C3, IYGB, PAPER N

1. BY LONG-DIVISION

$$\begin{array}{r} 2x-1 \\ x^2+x-2 \,\big)\, \overline{2x^3+x^2-4x+1} \\ -2x^3-2x^2+4x \\ \hline -x^2 \quad +1 \\ +x^2+x-2 \\ \hline x-1 \end{array}$$

THUS

$$\frac{2x^3+x^2-4x+1}{x^2+x-2} = 2x-1+\frac{x-1}{x^2+x-2}$$

$$= 2x-1+\frac{\cancel{x-1}}{\cancel{(x-1)}(x+2)}$$

$$= 2x-1+\frac{1}{x+2}$$

i.e.
$A=2$
$B=-1$
$C=1$
$D=2$

2. $y = xe^{2x}$

$$\frac{dy}{dx} = 1\times e^{2x} + x\times(2e^{2x})$$

$$\frac{dy}{dx} = e^{2x}+2xe^{2x}$$

$$\left.\frac{dy}{dx}\right|_{x=\frac{1}{2}} = e^{2\times\frac{1}{2}} + 2\times\tfrac{1}{2}\times e^{2\times\frac{1}{2}}$$

$$\left.\frac{dy}{dx}\right|_{x=\frac{1}{2}} = e^1+e^1 = 2e$$

WHEN $x=\frac{1}{2}$, $y = \frac{1}{2}\times e^{2\times\frac{1}{2}} = \frac{1}{2}e$

$\therefore \left(\frac{1}{2}, \frac{1}{2}e\right)$

TANGENT

$$y-y_0 = m(x-x_0)$$

$$\Rightarrow y-\tfrac{1}{2}e = 2e\left(x-\tfrac{1}{2}\right)$$

$$\Rightarrow 2y-e = 4e\left(x-\tfrac{1}{2}\right)$$

$$\Rightarrow 2y-e = 4ex-2e$$

$$\Rightarrow 2y = 4ex-e$$

$$\Rightarrow 2y = e(4x-1)$$

AS REQUIRED

3. a) $x^3 = 5x+1$

$$x^3-5x-1=0$$

LET $f(x) = x^3-5x-1$

$$f(2) = -3 < 0$$

$$f(3) = 11 > 0$$

AS $f(x)$ IS CONTINUOUS AND CHANGES SIGN IN THE INTERVAL $[2,3]$, THERE MUST BE A ROOT IN THE INTERVAL

b) $\boxed{x_{n+1} = \sqrt[3]{5x_n + 1}}$

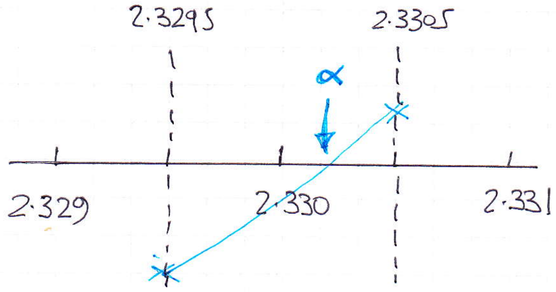
$x_1 = 2$

$x_2 = 2.22$

$x_3 = 2.30$

$x_4 = 2.32$

c)



$f(x) = x^3 - 5x - 1$

$f(2.3295) = -0.0063 < 0$

$f(2.3305) = 0.0050 > 0$

CONTINUITY & CHANGE OF SIGN IMPLY THAT

$2.3295 < \alpha < 2.3305$

$\therefore \alpha = 2.330$ CORRECT TO 3 d.p.

