C3, IYGB, PAPER K

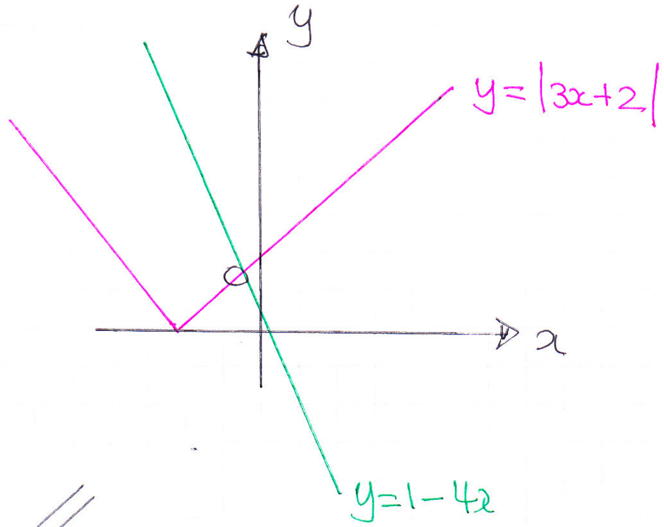
1.

$$4x + |3x+2| = 1$$
$$|3x+2| = 1 - 4x$$

$$\begin{cases} 3x+2 = 1-4x \\ 3x+2 = -1+4x \end{cases}$$

$$\begin{cases} 7x = -1 \\ 3 = x \end{cases}$$

$$x = \begin{cases} -\frac{1}{7} \\ \cancel{3} \text{ NOT ok.} \end{cases}$$



2. a) I)

$$\left.\begin{array}{l} y = 2^x \\ y = 3 - 2x \end{array}\right\} \Rightarrow 2^x = 3 - 2x$$
$$\Rightarrow 2^x + 2x - 3 = 0$$
$$\Rightarrow f(x) = 2^x + 2x - 3$$

$$\left.\begin{array}{l} f(0.5) = -0.586\ldots < 0 \\ f(1) = 1 > 0 \end{array}\right\}$$

AS $f(x)$ IS CONTINUOUS AND CHANGES SIGN BETWEEN 0.5 & 1 THERE MUST BE A SOLUTION IN THE INTERVAL

II) USING $2^x = 3 - 2x$

$$\ln 2^x = \ln(3-2x)$$
$$x \ln 2 = \ln(3-2x)$$
$$x = \frac{\ln(3-2x)}{\ln 2}$$

b) $$x_{n+1} = \frac{\ln(3-2x_n)}{\ln 2}$$

$x_0 = 0.5$

$x_1 = 1$

$x_2 = 0$

$x_3 = 1.585\ldots$

$x_4 =$ NOT POSSIBLE BECAUSE ARGUMENT OF $\ln(3-2x_n)$ BECOMES

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c) $$\boxed{x_{n+1} = \frac{3 - 2^{x_n}}{2}}$$

$x_0 = 0.5$
$x_1 = 0.79289$
$x_2 = 0.63373$
$x_3 = 0.72421$
$x_4 = 0.67400$
$x_5 = 0.70226$
$x_6 = 0.68648$
$x_7 = 0.69533$
$x_8 = 0.69037$
$x_9 = 0.69315$

∴ SEE ITERATION OPPOSITE

∴ $\alpha = 0.69$

2 d.p

3. a) $$\boxed{g(x) = \frac{5x}{2x-1}}$$

$\Rightarrow y = \frac{5x}{2x-1}$

$\Rightarrow 2yx - y = 5x$

$\Rightarrow 2xy - 5x = y$

$\Rightarrow x(2y-5) = y$

$\Rightarrow x = \frac{y}{2y-5}$

$$\boxed{\therefore g^{-1}(x) = \frac{x}{2x-5}}$$

$f(g^{-1}(3)) = f\left(\frac{3}{2\times3-5}\right)$
$= f(3)$
$= 4 - 3^2$
$= 4 - 9$
$= -5$

b) $g^{-1}(f(x)) = \frac{7}{5}$

$\Rightarrow g^{-1}[4 - x^2] = \frac{7}{5}$

$\Rightarrow \frac{4-x^2}{2(4-x^2)-5} = \frac{7}{5}$

$\Rightarrow \frac{4-x^2}{8-2x^2-5} = \frac{7}{5}$

$\Rightarrow \frac{4-x^2}{3-2x^2} = \frac{7}{5}$

$\Rightarrow 20 - 5x^2 = 21 - 14x^2$

$\Rightarrow 9x^2 = 1$

$\Rightarrow x^2 = \frac{1}{9}$

$\Rightarrow x = \frac{1}{3}, \; -\frac{1}{3}$

BOTH O.K.

