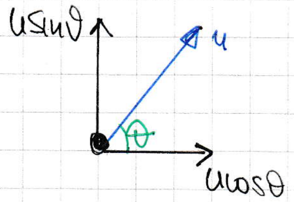
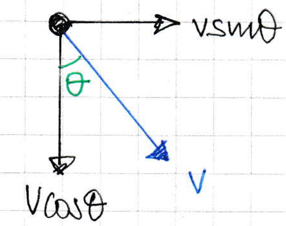
IYGB - FM1 PAPER P - QUESTION 1

a) STARTING WITH A BEFORE & AFTER DIAGRAM - LET THE "BOUNCING SPEED" BE V

BEFORE



AFTER



NO MOMENTUM EXCHANGE IN A DIRECTION PARALLEL TO THE WALL

$$\Longrightarrow u\cos\theta = v\sin\theta$$

$$\Longrightarrow \frac{u}{v} = \frac{\sin\theta}{\cos\theta}$$

$$\Longrightarrow \tan\theta = \frac{u}{v}$$

BY RESTITUTION, PERPENDICULAR TO THE WALL

$$\Longrightarrow e = \frac{\text{SEP}}{\text{APP}}$$

$$\Longrightarrow e = \frac{v\cos\theta}{u\sin\theta}$$

$$\Longrightarrow \frac{1}{e} = \frac{u\sin\theta}{v\cos\theta}$$

$$\Longrightarrow \frac{1}{e} = \frac{u}{v}\tan\theta$$

COMBINING EQUATIONS FROM ABOVE

$$\frac{1}{e} = \tan^2\theta$$

$$\tan\theta = +\sqrt{\frac{1}{e}} \qquad (\theta \text{ acute})$$

$$\tan\theta = \frac{1}{\sqrt{e}}$$

// AS REQUIRED

b) FINALLY WE HAVE

$$0 < e < 1$$

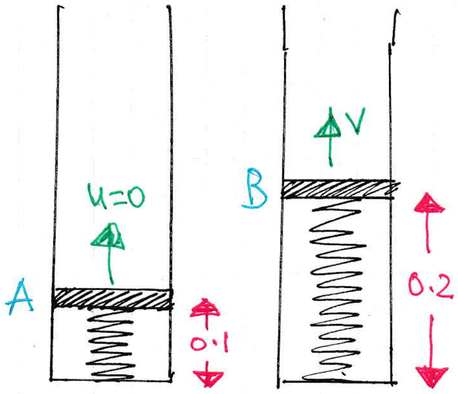
$$0 < \sqrt{e} < 1$$

$$1 < \frac{1}{\sqrt{e}} < \infty$$

$$\therefore \tan\theta > 1$$

$$\therefore 45 < \theta < 90$$ //

IYGB - FMI PAPER O - QUESTION 2



$\lambda = 2000$ N
$l = 0.2$ m
$m = 1.5$ kg

BY ENERGIES TAKING THE LEVEL OF "A" AS THE ZERO GRAVITATIONAL POTENTIAL LEVEL

$$\Rightarrow \cancel{KE_A} + \cancel{PE_A} + EE_A + \cancel{W_{IN}} - W_{OUT} = KE_B + PE_B + \cancel{EE_B}$$

$$\Rightarrow \frac{\lambda}{2l}x^2 - F \times d = \frac{1}{2}mv^2 + mgh$$

$$\Rightarrow \frac{2000}{2(0.2)}(0.1)^2 - (5.3)(0.1) = \frac{1}{2}(1.5)v^2 + (1.5)(9.8)(0.1)$$

