

C3, 1YGB, PAPER H

— 1 —

1. a) $x^3 - x^2 = 6x + 6$
 $x^3 - x^2 - 6x - 6 = 0$
 $f(x) = x^3 - x^2 - 6x - 6$

$f(3) = -6 < 0$

$f(4) = 18 > 0$

As $f(x)$ is continuous & changes sign between 3 & 4, there must be at least one root in this interval

b) $x^3 - x^2 = 6x + 6$

$x^2(x-1) = 6x+6$

$x^2 = \frac{6x+6}{x-1}$

$x = \pm \sqrt{\frac{6x+6}{x-1}}$

$x = \sqrt{\frac{6x+6}{x-1}}$

c) $x_{n+1} = \sqrt{\frac{6x_n+6}{x_n-1}}$

• $x_0 = 1$ DOES NOT PRODUCE x_1 (ZERO DENOMINATOR)

• $x_0 = 0.5$ DOES NOT PRODUCE x_1 (NEGATIVE IN THE ROOT)

d) $x_0 = 3.3$

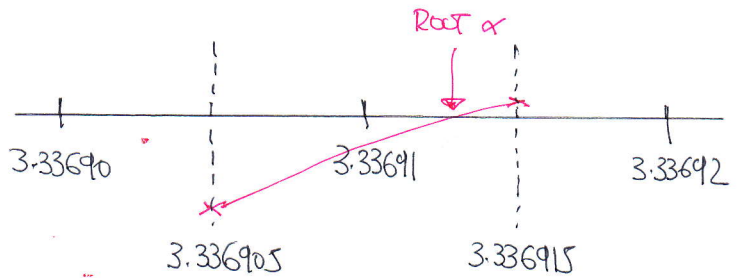
$x_1 \approx 3.340$

$x_2 \approx 3.333$

$x_3 \approx 3.338$

$x_4 \approx 3.336$

e)



$f(x) = x^3 - x^2 - 6x - 6$

$f(3.336905) = -0.00014 < 0$

$f(3.336915) = 0.00006 > 0$

CHANGE OF SIGN $\Rightarrow 3.336905 < \alpha < 3.336915$

$\Rightarrow \alpha \approx 3.33691$

5 d.p

C3, IYGB, PAPER 11

2. a) when $t=0$ $T = 20 + 50e^0 = 20 + 50$
 $\therefore T = 70$

b) when $t=30$ $T = 20 + 50e^{-\frac{30}{15}} = 20 + 50e^{-2}$
 $\therefore T \approx 27$ ($\approx 26.76676\dots$)

c) $T = 35$ $35 = 20 + 50e^{-\frac{t}{15}}$
 $15 = 50e^{-\frac{t}{15}}$
 $\frac{3}{10} = e^{-\frac{t}{15}}$
 $\frac{10}{3} = e^{\frac{t}{15}}$
 $\ln \frac{10}{3} = \frac{t}{15}$
 $t = 15 \ln \frac{10}{3} \approx 18$

3. $y = \frac{1}{6}(x^2 + 5)^{\frac{3}{2}}$
 $\frac{dy}{dx} = \frac{3}{2} \times \frac{1}{6}(x^2 + 5)^{\frac{1}{2}} \times 2x$
 $\frac{dy}{dx} = \frac{1}{2}x(x^2 + 5)^{\frac{1}{2}}$
 $\left. \frac{dy}{dx} \right|_{x=2} = \frac{1}{2} \times 2 \times 9^{\frac{1}{2}} = 3$

when $x=2$
 $y = \frac{1}{6}(2^2 + 5)^{\frac{3}{2}}$
 $y = \frac{1}{6} \times 9^{\frac{3}{2}}$
 $y = \frac{9}{2}$
 $\therefore P(2, \frac{9}{2})$

Thus $y - y_0 = m(x - x_0)$
 $y - \frac{9}{2} = 3(x - 2)$
 $y - \frac{9}{2} = 3x - 6$
 $y = 3x - \frac{3}{2}$

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$$b) f(g(x)) = f(7-2x^2) = \frac{(7-2x^2)+6}{(7-2x^2)+2} = \frac{13-2x^2}{9-2x^2}$$

$$c) \text{ Let } y = \frac{x+6}{x+2}$$

$$yx + 2y = x + 6$$

$$yx - x = 6 - 2y$$

$$x(y-1) = 6-2y$$

$$x = \frac{6-2y}{y-1}$$

$$\therefore f(x) = \frac{6-2x}{x-1}$$

$$d) \frac{6-2x}{x-1} = \frac{x+6}{x+2}$$

$$\Rightarrow (6-2x)(x+2) = (x-1)(x+6)$$

$$\Rightarrow 6x+12-2x^2-4x = x^2+5x-6$$

$$\Rightarrow 0 = 3x^2 + 3x - 18$$

$$\Rightarrow 0 = x^2 + x - 6$$

$$\Rightarrow 0 = (x-2)(x+3)$$

$$\Rightarrow x = \begin{matrix} 2 \\ -3 \end{matrix}$$

7.

