1. a) LET $f(x) = x^3 + 10x - 4$

$f(0) = -4$
$f(1) = 7$
AS $f(x)$ IS CONTINUOUS AND CHANGES SIGN, THERE MUST BE A ROOT α BETWEEN 0 & 1

b) $x_0 = 0.3$

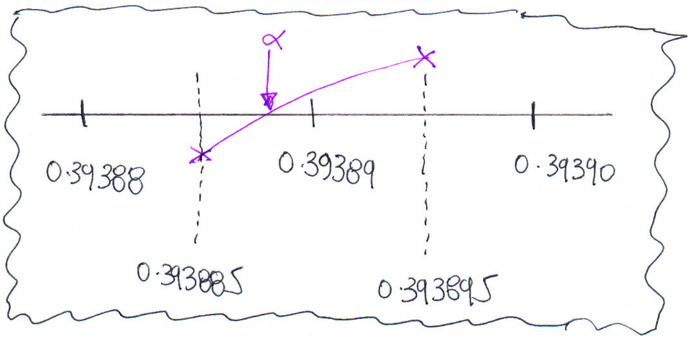
$x_1 = 0.3973$

$x_2 = 0.3937$

$x_3 = 0.3939$

$x_4 = 0.3939$

c)



$f(0.393885) = -0.000041$

$f(0.393895) = 0.000064$

CHANGE OF SIGN $\Longrightarrow$

$0.393885 < \alpha < 0.393895$

$\therefore \alpha = 0.39389$ 5 d.p

2.

- $y = \sqrt{x-3}$
- $y = (x-3)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(x-3)^{-\frac{1}{2}} = \frac{1}{2} \times \frac{1}{\sqrt{x-3}}$

$\left.\frac{dy}{dx}\right|_{x=7} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

NORMAL GRADIENT IS -4

WHEN $x = 7$ $y = \sqrt{7-3} = 2$ $\therefore (7, 2)$

THUS $y - y_0 = m(x - x_0)$

$y - 2 = -4(x - 7)$

$y - 2 = -4x + 28$

$4x + y = 30$

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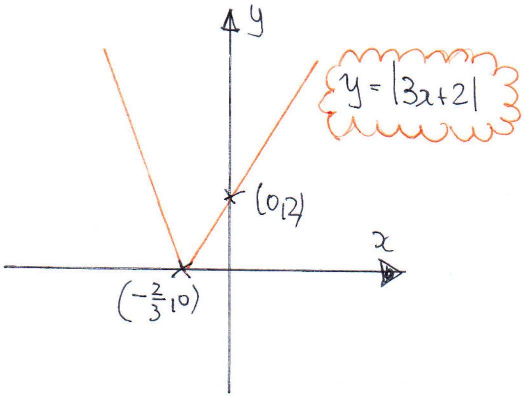
3. (a)



(b) $f(x)=1$

$|3x+2|=1$

$3x+2=1$ $\qquad$ $3x+2=-1$

$3x=-1$ $\qquad$ $3x=-3$

$x=-\frac{1}{3}$ $\qquad$ $x=-1$

(BOTH OK)

4. (a) $\sqrt{3}\sin x+\cos x \equiv R\cos(x-\alpha)$

$\equiv R\cos x\cos\alpha + R\sin x\sin\alpha$

$\equiv (R\cos\alpha)\cos x + (R\sin\alpha)\sin x$

$R\cos\alpha = 1$
$R\sin\alpha=\sqrt{3}$

- SQUARE & ADD: $R=\sqrt{1^2+(\sqrt{3})^2}=\sqrt{4}=2$
- DIVIDE EQUATIONS: $\tan\alpha=\sqrt{3}$

$\therefore \alpha=\frac{\pi}{3}$

$\therefore f(x)=2\cos\left(x-\frac{\pi}{3}\right)$

(b) MAX VALUE OF $f(x)$ IS 2

IT OCCURS WHEN $\cos\left(x-\frac{\pi}{3}\right)=1$

$x-\frac{\pi}{3}=0$

$x=\frac{\pi}{3}$

(c) $f(x)=\sqrt{3}$

$\Rightarrow 2\cos\left(x-\frac{\pi}{3}\right)=\sqrt{3}$

$\Rightarrow \cos\left(x-\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}$

