAR INTEGRAL WE TRY  $y = Px + Q + Re^x$

$$\frac{dy}{dx} = P + Re^x$$

$$\frac{d^2y}{dx^2} = Re^x$$

SUB INTO THE O.D.E

$$(Re^x) + 5(P + Re^x) + 6(Px + Q + Re^x) \equiv 12x + 12e^x$$

$$6Px + (5P + 6Q) + e^x(R + 5R + 6R) \equiv 12x + 12e^x$$

$$\begin{aligned} \therefore P &= 2 & R &= 1 & \& \quad 5P + 6Q &= 0 \\ & & & & & 10 + 6Q &= 0 \\ & & & & & Q &= -\frac{5}{3} \end{aligned}$$

HENCE THE GENERAL SOLUTION IS

$$y = Ae^{-2x} + Be^{-3x} + e^x + 2x - \frac{5}{3}$$

IYGB - FP2 PAPER M - QUESTION 2

$$\int_e^{\infty} \frac{1 - \ln x}{x^2} dx = \dots \text{ INTEGRATION BY PARTS}$$

$1 - \ln x$	$-\frac{1}{x}$
$-\frac{1}{x}$	$\frac{1}{x^2}$

$$= \left[ -\frac{1}{x} (1 - \ln x) \right]_e^{\infty} - \int_e^{\infty} \frac{1}{x^2} dx$$

$$= \left[ -\frac{1}{x} + \frac{1}{x} \ln x \right]_e^{\infty} - \left[ -\frac{1}{x} \right]_e^{\infty}$$

$$= \left[ \cancel{-\frac{1}{x}} + \frac{1}{x} \ln x + \cancel{\frac{1}{x}} \right]_e^{\infty}$$

$$= \left[ \frac{\ln x}{x} \right]_e^{\infty}$$

$$= \lim_{k \rightarrow \infty} \left[ \left[ \frac{\ln x}{x} \right]_e^k \right]$$

$$= \lim_{k \rightarrow \infty} \left[ \frac{\ln k}{k} - \frac{\ln e}{e} \right]$$

$$= 0 - \frac{1}{e}$$

$$= -\frac{1}{e}$$

$\frac{\ln k}{k} \rightarrow 0$  AS  $k \rightarrow \infty$   
AS  $\frac{1}{k} \rightarrow 0$  FASTER THAN  $\ln k \rightarrow \infty$

- 1 -

IYGB - FP2 PAPER 1 - QUESTION 3

a)  $f(r) = r^2(r+1)^2 - (r-1)r^2$   
 $= r^2[(r+1)^2 - (r-1)^2]$   
 $= r^2(r+1+r-1)(r+1-r+1)$   
 $= r^2 \times 2r \times 2$   
 $= \underline{4r^3}$

b) USING PART (a)

$4r^3 \equiv r^2(r+1)^2 - (r-1)^2r^2$

IF $r=1$	$4 \times 1^3 =$	<del><math>1^2 \times 2^2</math></del> $-$ <del><math>0^2 \times 1^2</math></del>
IF $r=2$	$4 \times 2^3 =$	<del><math>2^2 \times 3^2</math></del> $-$ <del><math>1^2 \times 2^2</math></del>
IF $r=3$	$4 \times 3^3 =$	<del><math>3^2 \times 4^2</math></del> $-$ <del><math>2^2 \times 3^2</math></del>
IF $r=4$	$4 \times 4^3 =$	<del><math>4^2 \times 5^2</math></del> $-$ <del><math>3^2 \times 4^2</math></del>
⋮	⋮	⋮
⋮	⋮	⋮
⋮	⋮	⋮
IF $r=20$	$4 \times 20^3 =$	<del><math>20^2 \times 21^2</math></del> $-$ <del><math>19^2 \times 20^2</math></del>

ADDING  $\sum_{r=1}^{20} 4r^3 = 20^2 \times 21^2$

$\Rightarrow 4 \sum_{r=1}^{20} r^3 = 20^2 \times 21^2$

$\Rightarrow \sum_{r=1}^{20} r^3 = \frac{20^2 \times 21^2}{4}$

$\Rightarrow \underline{\sum_{r=1}^{20} r^3 = 44100}$

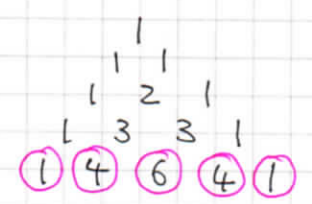
# 1YGB - FP2 PAPER M - QUESTION 4

Let  $\cos\theta + i\sin\theta \equiv C + iS$

$\Rightarrow (\cos\theta + i\sin\theta)^4 = (C + iS)^4$

$\Rightarrow \cos 4\theta + i\sin 4\theta = C^4 + 4iC^3S - 6C^2S^2 - 4iCS^3 + S^4$

NOTE THE PATTERN  
 + + - - + + ...  
 Re Im Re Im Re Im ...



