

C3, IYGB, PAPER W

- 1 -

1. a)

$$y = \frac{kx^2 - a}{kx^2 + a}$$

$$\frac{dy}{dx} = \frac{(kx^2 + a)(2kx) - (kx^2 - a)(2kx)}{(kx^2 + a)^2}$$

$$\frac{dy}{dx} = \frac{2k^2x^3 + 2akx - 4k^2x^3 + 2akx}{(kx^2 + a)^2}$$

$$\frac{dy}{dx} = \frac{4akx}{(kx^2 + a)^2}$$

b)

$$\frac{dy}{dx} = 0 \quad \text{Hence } 4akx = 0$$

$$\Rightarrow x = 0 \quad \text{For all } a, k$$

$$\text{if } y = \frac{-a}{a} = -1$$

$$\therefore (0, -1)$$

2. a)

$$\text{LHS} = \frac{2 \tan x}{1 + \tan^2 x} = \frac{2 \tan x}{\sec^2 x} = 2 \tan x \cos^2 x$$

$$= 2 \times \frac{\sin x}{\cos x} \times \cos^2 x = 2 \sin x \cos x = \sin 2x$$

$$= \text{RHS}$$

b)

USE $x = 15^\circ$ IN THE ABOVE IDENTITY

$$\frac{2 \tan 15}{1 + \tan^2 15} = \sin 30$$

$$\Rightarrow \frac{2T}{1 + T^2} = \frac{1}{2} \quad (T = \tan 15)$$

$$\begin{aligned} \Rightarrow 1+T^2 &= 4T \\ \Rightarrow T^2-4T+1 &= 0 \\ \Rightarrow (T-2)^2-4+1 &= 0 \\ \Rightarrow (T-2)^2 &= 3 \\ \Rightarrow T-2 &= \pm\sqrt{3} \end{aligned}$$

$$\begin{aligned} \Rightarrow T &= 2 \pm \sqrt{3} \\ \Rightarrow \tan 15 &= \begin{cases} 2+\sqrt{3} > 1 \\ 2-\sqrt{3} < 1 \end{cases} \end{aligned}$$

$\tan 45 = 1$ & $\tan x$ is
AN INCREASING FUNCTION

3. $7\sin^2 x + \sin x \cos x = 6$

$$\Rightarrow \frac{7\sin^2 x}{\cos^2 x} + \frac{\sin x \cos x}{\cos^2 x} = \frac{6}{\cos^2 x}$$

$$\Rightarrow 7\tan^2 x + \tan x = 6\sec^2 x$$

$$\Rightarrow 7\tan^2 x + \tan x = 6(1+\tan^2 x)$$

$$\Rightarrow 7\tan^2 x + \tan x = 6 + 6\tan^2 x$$

$$\Rightarrow \tan^2 x + \tan x - 6 = 0$$

$$\Rightarrow (\tan x - 2)(\tan x + 3) = 0$$

$$\tan x = \begin{cases} 2 \\ -3 \end{cases}$$

$$\bullet \arctan(2) \approx 63.4^\circ$$

$$x = 63.4^\circ \pm 180n$$

$$n = 0, 1, 2, 3, \dots$$



$$\bullet \arctan(-3) = -71.57^\circ$$

$$x = -71.6^\circ \pm 180n$$

$$n = 0, 1, 2, 3, \dots$$



$$x = 63.4^\circ, 243.4^\circ, 108.4^\circ, 288.4^\circ$$

C3, 1YGB, PAPER W

ALTERNATIVE

$$7\sin^2 x + \sin x \cos x = 6$$

$$7\left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) + \frac{1}{2}(2\sin x \cos x) = 6$$