4. a) $\text{LHS} = \dfrac{1 - \cos 2\theta}{\sin 2\theta} = \dfrac{1 - (1 - 2\sin^2\theta)}{2\sin\theta\cos\theta} = \dfrac{\cancel{2}\sin^{\cancel{2}}\theta}{\cancel{2\sin\theta}\cos\theta}$

$= \dfrac{\sin\theta}{\cos\theta} = \tan\theta = \text{RHS}$

b) $\boxed{\dfrac{1 - \cos 2\theta}{\sin 2\theta} = \tan\theta}$

LET $\theta = 15$

$\dfrac{1 - \cos 30}{\sin 30} = \tan 15$

$\tan 15 = \dfrac{1 - \frac{\sqrt{3}}{2}}{\frac{1}{2}}$ MULTIPLY TOP/BOTTOM BY 2

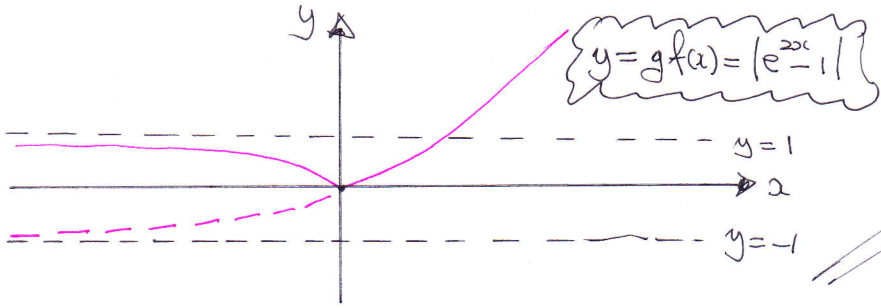
$\tan 15 = \dfrac{2 - \sqrt{3}}{1}$

$\tan 15 = 2 - \sqrt{3}$

AS REQUIRED

C3, IYGB, PAPER N

5. a) $gf(x) = g(f(x)) = g(e^{2x}-1) = |e^{2x}-1|$



b) FROM GRAPH THE ONLY SOLUTION COMES FROM

$e^{2x} - 1 = 1$

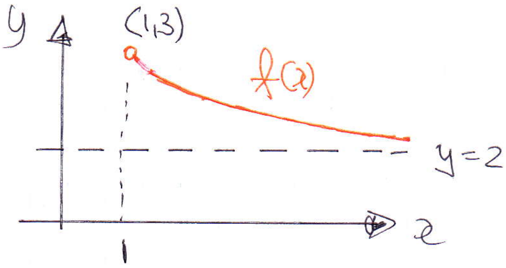
$e^{2x} = 2$

$2x = \ln 2$

$x = \frac{1}{2}\ln 2$

6. a) ASYMPTOTE IS $y = 2$

b)



RANGE OF $f(x)$

$2 < f(x) < 3$

c) $\boxed{f(x) = \frac{1}{x} + 2}$

$y = \frac{1}{x} + 2$

$y - 2 = \frac{1}{x}$

$\frac{1}{y-2} = \frac{x}{1}$

$x = \frac{1}{y-2}$

$\therefore f^{-1}(x) = \frac{1}{x-2}$

d)