EQUATE REAL & IMAGINARY PARTS

$\cos 4\theta = C^4 - 6C^2S^2 + S^4$

$\sin 4\theta = 4C^3S - 4CS^3$

FORMING THE  $\tan 4\theta$

$\Rightarrow \tan 4\theta = \frac{\sin 4\theta}{\cos 4\theta} = \frac{4C^3S - 4CS^3}{C^4 - 6C^2S^2 + S^4}$

$\Rightarrow \tan 4\theta = \frac{\frac{4C^3S}{C^4} - \frac{4CS^3}{C^4}}{\frac{C^4}{C^4} - \frac{6C^2S^2}{C^4} + \frac{S^4}{C^4}}$

$\Rightarrow \tan 4\theta = \frac{4T - 4T^3}{1 - 6T^2 + T^4}$

$\therefore \tan 4\theta = \frac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$

IYGB - FP2 PAPER M - QUESTION 5

a) USING STANDARD EXPANSIONS

•  $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + o(x^4)$

$e^{2x} = 1 + (2x) + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} + o(x^4)$

$e^{2x} = 1 + 2x + 2x^2 + \frac{4}{3}x^3 + o(x^4)$

•  $\sin x = x - \frac{x^3}{3!} + o(x^5)$

$\sin 3x = (3x) - \frac{(3x)^3}{3!} + o(x^5)$

$\sin 3x = 3x - \frac{9}{2}x^3 + o(x^5)$

COMBINING RESULTS

$\Rightarrow y = e^{2x} \sin 3x = \left[ 1 + 2x + 2x^2 + \frac{4}{3}x^3 + o(x^4) \right] \left[ 3x - \frac{9}{2}x^3 + o(x^5) \right]$

$\Rightarrow y = \begin{matrix} 3x & & -\frac{9}{2}x^3 & & + o(x^5) \\ & 6x^2 & & -9x^4 & + o(x^6) \\ & & 6x^3 & & + o(x^5) \\ & & & 4x^4 & + o(x^6) \end{matrix}$

$\Rightarrow y = \underline{3x + 6x^2 + \frac{3}{2}x^3 - 5x^4 + o(x^5)}$

1YGB - FP2 PAPER N1 - QUESTION 5

b) USING PART (a)

$$\begin{aligned} \int_0^{0.1} e^{2x} \sin 3x \, dx &\approx \int_0^{0.1} 3x + 6x^2 + \frac{3}{2}x^3 - 5x^4 \, dx \\ &\approx \left[ \frac{3}{2}x^2 + 2x^3 + \frac{3}{8}x^4 - x^5 \right]_0^{0.1} \\ &\approx \left( \frac{3}{200} + \frac{1}{500} + \frac{3}{80000} - \frac{1}{100000} \right) - (0) \\ &\approx \underline{0.0170275\dots} \end{aligned}$$

1YG-B, FP2 PAPER M, QUESTION 6

a) SOLVING THE EQUATIONS SIMULTANEOUSLY

$$\left. \begin{aligned} r &= 1 + \sin 2\theta \\ r &= 1.5 \end{aligned} \right\} \Rightarrow 1 + \sin 2\theta = 1.5$$