$$\frac{7}{2} - \frac{7}{2}\cos 2x + \frac{1}{2}\sin 2x = 6$$

$$7 - 7\cos 2x + \sin 2x = 12$$

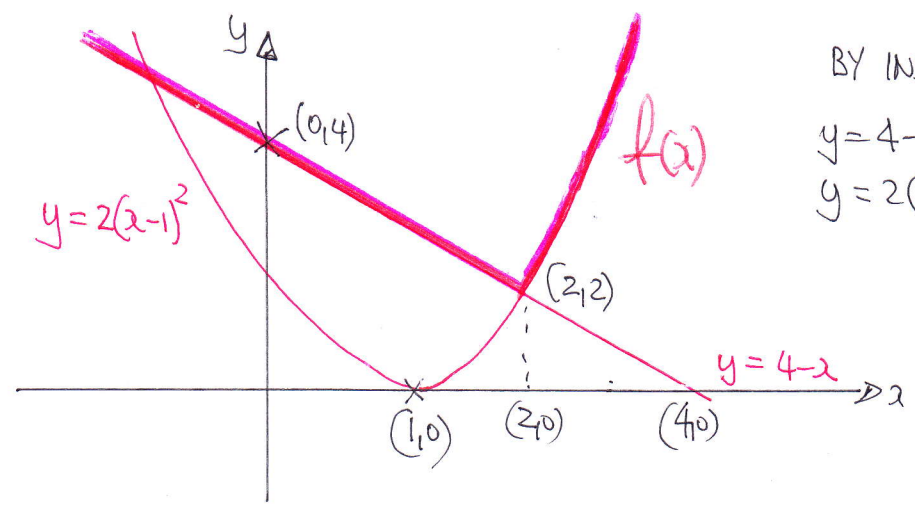
$$\sin 2x - 7\cos 2x = 5$$

↓

WRITE AS $R\sin(2x - \alpha) = 5$

ETC ETC

4. a)



BY INSPECTION
 $y = 4 - 2 = 2$
 $y = 2(2 - 1)^2 = 2$

b) $f(x) \geq 2$

c) $f(x) = 18$
 $4 - x = 18$
 $x = -14$

$f(x) = 18$
 $2(x-1)^2 = 18$
 $(x-1)^2 = 9$
 $x-1 < 3$
 -3
 $x = 4$
 ~~$x = -2$~~

C3, 1YGB, PAGE IV

- 4 -

So a) If $f(x) = |f(x)| \Rightarrow f(x) \geq 0$

So either

$$\begin{aligned} f(x) &= 9 \left[x^2 + \frac{2}{3}x + \frac{2}{9} \right] \\ &= 9 \left[\left(x + \frac{1}{3}\right)^2 - \frac{1}{9} + \frac{2}{9} \right] \\ &= 9 \left[\left(x + \frac{1}{3}\right)^2 + \frac{1}{9} \right] \\ &= 9 \left(x + \frac{1}{3}\right)^2 + 1 \geq 1 > 0 \\ \therefore f(x) &= |f(x)| \end{aligned}$$

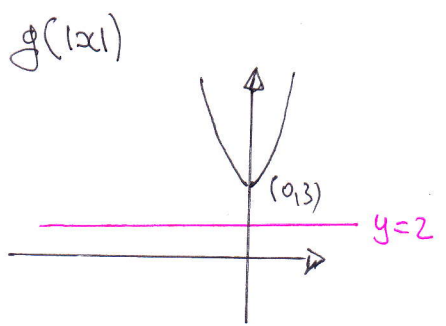
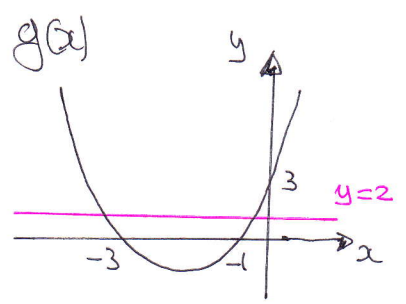
OR

USE DISCRIMINANT

$$\begin{aligned} b^2 - 4ac &= 6^2 - 4 \times 9 \times 2 \\ &= 36 - 72 = -36 < 0 \end{aligned}$$