$$\Rightarrow 50 - 0.53 = 0.75v^2 + 1.47$$

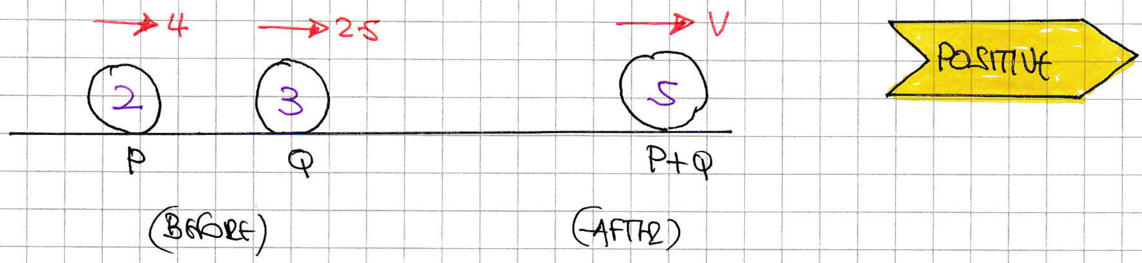
$$\Rightarrow 48 = 0.75v^2$$

$$\Rightarrow v^2 = 64$$

$$\Rightarrow |v| = 8 \text{ ms}^{-1}$$

IYGB - FM1 PAPER O - QUESTION 3

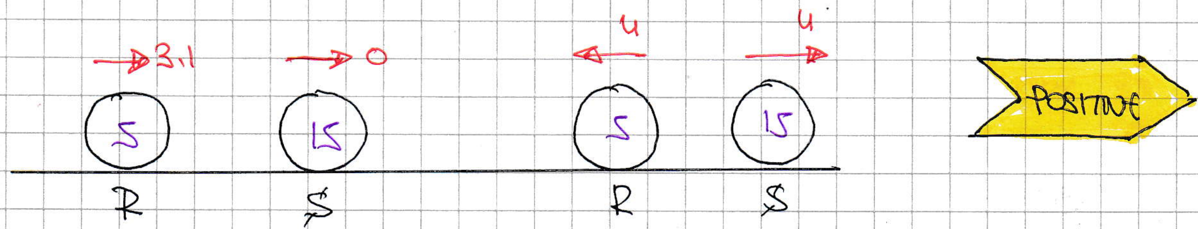
a) START WITH A COLLISION DIAGRAM



BY CONSERVATION OF MOMENTUM

$$(2\times4)+(3\times2.5) = 5V$$
$$8+7.5 = 5V$$
$$5V = 15.5$$
$$V = 3.1 \text{ ms}^{-1}$$