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4. a) $\boxed{y = 4(2x-1)^{-2}}$

$$\frac{dy}{dx} = -8(2x-1)^{-3} \times 2$$

$$\frac{dy}{dx} = -16(2x-1)^{-3}$$

OR $\frac{dy}{dx} = -\frac{16}{(2x-1)^3}$

b) $\boxed{y = x^3 e^{-2x}}$

$$\frac{dy}{dx} = 3x^2 e^{-2x} + x^3(e^{-2x} \times -2)$$

$$\frac{dy}{dx} = 3x^2 e^{-2x} - 2x^3 e^{-2x}$$

OR $\frac{dy}{dx} = x^2 e^{-2x}(3-2x)$

c) $\boxed{y = \frac{2x^2+1}{3x^2+1}}$

$$\frac{dy}{dx} = \frac{(3x^2+1)(4x) - (2x^2+1)(6x)}{(3x^2+1)^2} = \frac{\cancel{12x^3} + 4x - \cancel{12x^3} - 6x}{(3x^2+1)^2}$$

$$= -\frac{2x}{(3x^2+1)^2}$$

5. a) $\frac{4x-1}{2(x-1)} - 2 - \frac{3}{2(x-1)(2x-1)}$

$$= \frac{(4x-1)(2x-1) - 2\times 2(x-1)(2x-1) - 3}{2(x-1)(2x-1)}$$

$$= \frac{8x^2 - 4x - 2x + 1 - 4(2x^2 - x - 2x + 1) - 3}{2(x-1)(2x-1)}$$

$$= \frac{\cancel{8x^2} - 6x + 1 - \cancel{8x^2} + 12x - 4 - 3}{2(x-1)(2x-1)} = \frac{6x-6}{2(x-1)(2x-1)}$$

$$= \frac{6\cancel{(x-1)}}{2\cancel{(x-1)}(2x-1)} = \frac{6}{2(2x-1)} = \frac{3}{2x-1}$$

P.T.O

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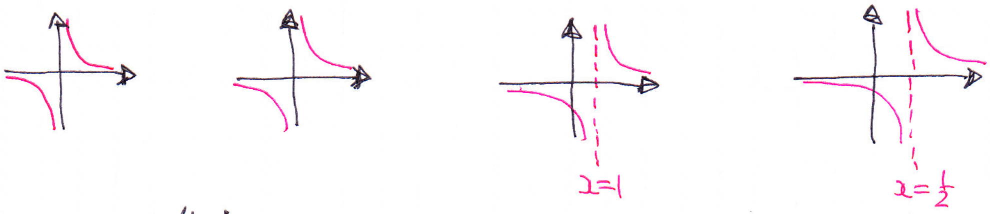
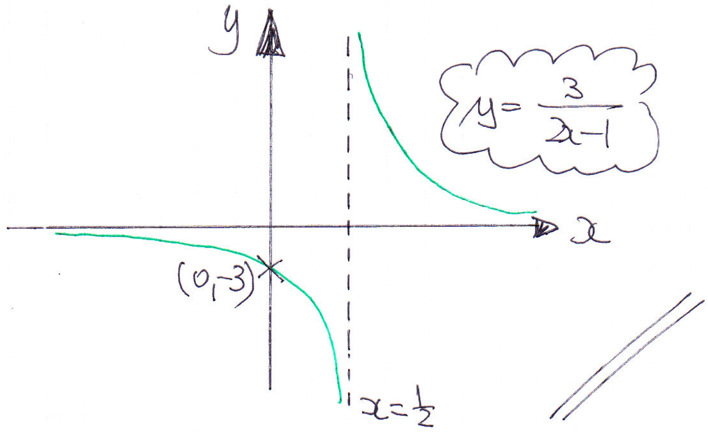
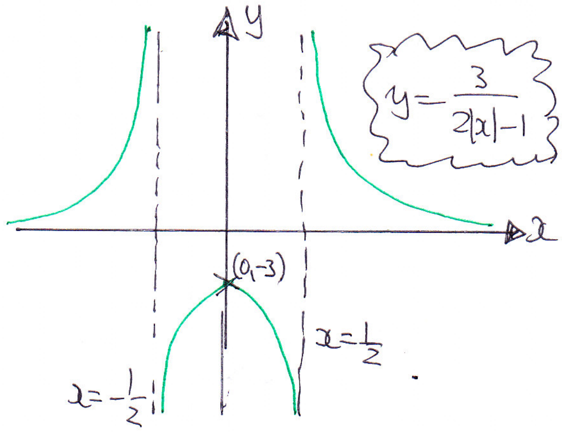
b) $\frac{1}{x} \longmapsto \frac{3}{x} \longmapsto \frac{3}{(x-1)} \longmapsto \frac{3}{(2x)-1}$





c) IF $f(x) = \frac{3}{2x-1}$

THEN $f(|x|) = \frac{3}{2|x|-1}$



6. I) $\frac{\sec^2 x + 8}{4 - \tan x} = 3\tan x$

$\Rightarrow \sec^2 x + 8 = 3\tan x(4 - \tan x)$

$\Rightarrow \sec^2 x + 8 = 12\tan x - 3\tan^2 x$

$\Rightarrow (1 + \tan^2 x) + 8 = 12\tan x - 3\tan^2 x$

$\Rightarrow 9 + \tan^2 x = 12\tan x - 3\tan^2 x$

$\Rightarrow 4\tan^2 x - 12\tan x + 9 = 0$

$\Rightarrow (2\tan x - 3)^2 = 0$

$\tan x = \frac{3}{2}$

$\arctan\left(\frac{3}{2}\right) = 0.983^c$

$x = 0.983^c \pm n\pi$

$n = 0, 1, 2, 3 \ldots$

$\therefore x_1 = 0.983^c$

$x_2 = 4.12^c$

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II)

$$\cos 2\theta = \sin\theta$$
$$(1 - 2\sin^2\theta) = \sin\theta$$
$$0 = 2\sin^2\theta + \sin\theta - 1$$
$$(2\sin\theta - 1)(\sin\theta + 1) = 0$$
$$\sin\theta = \begin{cases} -1 \\ \frac{1}{2} \end{cases}$$