$$y = e^{-x} \sin(\sqrt{3}x)$$

$$\frac{dy}{dx} = -e^{-x} \sin(\sqrt{3}x) + e^{-x} \times \sqrt{3} \cos(\sqrt{3}x)$$

$$= e^{-x} [\sqrt{3} \cos \sqrt{3}x - \sin \sqrt{3}x]$$

$$\text{Now } \sqrt{3} \cos \sqrt{3}x - \sin \sqrt{3}x \equiv R \cos(\sqrt{3}x + \alpha)$$

$$\equiv R \cos \sqrt{3}x \cos \alpha - R \sin \sqrt{3}x \sin \alpha$$

$$\equiv (R \cos \alpha) \cos \sqrt{3}x - (R \sin \alpha) \sin \sqrt{3}x$$

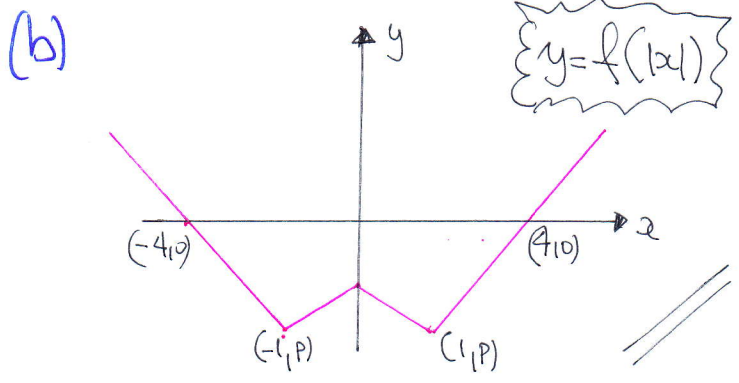
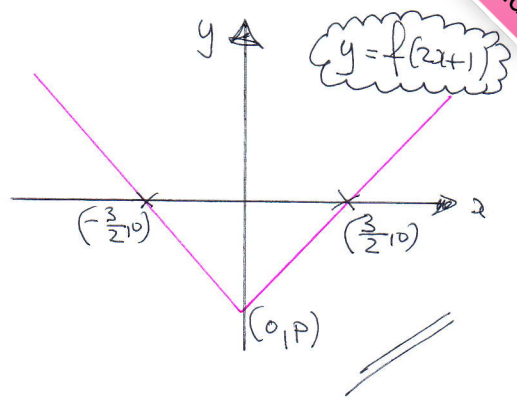
$$\begin{matrix} R \sin \alpha = 1 \\ R \cos \alpha = \sqrt{3} \end{matrix}$$

$$\bullet R = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3+1} = 2$$

$$\bullet \tan \alpha = \frac{1}{\sqrt{3}} \quad \alpha = \frac{\pi}{6}$$

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8. (a) TRANSLATION LEFT BY 1 UNIT.
FOLLOWED BY HORIZONTAL
STRETCH BY SCALE FACTOR $\frac{1}{2}$



(c) w/h/v $x=0$ $|0-1|-3 = 1-3 = -2$ $\therefore C(0, -2)$
w/h/v $x=1$ $|1-1|-3 = -3$ $\therefore P(1, -3)$

(d) $f(x) = 4x$
 $\Rightarrow |x-1|-3 = 4x$
 $\Rightarrow |x-1| = 4x+3$
 $\begin{cases} x-1 = 4x+3 \\ x-1 = -4x-3 \end{cases}$
 $\begin{cases} -4 = 3x \\ 5x = -2 \end{cases}$
 $x = \begin{cases} -\frac{4}{3} \\ -\frac{2}{5} \end{cases}$

check solutions
 $\bullet \left| -\frac{2}{5} - 1 \right| - 3 = \frac{7}{5} - 3 = -\frac{8}{5}$
 $4\left(-\frac{2}{5}\right) = -\frac{8}{5}$
 $\bullet \left| -\frac{4}{3} - 1 \right| - 3 = -\frac{2}{3}$
 $4\left(-\frac{4}{3}\right) = -\frac{16}{3}$

\therefore ONLY SOLUTION $x = -\frac{2}{5}$

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9.

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\cot \theta = 2$$
$$\tan \theta = \frac{1}{2}$$

$$\cot 2\theta = \frac{1}{\tan 2\theta} = \frac{1 - \tan^2 \theta}{2 \tan \theta} = \frac{1 - \left(\frac{1}{2}\right)^2}{2 \times \frac{1}{2}} = \frac{3}{4}$$

$$\begin{aligned} \tan 4\theta &= \tan [2 \times 2\theta] = \frac{2 \tan 2\theta}{1 - \tan^2 2\theta} \\ &= \frac{2 \times \frac{4}{3}}{1 - \left(\frac{4}{3}\right)^2} = \frac{\frac{8}{3}}{1 - \frac{16}{9}} = -\frac{24}{7} \end{aligned}$$

BUT $\cot 2\theta = \frac{3}{4}$
 $\tan 2\theta = \frac{4}{3}$

$$\therefore \tan \theta \cot 2\theta \tan 4\theta = \frac{1}{2} \times \frac{3}{4} \times \frac{-24}{7} = -\frac{9}{7}$$

As Required