b) DRAWING A NEW DIAGRAM



BY MOMENTUM CONSERVATION

$$(5\times3.1)+0 = -5u+15u$$
$$15.5 = 10u$$
$$u = 1.55 \text{ ms}^{-1}$$

FINALLY AS THE PARTICLES MOVE IN OPPOSITE DIRECTION, WITH EQUAL SPEEDS OF 1.55 ms^{-1}

EVERY SECOND THEY MOVE $(1.55+1.55)$ METRES APART

$$\therefore d = 3.6\times2\times1.55$$
$$d = 11.16 \text{ m}$$

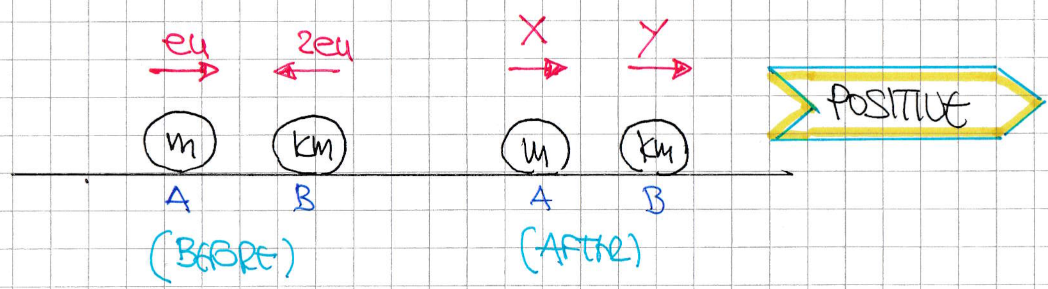
IYGB - FM1 PAPER O - QUESTION 4

- STARTING WITH THE COLLISIONS WITH THE WALLS

AFTER REBOUNDING

$V_A = eu$ & $V_B = 2eu$

- NEXT THE COLLISION BETWEEN THEM



- BY CONSERVATION OF MOMENTUM

$\Rightarrow meu - 2kmeu = mX + kmY$

$\Rightarrow \boxed{eu - 2keu = X + kY}$

- BY THE RESTITUTION COEFFICIENT

$\Rightarrow \dfrac{Y - X}{eu + 2eu} = e$

$\Rightarrow -X + Y = 3e^2u$ $\quad \times(-k)$

$\Rightarrow \boxed{kX - kY = -3ke^2u}$

- ADDING THE EQUATIONS

$kX - kY = -3ke^2u$

$X + kY = eu - 2keu$

$(k+1)X = eu - 2keu - 3ke^2u$

$X = \dfrac{eu(1 - 2k - 3ke)}{(k+1)}$

- A DOES NOT REVERSE ITS DIRECTION

$X > 0$

$\Rightarrow \dfrac{eu(1 - 2k - 3ke)}{k+1} > 0$

$\Rightarrow 1 - 2k - 3ke > 0 \qquad [eu > 0,\ k+1 > 0]$

$\Rightarrow -3ke > 2k - 1$

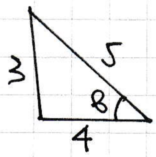
$\Rightarrow e < \dfrac{1-2k}{3k}$

// AS REQUIRED

IYGB – FM1 PAPER P – QUESTION 5

AUXILIARIES FIRST

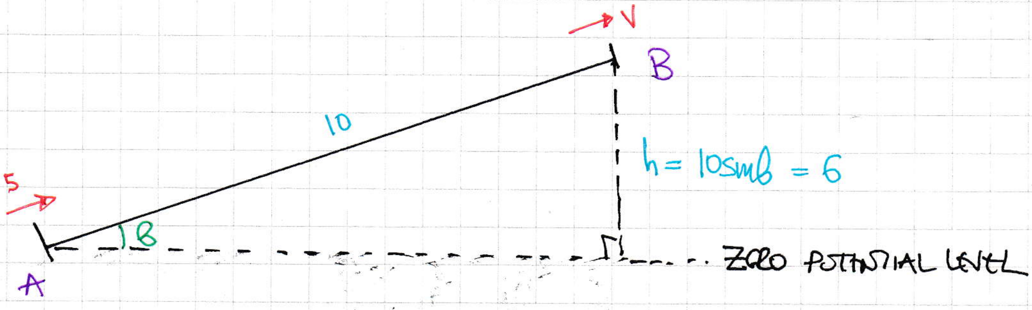
$\tan\theta = \frac{3}{4} \Longrightarrow$



$\Longrightarrow \begin{cases} \sin\theta = \frac{3}{5} \\ \cos\theta = \frac{4}{5} \end{cases}$

ONLY THE COMPONENT OF THE TENSION PARALLEL TO THE PLANE PUTS ENERGY INTO THE SYSTEM, IE. $1500\cos\theta = 1500 \times \frac{4}{5} = 1200$

NOW LOOKING AT AN ENERGY DIAGRAM, AND TAKING THE LEVEL OF "A", AS THE ZERO GRAVITATIONAL POTENTIAL LEVEL



$\Longrightarrow K.E_A + \cancel{P.E_A} + W_{IN} - W_{OUT} = K.E._B + P.E_B$

$\Longrightarrow \frac{1}{2}(120)\times 5^2 + 0 + (1200\times 10) - (180\times 10) = \frac{1}{2}(120)v^2 + 120g\times 6 \quad \leftarrow \text{"mgh"}$