$$\Rightarrow \sin 2\theta = 0.5$$

$$\Rightarrow 2\theta = \left\langle \frac{\pi}{6}, \dots \right.$$

$$\left. \frac{5\pi}{6}, \dots \right.$$

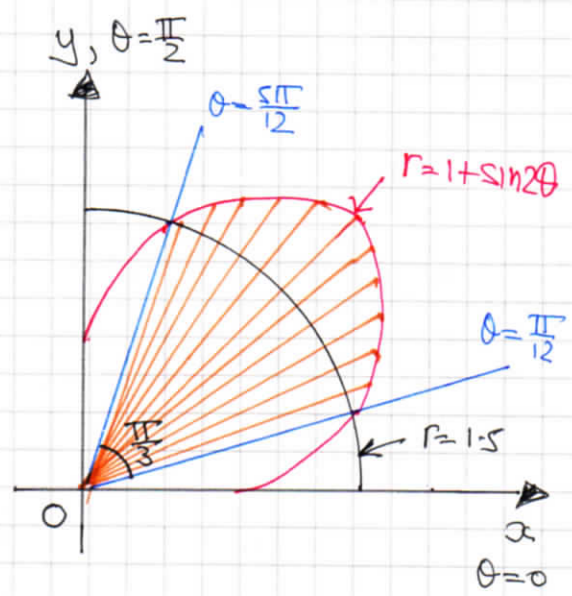
$$\Rightarrow \theta = \left\langle \frac{\pi}{12}, \dots \right.$$

$$\left. \frac{5\pi}{12}, \dots \right.$$

$\therefore (r, \theta) = (1.5, \frac{\pi}{12})$  or  $(r, \theta) = (1.5, \frac{5\pi}{12})$

b) AREA OF  $\frac{1}{6}$  OF A CIRCLE OF RADIUS 1.5.  
( $\frac{\pi}{3} = \frac{1}{6}$  OF A CIRCLE)

$$\begin{aligned} \text{AREA} &= \frac{1}{6} \times \pi r^2 \\ &= \frac{1}{6} \times \pi \times \left(\frac{3}{2}\right)^2 \\ &= \frac{3}{8}\pi \end{aligned}$$



AREA OF POLAR SECTOR DEFINED BY  $r = 1 + \sin 2\theta$

$$\text{AREA} = \int_{\theta_1}^{\theta_2} \frac{1}{2} r^2 d\theta$$

$$\text{AREA} = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{1}{2} (1 + \sin 2\theta)^2 d\theta$$

1YGB - FP2 PAGE 11 - QUESTION 6

$$\text{AREA} = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{1}{2} (1 + 2\sin 2\theta + \sin^2 2\theta) d\theta$$

$$\text{AREA} = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{1}{2} \left[ 1 + 2\sin 2\theta + \frac{1}{2} + \frac{1}{2} \cos 4\theta \right] d\theta$$

$$\text{AREA} = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{3}{4} + \sin 2\theta - \frac{1}{4} \cos 4\theta d\theta$$

$$\text{AREA} = \left[ \frac{3}{4}\theta - \frac{1}{2} \cos 2\theta - \frac{1}{16} \sin 4\theta \right]_{\frac{\pi}{12}}^{\frac{5\pi}{12}}$$

$$\text{AREA} = \left[ \frac{5}{16}\pi - \frac{1}{2} \left( -\frac{\sqrt{3}}{2} \right) - \frac{1}{16} \left( -\frac{\sqrt{3}}{2} \right) \right] - \left[ \frac{1}{16}\pi - \frac{1}{2} \left( \frac{\sqrt{3}}{2} \right) - \frac{1}{16} \left( \frac{\sqrt{3}}{2} \right) \right]$$

$$\text{AREA} = \frac{5}{16}\pi + \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{32} - \frac{1}{16}\pi + \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{32}$$

$$\text{AREA} = \frac{1}{4}\pi + \frac{9}{16}\sqrt{3}$$

HENCE THE REQUIRED AREA CAN BE FOUND

$$\text{REQUIRED AREA} = \left( \frac{1}{4}\pi + \frac{9}{16}\sqrt{3} \right) - \frac{3}{8}\pi$$

$$= \frac{9}{16}\sqrt{3} - \frac{1}{8}\pi$$

$$= \frac{1}{16} (9\sqrt{3} - 2\pi)$$



## 1YGB - FP2 PAPER M - QUESTION 7

- START BY USING THE "COMPOUND ANGLE" IDENTITIES IN HYPERBOLIC

$$\begin{aligned}
 5\cosh\alpha + 3\sinh\alpha &\equiv R\cosh(\alpha + \alpha) \\
 &\equiv R\cosh\alpha\cosh\alpha + R\sinh\alpha\sinh\alpha \\
 &\equiv (R\cosh\alpha)\cosh\alpha + (R\sinh\alpha)\sinh\alpha
 \end{aligned}$$

