IYGB GCE

Core Mathematics C3

Advanced

Practice Paper Q

Difficulty Rating: 3.62/1.6807

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

Differentiate each of the following expressions with respect to x, simplifying the final answers where possible.

a) $y=\sqrt{x^2-1}$ **(2)**

b) $y=x^4\ln x$ **(3)**

c) $y=\dfrac{e^x-1}{e^x+1}$ **(3)**

Question 2

Show clearly that

$$\frac{\left[(3x-1)(2x+3)-2(4x-1)\right](3x-1)}{3x+1}\equiv ax^2+bx+c\,,$$

where a, b and c are integers to be found. **(4)**

Question 3

Solve each of the following trigonometric equations.

i. $2\sec\theta-1=2\sec\theta\sin^2\theta\,,\qquad 0°\le\theta<180°\,,\ \ \theta\ne 90°$ **(5)**

ii. $4\cot^2 x-9\operatorname{cosec}x+6=0\,,\qquad 0°\le x<360°\,,\ \ x\ne 0°,180°$ **(6)**

Question 4

The curve C has equation

$$y = x^2 e^x, \ x \in \mathbb{R}.$$

a) Find the exact coordinates of the stationary points of C. **(6)**

b) By considering the sign of $\frac{d^2y}{dx^2}$ at each of these points determine their nature. **(4)**

Question 5

The functions f and g are defined as

$$f(x) = |2x-1|, \ x \in \mathbb{R},$$

$$g(x) = \ln(x+2), \ x \in \mathbb{R}, \ x > -2.$$

a) State the range of $f(x)$. **(1)**

b) Find, in exact form, the solutions of the equation

$$gf(x) = 2.$$ **(6)**

c) Show that the equation $f(x) = g(x)$ has a solution between 1 and 2. **(3)**

d) Use the iteration formula

$$x_{n+1} = \tfrac{1}{2}\left[1 + \ln(x_n + 2)\right], \ x_1 = 1,$$

to find the values of x_2, x_3 and x_4, correct to three decimal places. **(2)**

Question 6

$$f(x) = \sec x, \; x \in \mathbb{R}, \; 0 \le x \le 4\pi.$$

a) Sketch the graph of $f(x)$, showing clearly the coordinates of any stationary points and equations of asymptotes. **(3)**

It is now given that

$$\sec\theta = \sec\varphi,$$

where $0 < \theta < \frac{\pi}{2}$ and $\frac{7\pi}{2} < \varphi < 4\pi$.

b) Express φ in terms of θ. **(1)**

Question 7

Find, in exact form if appropriate, the solution of the following simultaneous equations

$$x + e^y = 5$$

$$\ln(x+1)^2 = 2y. \qquad \textbf{(7)}$$

Question 8

$$f(x) \equiv \cos x + \sqrt{3}\sin x, \; x \in \mathbb{R}.$$

a) Express $f(x)$ in the form $R\cos(x-\alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. **(3)**

b) Hence solve the equation

$$\cos 2\theta + \sqrt{3}\sin 2\theta = 2\cos\theta, \; 0 \le \theta < 2\pi. \qquad \textbf{(6)}$$

Question 9

The functions f and g are defined as

$$f(x) = 4a^2 - x^2, \ x \in \mathbb{R},$$

$$g(x) = |4x - a|, \ x \in \mathbb{R},$$

where a is a constant, such that $a \geq 1$.

a) Sketch in the same diagram the graph of $f(x)$ and the graph of $g(x)$. The sketch must include the coordinates of any points where each of the graphs meets the coordinate axes. **(4)**

b) Find, in exact form where appropriate, the solutions of the equation

$$4 - x^2 = |4x - 1|.$$ **(6)**