$\Longrightarrow 1500 + 12000 - 1800 = 60v^2 + 7056$

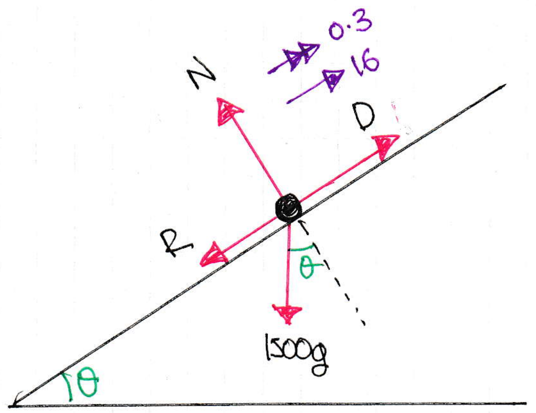
$\Longrightarrow \quad 4644 = 60v^2$

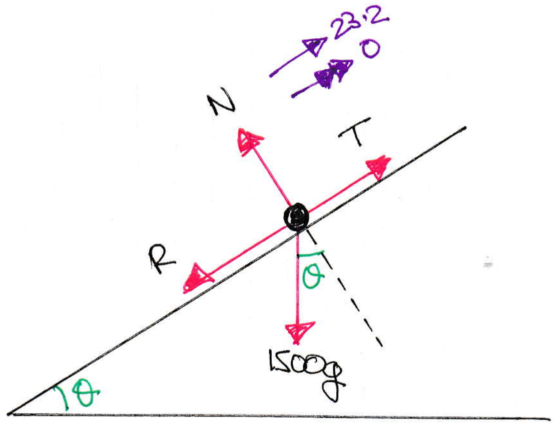
$\Longrightarrow \quad v^2 = 77.4$

$\Longrightarrow \quad |v| \simeq 8.80 \text{ ms}^{-1}$

IYGB - FM1 PAPER O - QUESTION 6

START WITH TWO SEPARATE DIAGRAMS, TRYING TO FORM EQUATIONS





"P = Dv"

$$\Rightarrow P = D \times 16$$

$$\Rightarrow D = \frac{P}{16}$$

EQUATION OF MOTION

$\Rightarrow$ "F = ma"

$$\Rightarrow D - R - 1500g\sin\theta = 1500a$$

$$\Rightarrow \frac{P}{16} - R - 1500g\left(\frac{2}{49}\right) = 1500(0.3)$$

$$\Rightarrow \frac{P}{16} - R - 600 = 450$$

$$\Rightarrow \frac{P}{16} - R = 1050$$

"P = Tv"

$$\Rightarrow P = T \times 23.2$$

$$\Rightarrow T = \frac{P}{23.2}$$

NO ACCELERATION (EQUILIBRIUM)

$$\Rightarrow T = R + 1500g\sin\theta$$

$$\Rightarrow \frac{P}{23.2} = R + 1500(9.8)\left(\frac{2}{49}\right)$$

$$\Rightarrow \frac{P}{23.2} = R + 600$$

$$\Rightarrow \frac{P}{23.2} - R = 600$$

COMBINING EQUATIONS BY SUBTRACTION

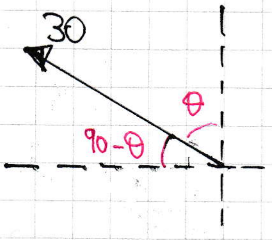
$$\frac{P}{16} - \frac{P}{23.2} = 450 \Rightarrow 23.2P - 16P = 167040$$

$$\Rightarrow 7.2P = 167040$$

$$P = 23200$$

IYGB – FM1 PAPER P – QUESTION 7

STARTING WITH A DIAGRAM



IMPULSE = MOMENTUM AFTER − MOMENTUM BEFORE

$$\underline{I} = m\underline{v} - m\underline{u}$$

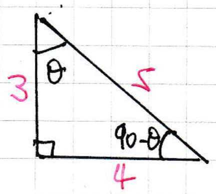
$$(-30\sin\theta)\underline{i} + (30\cos\theta)\underline{j} = 0.5\underline{v} - 0.5\times 40\underline{i}$$

$$-30(0.8)\underline{i} + 30\times 0.6\underline{j} = 0.5\underline{v} - 20\underline{i}$$

$$-24\underline{i} + 18\underline{j} = 0.5\underline{v} - 20\underline{i}$$

$$-4\underline{i} + 18\underline{j} = 0.5\underline{v}$$

$$\underline{v} = -8\underline{i} + 36\underline{j}$$



$\tan\theta = \frac{4}{3}$

$\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

$$\therefore \ |\underline{v}| = \sqrt{(-8)^2 + 36^2} = \sqrt{1360} \simeq 36.9 \text{ ms}^{-1}$$