$\arccos\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi}{6}$

THUS $\begin{cases} x-\frac{\pi}{3}=\frac{\pi}{6}\pm 2n\pi \\ x-\frac{\pi}{3}=\frac{11\pi}{6}\pm 2n\pi \end{cases}$ $\quad n=0,1,2,3,\ldots$

$\begin{cases} x=\frac{\pi}{2}\pm 2n\pi \\ x=\frac{13\pi}{6}\pm 2n\pi \end{cases}$

FOR $0\le x<2\pi$ $\quad x=\frac{\pi}{2}, \frac{\pi}{6}$

5. (a) $y = (x^2-4)^3$

$$\frac{dy}{dx} = 3(x^2-4)^2 \times 2x$$

$$\frac{dy}{dx} = 6x(x^2-4)^2$$

(b) $y = x\cos 2x$

$$\frac{dy}{dx} = 1 \times \cos 2x + x \times [-\sin 2x \times 2]$$

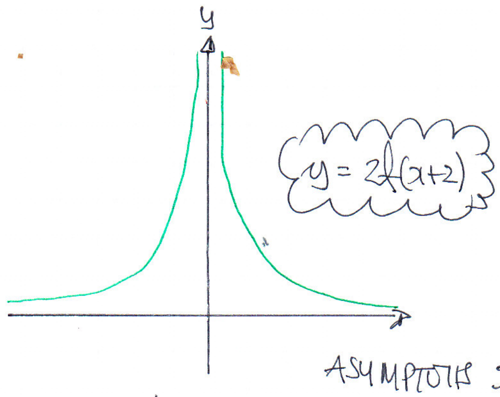
$$\frac{dy}{dx} = \cos 2x - 2x\sin 2x$$

(c) $y = \frac{\sin x}{x}$

$$\frac{dy}{dx} = \frac{x(\cos x) - \sin x \times 1}{x^2}$$

$$\frac{dy}{dx} = \frac{x\cos x - \sin x}{x^2}$$

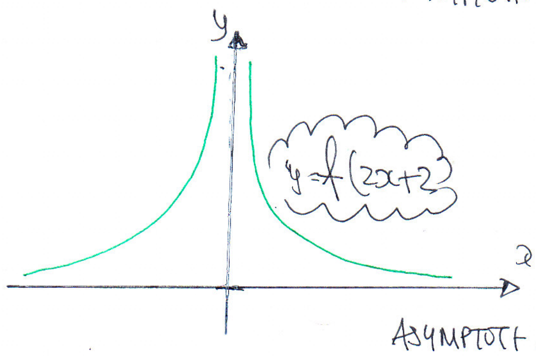
6.



- TRANSLATION "LEFT" BY 2 UNITS
- VERTICAL STRETCH, SCALE FACTOR 2

(EITHER ORDER)

ASYMPTOTES $x=0$
$y=0$

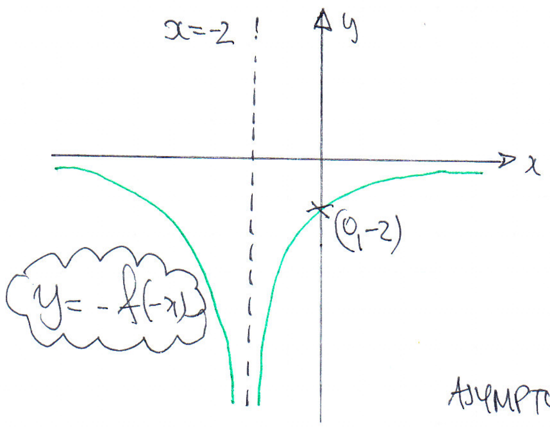


- TRANSLATION, 2 UNITS TO THE "LEFT"

FOLLOWED BY

- HORIZONTAL STRETCH BY SCALE FACTOR $\frac{1}{2}$

ASYMPTOTE $x=0$
$y=0$



REFLECTION IN THE x AXIS FOLLOWED BY REFLECTION IN THE y AXIS (EITHER ORDER)

ASYMPTOTES $x=-2$
$y=0$

7. (a)
$$f(x) = \frac{2}{x-3} - \frac{4}{x^2-4x+3} = \frac{2}{x-3} - \frac{4}{(x-3)(x-1)}$$
$$= \frac{2(x-1)-4}{(x-3)(x-1)} = \frac{2x-2-4}{(x-3)(x-1)} = \frac{2x-6}{(x-3)(x-1)} = \frac{2\cancel{(x-3)}}{\cancel{(x-3)}(x-1)}$$
$$= \frac{2}{x-1}$$ // AS REQUIRED