	f	f^{-1}
D	$x > 1$	$2 < x < 3$
R	$2 < f(x) < 3$	$f^{-1}(x) > 1$

DOMAIN $2 < x < 3$

RANGE $f^{-1}(x) > 1$

7. a) $2\ln 56 - \left[\ln 168 - \ln\frac{3}{7}\right] = x\ln 2$

$\Rightarrow \ln 56^2 - \ln 168 + \ln\frac{3}{7} = x\ln 2$

$\Rightarrow \ln 3136 - \ln 168 + \ln\frac{3}{7} = x\ln 2$

$\Rightarrow \ln\left[\frac{3136 \times \frac{3}{7}}{168}\right] = x\ln 2$

$\Rightarrow \ln 8 = x\ln 2$

$\Rightarrow 3\ln 2 = x\ln 2$

$\Rightarrow x = 3$

OR

$\ln 8 = \ln(2^x)$

$8 = 2^x$

$x = 3$

b) $e^y \times 3^e = 3$

$\Rightarrow e^y = \frac{3}{3^e}$

$\Rightarrow e^y = \frac{3^1}{3^e}$

$\Rightarrow e^y = 3^1 \times 3^{-e}$

$\Rightarrow e^y = 3^{1-e}$

$\Rightarrow y = \ln 3^{1-e}$

$\Rightarrow y = (1-e)\ln 3$

ALTERNATIVE

$\Rightarrow \ln\left[e^y \times 3^e\right] = \ln 3$

$\Rightarrow \ln e^y + \ln 3^e = \ln 3$

$\Rightarrow y + e\ln 3 = \ln 3$

$\Rightarrow y = \ln 3 - e\ln 3$

$\Rightarrow y = (\ln 3)(1-e)$

$\Rightarrow y = (1-e)\ln 3$

AS BEFORE

c) $e^{\cos(\ln w)} = 1$

$\Rightarrow \cos(\ln w) = \ln 1$

$\Rightarrow \cos(\ln w) = 0$

• $\arccos 0 = \frac{\pi}{2}$

$\Rightarrow \begin{cases} \ln w = \frac{\pi}{2} \pm 2n\pi \\ \ln w = \frac{3\pi}{2} \pm 2n\pi \end{cases}$ $n = 0, 1, 2, 3, \ldots$

$\Rightarrow \ln w = \ldots -\frac{5\pi}{2}, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

$\Rightarrow w = \ldots e^{-\frac{5\pi}{2}}, e^{-\frac{3\pi}{2}}, e^{-\frac{\pi}{2}}, e^{\frac{\pi}{2}}, e^{\frac{3\pi}{2}}, e^{\frac{5\pi}{2}}, \ldots$

0.0004, 0.009, 0.207, 4.81, 111.3, 2576

ONLY SOLUTION IF $1 \leqslant w < 5$

8. a) $y = \frac{x}{x^2+1}$

$\Rightarrow \frac{dy}{dx} = \frac{(x^2+1)\times 1 - x(2x)}{(x^2+1)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{x^2+1-2x^2}{(x^2+1)^2}$

$\Rightarrow \boxed{\frac{dy}{dx} = \frac{1-x^2}{(x^2+1)^2}}$

• NOW SET TO -1

$\Rightarrow -1 = \frac{1-x^2}{(x^2+1)^2}$

$\Rightarrow -(x^2+1)^2 = 1-x^2$

$\Rightarrow -x^4-2x^2-1 = 1-x^2$

$\Rightarrow 0 = x^4+x^2+2$

• DISCRIMINANT IN "x^2"

$b^2-4ac = 1^2-4\times1\times2 = -7$

NO SOLUTION $\Rightarrow$ NO POINT WITH GRADIENT -1

b) NOW $\frac{dy}{dx} = \frac{12}{25}$

$\frac{1-x^2}{(x^2+1)^2} = \frac{12}{25}$

$\Rightarrow 25-25x^2 = 12(x^2+1)^2$

$\Rightarrow 25-25x^2 = 12x^4+24x^2+12$

$\Rightarrow 0 = 12x^4+49x^2-13$

• QUADRATIC FORMULA

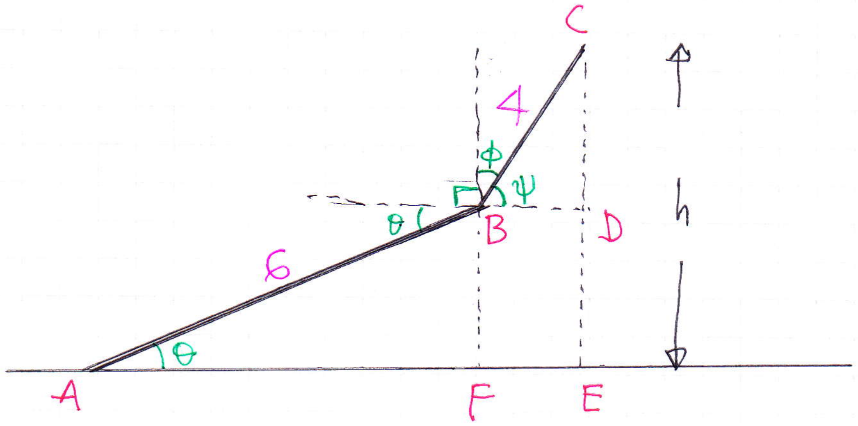
$\Rightarrow x^2 = \frac{-49 \pm \sqrt{49^2-4\times12(-13)}}{2\times12}$