AND THE REQUIRED ANGLE

$$\tan\psi = \frac{36}{8} = \frac{9}{2}$$

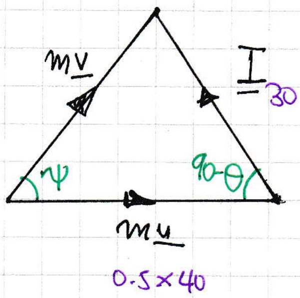
$$\psi \simeq 77.5°$$

ALTERNATIVE BY GEOMETRY

$$\underline{I} = m\underline{v} - m\underline{u}$$

$$m\underline{u} + \underline{I} = m\underline{v}$$

DRAWING A VECTOR TRIANGLE



IYGB - FM1 PAPER P - QUESTION 7

BY THE COSINE RULE

$$|m\underline{v}|^2 = |m\underline{u}|^2 + |\underline{I}|^2 - 2|m\underline{u}||\underline{I}|\cos(90-\theta)$$

$$|0.5\underline{v}|^2 = 20^2 + 30^2 - 2\times 20\times 30\times \sin\theta$$

$\cos(90-\theta) = \sin\theta$

$$\tfrac{1}{4}|\underline{v}|^2 = 400 + 900 - 1200\times \tfrac{4}{5} \leftarrow \text{(FROM BEFORE)}$$

$$\tfrac{1}{4}|v|^2 = 340$$

$$|v|^2 = 1360$$

$$|\underline{v}| = \sqrt{1360} \simeq 36.9 \text{ ms}^{-1}$$

AS BEFORE

AND BY THE SINE RULE

$$\frac{\sin\psi}{|\underline{I}|} = \frac{\sin(90-\theta)}{|m\underline{v}|} \Longrightarrow \frac{\sin\psi}{30} = \frac{\cos\theta}{\frac{1}{2}\sqrt{1360}}$$

$$\Longrightarrow \frac{\sin\psi}{30} = \frac{0.6}{\frac{1}{2}\sqrt{1360}}$$

$$\Longrightarrow \sin\psi = 0.976187\ldots$$

$$\Longrightarrow \psi \simeq 77.5^\circ$$

AS BEFORE

IYGB – FM1 PAPER P – QUESTION 8

BY HOOKE'S LAW – LET THE NATURAL LENGTH BE l

$$mg = \frac{\lambda}{l}(x-l) \qquad \& \qquad Mg = \frac{\lambda}{l}(y-l)$$

$$mgl = \lambda(x-l) \qquad\qquad Mgl = \lambda(y-l)$$

$$\frac{mgl}{\lambda} = x-l \qquad\qquad \frac{Mgl}{\lambda} = y-l$$

$$\frac{gl}{\lambda} = \frac{x-l}{m} \qquad\qquad \frac{gl}{\lambda} = \frac{y-l}{M}$$

EQUATING YIELDS

$$\Rightarrow \frac{x-l}{m} = \frac{y-l}{M}$$

$$\Rightarrow Mx - Ml = my - ml$$

$$\Rightarrow Mx - my = Ml - ml$$

$$\Rightarrow Mx - my = l(M-m)$$

$$\Rightarrow l = \frac{Mx-my}{M-m}$$

AS $l>0$ & $M>m$, SO THAT $M-m>0$, IT IMPLIES THAT

$$\Rightarrow Mx - my > 0$$

$$\Rightarrow Mx > my$$

AS REQUIRED