(b) LET $y = \frac{2}{x-1}$

$\Rightarrow y(x-1) = 2$

$\Rightarrow yx - y = 2$

$\Rightarrow yx = y + 2$

$\Rightarrow x = \frac{y+2}{y}$

$\therefore f^{-1}(x) = \frac{x+2}{x}$ //

(c) $f(g(x)) = \frac{4}{7}$

$\Rightarrow f(2x^2+4) = \frac{4}{7}$

$\Rightarrow \frac{2}{(2x^2+4)-1} = \frac{4}{7}$

$\Rightarrow \frac{2}{2x^2+3} = \frac{4}{7}$

$\Rightarrow 8x^2+12 = 14$

$\Rightarrow 8x^2 = 2$

$\Rightarrow x^2 = \frac{1}{4}$

$\Rightarrow x = \pm\frac{1}{2}$ // BOTH OK

8. $6\sec^2 2x + 5\tan 2x = 12$

$6(1+\tan^2 2x) + 5\tan 2x = 12$ $\quad 1+\tan^2\theta \equiv \sec^2\theta$

$6\tan^2 2x + 5\tan 2x - 6 = 0$

LET $t = \tan 2x$

$6t^2 + 5t - 6 = 0$ FACTORIZE OR QUADRATIC FORMULA

$$t = \frac{-5 \pm \sqrt{5^2 - 4\times6\times(-6)}}{2\times6} = \frac{-5\pm\sqrt{169}}{12} = \begin{cases} \frac{2}{3} \\ -\frac{3}{2} \end{cases}$$

$$\therefore \tan 2x = \begin{cases} 2/3 \\ -3/2 \end{cases}$$

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$\arctan\left(\frac{2}{3}\right) = 0.5880^c\ldots$

$2x = 0.588 \pm n\pi$

$x = 0.294 \pm \frac{n\pi}{2}$

$\arctan\left(-\frac{3}{2}\right) = -0.983^c\ldots$

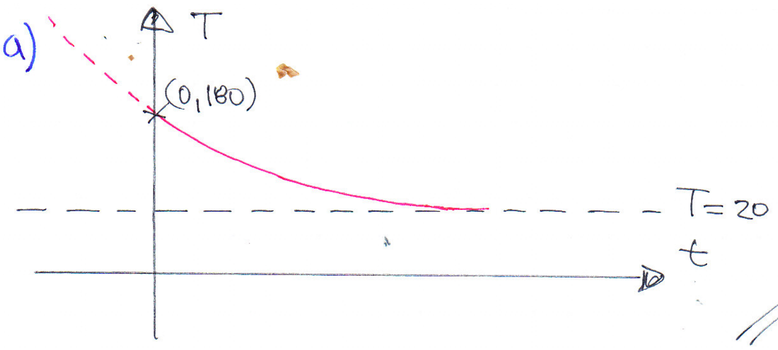
$2x = -0.983^c \pm n\pi$

$x = -0.491 \pm \frac{n\pi}{2}$

$x = 0.294^c, 1.864^c, 1.079^c, 2.650^c$

$\therefore x = 0.29^c, 1.08^c, 1.86^c, 2.65^c$

9. a)



(b) $T = 100$

$100 = 20 + 160e^{-0.1t}$

$80 = 160e^{-0.1t}$

$\frac{1}{2} = e^{-0.1t}$

$e^{0.1t} = 2$

$0.1t = \ln 2$

$\frac{1}{10}t = \ln 2$

$t = 10\ln 2$

≈ 6.93 min

(c) $T = 20 + 160e^{-0.1t}$

$\frac{dT}{dt} = -0.1 \times 160e^{-0.1t}$

$\frac{dT}{dt} = -16e^{-0.1t}$

(d) $\frac{dT}{dt} = -2$

$-2 = -16e^{-0.1t}$

$\boxed{e^{-0.1t} = \frac{1}{8}}$

SUB INTO $T = 20 + 160e^{-0.1t}$

$T = 20 + 160 \times \frac{1}{8}$

$T = 40$