- HENCE WE HAVE

$$\begin{aligned}
 \left. \begin{aligned} R\cosh\alpha &= 5 \\ R\sinh\alpha &= 3 \end{aligned} \right\} &\Rightarrow \begin{aligned} R^2\cosh^2\alpha &= 25 \\ R^2\sinh^2\alpha &= 9 \end{aligned} \\
 &\Rightarrow R^2\cosh^2\alpha - R^2\sinh^2\alpha = 16 && \leftarrow \underline{\text{SUBTRACT}} \\
 &\Rightarrow R^2(\cosh^2\alpha - \sinh^2\alpha) = 16 \\
 &\Rightarrow R^2 = 16 \\
 &\Rightarrow \underline{R = +4}
 \end{aligned}$$

$$\& R\sinh\alpha = 3$$

$$\Rightarrow 4\sinh\alpha = 3$$

$$\Rightarrow \sinh\alpha = \frac{3}{4}$$

$$\Rightarrow \alpha = \operatorname{arsinh}\left(\frac{3}{4}\right) = \ln\left[\frac{3}{4} + \sqrt{\frac{9}{16} + 1}\right]$$

$$\Rightarrow \alpha = \ln\left[\frac{3}{4} + \sqrt{\frac{25}{16}}\right] = \ln\left[\frac{3}{4} + \frac{5}{4}\right]$$

$$\Rightarrow \underline{\alpha = \ln 2}$$

- HENCE THE EQUATION BECOMES

$$\Rightarrow 5\cosh\alpha + 3\sinh\alpha = 12$$

$$\Rightarrow 4\cosh(\alpha + \ln 2) = 12$$

$$\Rightarrow \cosh(\alpha + \ln 2) = 3$$

$$\Rightarrow \alpha + \ln 2 = \pm \operatorname{arcosh} 3$$

NYGB - FP2 PAPER M - QUESTION 7

$$\Rightarrow x + \ln 2 = \pm \ln[3 + \sqrt{3^2 - 1}]$$

$$\Rightarrow x + \ln 2 = \pm \ln[3 + 2\sqrt{2}]$$

$$\begin{aligned} \Rightarrow x + \ln 2 &= \begin{cases} \ln(3 + 2\sqrt{2}) \\ -\ln(3 + 2\sqrt{2}) = \ln\left(\frac{1}{3 + 2\sqrt{2}}\right) \\ &= \ln\left(\frac{3 - 2\sqrt{2}}{(3 + 2\sqrt{2})(3 - 2\sqrt{2})}\right) \\ &= \ln\left(\frac{3 - 2\sqrt{2}}{9 - 8}\right) \\ &= \ln(3 - 2\sqrt{2}) \end{cases} \end{aligned}$$

$$\Rightarrow x = \begin{cases} -\ln 2 + \ln(3 + 2\sqrt{2}) \\ -\ln 2 + \ln(3 - 2\sqrt{2}) \end{cases}$$

$$\Rightarrow x = \begin{cases} \ln\left(\frac{3 + 2\sqrt{2}}{2}\right) \\ \ln\left(\frac{3 - 2\sqrt{2}}{2}\right) \end{cases}$$

$$\Rightarrow x = \begin{cases} \ln\left(\frac{3}{2} + \sqrt{2}\right) \\ \ln\left(\frac{3}{2} - \sqrt{2}\right) \end{cases} //$$

IYGB - FP2 PAPER 1 - QUESTION 8

a) STARTING AS SUGGESTED

$\Rightarrow y = \arctan x$

$\Rightarrow \tan y = x$

$\Rightarrow x = \tan y$

DIFFERENTIATE W.R.T y

$\Rightarrow \frac{dx}{dy} = \sec^2 y$

$\Rightarrow \frac{dx}{dy} = 1 + \tan^2 y$

$\Rightarrow \frac{dx}{dy} = 1 + x^2$

$\Rightarrow \frac{dy}{dx} = \frac{1}{1+x^2}$  ~~AS REQUIRED~~

b)  $f(x) = \arctan(x^{\frac{1}{2}})$

$\Rightarrow f'(x) = \frac{1}{1+(x^{\frac{1}{2}})^2} \times \frac{1}{2}x^{-\frac{1}{2}} = \frac{\frac{1}{2}x^{-\frac{1}{2}}}{1+x} = \frac{1}{2}x^{-\frac{1}{2}}(1+x)^{-1}$