$\Rightarrow x^2 = \begin{cases} \frac{1}{4} \\ \cancel{-\frac{13}{3}} \end{cases}$

$\Rightarrow x = \begin{cases} \frac{1}{2} \\ -\frac{1}{2} \end{cases}$ $y = \begin{cases} \frac{2}{5} \\ -\frac{2}{5} \end{cases}$

$\therefore \left(\frac{1}{2}, \frac{2}{5}\right) \& \left(\frac{1}{2}, -\frac{2}{5}\right)$

9. a) I)



RELABEL ANGLES AROUND B

- $\theta + 90 + \phi = 120°$

  $\theta + \phi = 30$

  $\boxed{\phi = 30 - \theta}$

- $\phi + \psi = 90°$

  $30 - \theta + \psi = 90$

  $\psi = \theta + 60$

  ie $\widehat{DBC} = \theta + 60°$

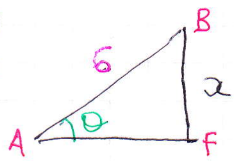
  AS REQUIRED

II) $h = |DE| + |CD|$

$\Rightarrow h = |BF| + |CD|$

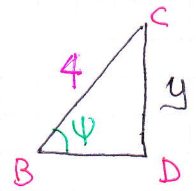
$\Rightarrow h = 6\sin\theta + 4\sin\psi$

$\Rightarrow h = 6\sin\theta + 4\sin(\theta + 60)$



$\frac{x}{6} = \sin\theta$

$x = 6\sin\theta$



$\frac{y}{4} = \sin\psi$

$y = 4\sin\psi$

- USING COMPOUND ANGLE

$\Rightarrow h = 6\sin\theta + 4\sin\theta\cos60 + 4\cos\theta\sin60$

$\Rightarrow h = 6\sin\theta + 4\sin\theta \times \frac{1}{2} + 4\cos\theta \times \frac{\sqrt{3}}{2}$

$\Rightarrow h = 8\sin\theta + 2\sqrt{3}\cos\theta$

AS REQUIRED

b) $\boxed{h = 8\sin\theta + 2\sqrt{3}\cos\theta}$

$\Rightarrow h = R\cos(\theta - \alpha)$

$\Rightarrow h = R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$

$\Rightarrow h = (R\cos\alpha)\cos\theta - (R\sin\alpha)\sin\theta$

$\left.\begin{array}{l} R\cos\alpha = 2\sqrt{3} \\ R\sin\alpha = 8 \end{array}\right\} \Rightarrow R = \sqrt{(2\sqrt{3})^2 + 8^2} = \sqrt{76}$

$\tan\alpha = \dfrac{8}{2\sqrt{3}}$

$\alpha \simeq 66.6°$

$\Rightarrow h \approx \sqrt{76}\cos(\theta - 66.6°)$

- Now $h = 6$

$\Rightarrow 6 = \sqrt{76}\cos(\theta - 66.6°)$

$\Rightarrow \cos(\theta - 66.6°) = 0.6882\ldots$

$\arccos(0.6882\ldots) \simeq 46.5$

$\Rightarrow \begin{cases} \theta - 66.6° = 46.5 \pm 360n \\ \theta - 66.6° = 313.5 \pm 360n \end{cases}$ $\quad n = 0, 1, 2, 3 \ldots$

$\begin{cases} \theta = 113.1° \pm 360n \\ \theta = 380.1° \pm 360n \end{cases}$

$\therefore \theta = 113°$ OR $20°$