IE GRAPH HAS ENTIRELY ABOVE x AXIS

b) GRAPHICALLY



THE MINIMUM VALUE OF $g(|x|)$ IS 3, SO NO INTERSECTIONS & THENCE NO SOLUTIONS TO $g(|x|) = 2$

ALGEBRAICALLY

$$\begin{aligned} g(x) &= (x+1)(x+3) \\ g(x) &= x^2 + 4x + 3 \\ g(|x|) &= |x|^2 + 4|x| + 3 \end{aligned}$$

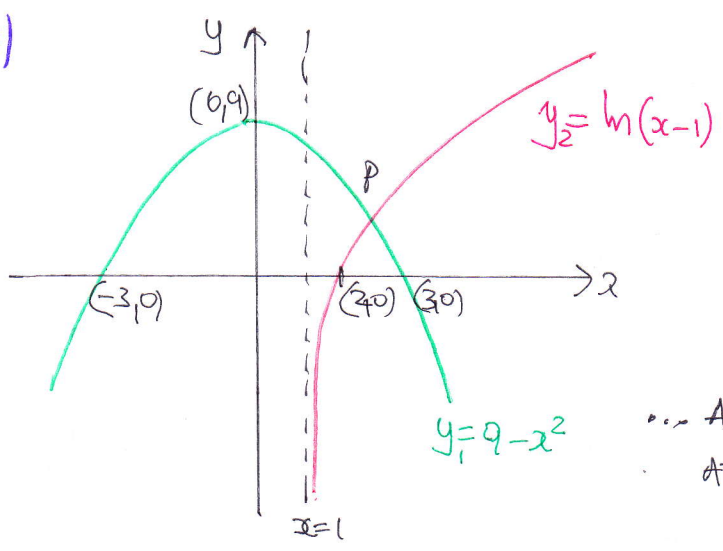
As $|x| \geq 0$

$$g(|x|) \geq 3$$

& SIMILAR ARGUMENT SHOWS

C3, 1YGB, PAPER W

6. a)



... AS THE GRAPHS INTERSECT AT ONE POINT ONLY

b) AS y_2 INTERCEPT AT (2, 0) & y_1 INTERCEPTS AT (3, 0) P MUST BE BETWEEN THESE VALUES, SHOWN BY THE CONFIGURATION OF THE GRAPHS

$$x_{n+1} = \sqrt{9 - \ln(x_n - 1)}$$

- $x_1 = 2.5$
- $x_2 = 2.93164$
- $x_3 = 2.88819$
- $x_4 = 2.89212$
- $x_5 = 2.89176$
- $x_6 = 2.89180$
- $x_7 = 2.89179$

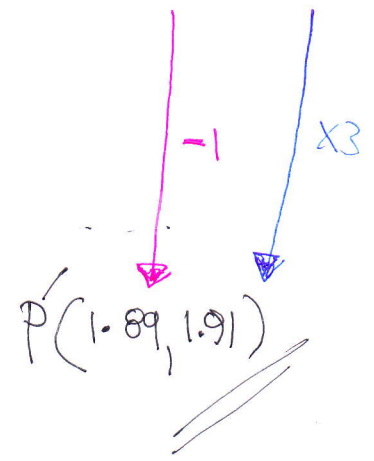
f7.

$\therefore x \approx 2.892$

C3, 1YGB, PAPER W - 6 -

d) $y \approx 9 - 2.89179^2 \approx 0.6375 \dots$ $P(2.892, 0.638)$

TRANSLATION "LEFT" BY 1
FOLLOWED BY VERTICAL STRETCH
BY SCALE FACTOR 3



7. a) $x = y(9 - 4y^2)^{\frac{1}{2}}$

$$\Rightarrow \frac{dx}{dy} = 1(9 - 4y^2)^{\frac{1}{2}} + y(9 - 4y^2)^{-\frac{1}{2}} \times \frac{1}{2} \times (-8y)$$

$$\Rightarrow \frac{dx}{dy} = (9 - 4y^2)^{\frac{1}{2}} - 4y^2(9 - 4y^2)^{-\frac{1}{2}}$$

$$\Rightarrow \frac{dx}{dy} = (9 - 4y^2)^{-\frac{1}{2}} [(9 - 4y^2)^1 - 4y^2]$$

$$\Rightarrow \frac{dx}{dy} = \frac{9 - 8y^2}{(9 - 4y^2)^{\frac{1}{2}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(9 - 4y^2)^{\frac{1}{2}}}{9 - 8y^2}$$