DIFFERENTIATE AGAIN VIA THE PRODUCT RULE

$\Rightarrow f''(x) = -\frac{1}{4}x^{-\frac{3}{2}}(1+x)^{-1} + \frac{1}{2}x^{-\frac{1}{2}} \times (-1)(1+x)^{-2}$

$\Rightarrow f''(x) = -\frac{1}{4}x^{-\frac{3}{2}}(1+x)^{-1} - \frac{1}{2}x^{-\frac{1}{2}}(1+x)^{-2}$

$\Rightarrow f''(x) = -\frac{1}{4}x^{-\frac{3}{2}}(1+x)^{-2} [(1+x) + 2x]$

$\Rightarrow f''(x) = -\frac{1}{4}x^{-\frac{3}{2}}(1+x)^{-2}(3x+1)$

~~AS REQUIRED~~

14GB - FP2 PAPER M - QUESTION 9

WRITE THE O.D.E IN THE USUAL ORDER

$$\Rightarrow \frac{dy}{dx} + \frac{y}{x} = \frac{5}{(x^2+2)(4x^2+3)}$$

INTEGRATING FACTOR CAN BE FOUND

$$e^{\int \frac{1}{x} dx} = e^{\ln x} = x$$

HENCE WE OBTAIN

$$\Rightarrow \frac{d}{dx}(yx) = \frac{5x}{(x^2+2)(4x^2+3)}$$

$$\Rightarrow yx = \int \frac{5x}{(x^2+2)(4x^2+3)} dx$$

PARTIAL FRACTIONS ARE NEEDED

$$\frac{5x}{(x^2+2)(4x^2+3)} \equiv \frac{Ax+B}{x^2+2} + \frac{Cx+D}{4x^2+3}$$

$$\boxed{5x \equiv (Ax+B)(4x^2+3) + (x^2+2)(Cx+D)}$$

$$5x \equiv 4Ax^3 + 4Bx^2 + 3Ax + 3B + Cx^3 + Dx^2 + 2Cx + 2D$$

$$5x \equiv (4A+C)x^3 + (4B+D)x^2 + (3A+2C)x + (3B+2D)$$

$$\left. \begin{matrix} 4A+C=0 \\ 3A+2C=5 \end{matrix} \right\} \Rightarrow \left. \begin{matrix} 8A+2C=0 \\ 3A+2C=5 \end{matrix} \right\} \Rightarrow \begin{matrix} A=-1 \\ C=4 \end{matrix}$$

$$\left. \begin{matrix} 4B+D=0 \\ 3B+2D=0 \end{matrix} \right\} \Rightarrow \left. \begin{matrix} 8B+2D=0 \\ 3B+2D=0 \end{matrix} \right\} \Rightarrow \begin{matrix} B=0 \\ D=0 \end{matrix}$$

## 1YGB - FP2 PAPER 1 - QUESTION 9

CARRYING OUT THE REQUIRED INTEGRATION

$$\Rightarrow yx = \int \frac{4x}{4x^2+3} - \frac{x}{x^2+2} dx$$

$$\Rightarrow 2yx = \int \frac{8x}{4x^2+3} - \frac{2x}{x^2+2} dx$$

$$\Rightarrow 2yx = \ln(4x^2+3) - \ln(x^2+2) + \ln A$$

$$\Rightarrow 2yx = \ln \left[ \frac{A(4x^2+3)}{x^2+2} \right]$$

APPLY CONDITION  $x=1$ ,  $y = \frac{1}{2} \ln \frac{7}{6}$

$$\Rightarrow 2 \times \frac{1}{2} \ln \frac{7}{6} \times 1 = \ln \left( \frac{7A}{3} \right)$$

$$\Rightarrow \ln \frac{7}{6} = \ln \frac{7A}{3}$$

$$\Rightarrow \frac{7}{6} = \frac{7A}{3}$$

$$\Rightarrow A = \frac{1}{2}$$

FINALLY WE HAVE

$$\Rightarrow 2yx = \ln \left[ \frac{4x^2+3}{2(x^2+2)} \right]$$

$$\Rightarrow y = \frac{1}{2x} \ln \left[ \frac{4x^2+3}{2x^2+4} \right]$$

AS REQUIRED