- $\arcsin(-1) = -90°$

$$\theta = -90 \pm 360n$$
$$\theta = 270 \pm 360n$$

$n = 0, 1, 2, 3 \ldots$

- $\arcsin\left(\frac{1}{2}\right) = 30°$

$$\theta = 30 \pm 360n$$
$$\theta = 150 \pm 360n$$

$n = 0, 1, 2, 3 \ldots$

$$\theta = 30°, 150°, 270°$$

7.

$$\left.\begin{aligned} y &= 2 + 3e^x \\ y &= -1 + e^{x+3} \end{aligned}\right\} \Longrightarrow \left.\begin{aligned} \frac{y-2}{3} &= e^x \\ y+1 &= e^{x+3} \end{aligned}\right\} \text{ DIVIDE EQUATIONS}$$

$$\Rightarrow \frac{e^{x+3}}{e^x} = \frac{y+1}{\frac{y-2}{3}}$$

$$\Rightarrow e^3 = \frac{3y+3}{y-2}$$

$$\Rightarrow ye^3 - 2e^3 = 3y + 3$$

$$\Rightarrow ye^3 - 3y = 2e^3 + 3$$

$$\Rightarrow y(e^3 - 3) = 2e^3 + 3$$

$$\Rightarrow y = \frac{2e^3 + 3}{e^3 - 3}$$

AS REQUIRED

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8. $\boxed{y = e^{x\sqrt{12}} \sin 6x}$

$$\Rightarrow \frac{dy}{dx} = \sqrt{12}\,e^{x\sqrt{12}} \sin 6x + e^{x\sqrt{12}}(6\cos 6x)$$

$$\Rightarrow \frac{dy}{dx} = e^{x\sqrt{12}}\left[\sqrt{12}\sin 6x + 6\cos 6x\right]$$

SET TO ZERO

$$\Rightarrow \sqrt{12}\sin 6x + 6\cos 6x = 0 \qquad \left[e^{x\sqrt{12}} \neq 0\right]$$

$$\Rightarrow \sqrt{12}\sin 6x = -6\cos 6x$$

$$\Rightarrow \sqrt{12}\,\frac{\sin 6x}{\cos 6x} = -\frac{6\cancel{\cos 6x}}{\cancel{\cos 6x}}$$

$$\Rightarrow \sqrt{12}\tan 6x = -6$$

$$\Rightarrow \tan 6x = -\sqrt{3}$$

$$\boxed{\arctan(-\sqrt{3}) = -\frac{\pi}{3}}$$

$$\Rightarrow 6x = -\frac{\pi}{3} \pm n\pi \qquad n = 0, 1, 2, 3, \ldots$$

$$x = -\frac{\pi}{18} \pm \frac{n\pi}{6}$$

$$x = \ldots -\frac{\pi}{18}, \frac{\pi}{9}, \frac{5\pi}{18}, \frac{4\pi}{9}, \ldots$$

FROM GRAPH

9. a) $f(x) = 2\sin x + 2\cos x \equiv R\sin(x+\alpha)$

$\equiv R\sin x\cos\alpha + R\cos x\sin\alpha$

$\equiv (R\cos\alpha)\sin x + (R\sin\alpha)\cos x$

$\left\{\begin{array}{l} R\cos\alpha = 2 \\ R\sin\alpha = 2 \end{array}\right\}$ $\therefore R = \sqrt{2^2+2^2} = \sqrt{8} = 2\sqrt{2}$

$\tan\alpha = 1$

$\alpha = \frac{\pi}{4}$

$\therefore f(x) = 2\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$

b)

	MIN	MAX
$y = f(x)$	$-2\sqrt{2}$	$2\sqrt{2}$
$y = f(x-\pi)$	$-2\sqrt{2}$	$2\sqrt{2}$
$y = 2f(x)+1$	$(-2\sqrt{2})\times 2+1$ $= 1-4\sqrt{2}$	$2\sqrt{2}\times 2+1$ $= 1+4\sqrt{2}$
$y = [f(x)]^2$	$0^2 = 0$	$(2\sqrt{2})^2 = 8$
$y = \frac{10}{f(x)+3\sqrt{2}}$	$\frac{10}{2\sqrt{2}+3\sqrt{2}} = \frac{10}{5\sqrt{2}}$ $= \sqrt{2}$	$\frac{10}{-2\sqrt{2}+3\sqrt{2}} = \frac{10}{\sqrt{2}}$ $= 5\sqrt{2}$