AS REQUIRED

b) BE INFINITE GRADIENT DENOMINATOR MUST EQUAL ZERO

$$9 - 8y^2 = 0$$

$$8y^2 = 9$$

$$y^2 = \frac{9}{8}$$

$$y^2 = \frac{18}{16}$$

$$\therefore y = \left\langle \begin{array}{l} \frac{3}{4}\sqrt{2} \\ -\frac{3}{4}\sqrt{2} \end{array} \right\rangle$$

- 7 -

C3, 1YGB, PAPER W

$$x = \begin{cases} \frac{3}{4}\sqrt{2} \sqrt{9 - 4 \times \frac{9}{8}} = \frac{9}{4} \\ -\frac{3}{4}\sqrt{2} \sqrt{9 - 4 \times \frac{9}{8}} = -\frac{9}{4} \end{cases}$$

$$\therefore \left(\frac{9}{4} \mid \frac{3}{4}\sqrt{2} \right) \mid \left(-\frac{9}{4} \mid -\frac{3}{4}\sqrt{2} \right)$$

8.

$$N = \frac{600}{1 + e^{-0.25t}} \quad t \geq 0$$

a) $t=0$ $N = \frac{600}{1 + e^0} = 300$

b) $N = 455$ $455 = \frac{600}{1 + e^{-0.25t}}$

$$\Rightarrow 1 + e^{-0.25t} = \frac{600}{455}$$

$$\Rightarrow e^{-0.25t} = \frac{29}{91}$$

$$\Rightarrow e^{0.25t} = \frac{91}{29}$$

$$\Rightarrow 0.25t = \ln\left(\frac{91}{29}\right)$$

$$\Rightarrow t = 4 \ln\left(\frac{91}{29}\right) \approx 4.57$$

(P.T.O)

C3, 1Y6B, PAPER W - 8 -

c) $N = 600(1 + e^{-0.25t})^{-1}$

$$\Rightarrow \frac{dN}{dt} = -600(1 + e^{-0.25t})^{-2} \times (-0.25e^{-0.25t})$$

$$\Rightarrow \frac{dN}{dt} = \frac{150 e^{-0.25t}}{(1 + e^{-0.25t})^2}$$

BUT $1 + e^{-0.25t} = \frac{600}{N}$
 $e^{-0.25t} = \frac{600}{N} - 1$

$$\Rightarrow \frac{dN}{dt} = \frac{150 \left(\frac{600}{N} - 1 \right)}{\frac{360000}{N^2}}$$

MULTIPLY TOP/BOTTOM BY N^2

$$\Rightarrow \frac{dN}{dt} = \frac{150(600N - N^2)}{360000}$$

$$\Rightarrow \frac{dN}{dt} = \frac{90000N}{360000} - \frac{150N^2}{360000}$$

$$\Rightarrow \frac{dN}{dt} = \frac{1}{4}N - \frac{1}{2400}N^2$$

* Answer

d) LET $\frac{dN}{dt} = f(N)$

↑
RATE OF GROWTH

$$\therefore f(N) = \frac{1}{4}N - \frac{1}{2400}N^2$$

C3, 1YGB, PAPER W

$$f'(N) = \frac{1}{4} - \frac{1}{1200}N$$

$$0 = \frac{1}{4} - \frac{1}{1200}N$$

$$N = 300$$

∴ MAX GROWTH AT t=0