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C2, IYGB, PAPER P

1. a) $(1+2x)^{12} = 1 + \frac{12}{1}(2x)^1 + \frac{12\times11}{1\times2}(2x)^2 + \frac{12\times11\times10}{1\times2\times3}(2x)^3 + \frac{12\times11\times10\times9}{1\times2\times3\times4}(2x)^4 + \cdots$

$(1+2x)^{12} = 1 + 24x + 264x^2 + 1760x^3 + 7920x^4 + \cdots$

b) LET $1+2x = 1.02$

$2x = 0.02$

$x = 0.01$

$[1+2(0.01)]^{12} = 1 + 24(0.01) + 264(0.01)^2 + 1760(0.01)^3 + 7920(0.01)^4 + \cdots$

$1.02^{12} \simeq 1 + 0.24 + 0.0264 + 0.00176 + 0.0000792$

$1.02^{12} \simeq 1.2682392$

c) ERROR $= 1.02^{12} - 1.2682392 \simeq 0.0000026$

2. $f(x) \equiv x^2 - 4x + 12$

$f(-k) = 3 \times f(k)$

$\Rightarrow (-k)^2 - 4(-k) + 12 = 3[k^2 - 4k + 12]$

$\Rightarrow k^2 + 4k + 12 = 3k^2 - 12k + 36$

$\Rightarrow 0 = 2k^2 - 16k + 24$

$\Rightarrow k^2 - 8k + 12 = 0$

$\Rightarrow (k-2)(k-6) = 0$

$k = \begin{cases} 2 \\ 6 \end{cases}$

3. a) $2\times 3^x = 900$

$3^x = 450$

$\log 3^x = \log 450$

$x \log 3 = \log 450$

$x = \frac{\log 450}{\log 3}$

$x \simeq 5.56$

b) $\log_2(7y-1) = 3 + \log_2(y-1)$

$\log_2(7y-1) - \log_2(y-1) = 3\log_2 2$

$\log_2\left(\frac{7y-1}{y-1}\right) = \log_2 8$

$\frac{7y-1}{y-1} = 8$

$7y - 1 = 8y - 8$

$7 = y$

$\therefore y = 7$

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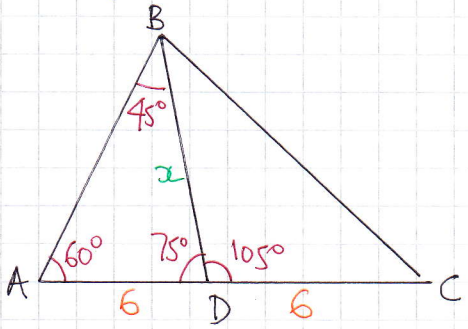
4.



a) BY THE SINE RULE ON $B\hat{A}D$

$$\frac{x}{\sin 60} = \frac{6}{\sin 45}$$

$$x = \frac{6\sin 60}{\sin 45}$$

$$x = \frac{3\sqrt{3}}{\frac{1}{\sqrt{2}}} = 3\sqrt{6}$$

$$x \simeq 7.35 \text{ cm}$$

b) $\text{AREA } B\hat{A}D = \frac{1}{2}|AD||BD|\sin 75 = \frac{1}{2}\times 6\times 3\sqrt{6}\times \sin 75 = \frac{27+9\sqrt{3}}{2}$

$$\simeq 21.3 \text{ cm}^2$$

c) THIS IS THE HEIGHT OF THE TRIANGLE $B\hat{A}D$

$$\frac{1}{2}\times \text{BASE}\times \text{HEIGHT} = \frac{27+9\sqrt{3}}{2}$$

$$\frac{1}{2}\times 6\times h = \frac{27+9\sqrt{3}}{2}$$

$$h = \frac{9+3\sqrt{3}}{2} \simeq 7.10 \text{ cm}$$

d) BY THE COSINE RULE ON $B\hat{D}C$: $|BC|^2 = |BD|^2+|DC|^2-2|BD||BC|\cos 105°$

$$|BC|^2 = x^2+6^2-2\times x\times 6\times \cos 105$$

$$|BC|^2 = (3\sqrt{6})^2+36-2\times 6\times 3\sqrt{6}\times \cos 105$$

$$|BC|^2 = 54+36-36\sqrt{6}\cos 105$$

$$|BC|^2 = 112.82\ldots$$

$$|BC| \simeq 10.6 \text{ cm}$$

5. a) $(x-2)^2+(y-5)^2 = (\sqrt{10})^2$

$$x^2-4x+4+y^2-10y+25 = 10$$

$$x^2+y^2-4x-10y+19 = 0$$

AS REQUIRED

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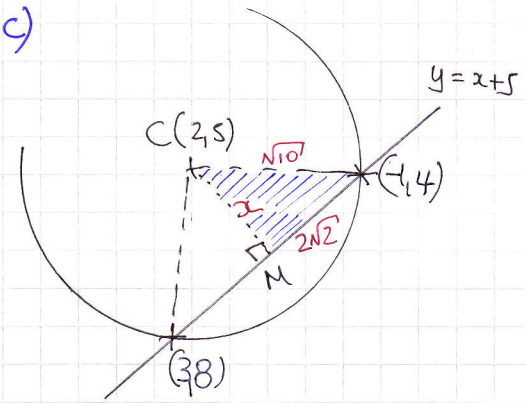
b) SOLVING SIMULTANEOUSLY

$$\left.\begin{aligned} y &= x+5 \\ (x-2)^2+(y-5)^2 &= 10 \end{aligned}\right\} \Rightarrow (x-2)^2+(x+5-5)^2 = 10$$

$$x^2-4x+4+x^2=10$$
$$2x^2-4x-6=0$$
$$x^2-2x-3=0$$
$$(x-3)(x+1)=0$$
$$x = \begin{cases} 3 \\ -1 \end{cases} \qquad y = \begin{cases} 8 \\ 4 \end{cases}$$

$\therefore (3,8)$ & $(-1,4)$ //

c)



DISTANCE $|PQ| = \sqrt{(4-8)^2+(-1-3)^2}$

$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$

BY PYTHAGORAS

$$x^2+(2\sqrt{2})^2 = (\sqrt{10})^2$$
$$x^2+8=10$$
$$x^2=2$$
$$x=\sqrt{2}$$

// AS REQUIRED

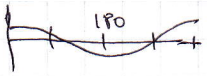
ALTERNATIVE

MIDPOINT $M\left(\frac{3-1}{2}, \frac{8+4}{2}\right) = M(1,6)$

$|MC| = \sqrt{(6-5)^2+(1-2)^2} = \sqrt{1+1} = \sqrt{2}$ //

6. a) $A(0,2)$ (when $x=0$)

$B(\pi,-4)$ //



GRAPH OF $y=\cos x$ HAS MIN AT $(180,-1) = (\pi,-1)$

$y=3\cos x$ HAS MIN AT $(\pi,-3)$

$y=3\cos x-1$ HAS MIN AT $(\pi,-4)$

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b) $y = 3\cos x - 1$

$0 = 3\cos x - 1$

$1 = 3\cos x$

$\boxed{\cos x = \frac{1}{3}}$

$\Rightarrow \arccos\left(\frac{1}{3}\right) = 1.23^c$

$\Rightarrow \begin{cases} x = 1.23^c \pm 2\pi n \\ x = 5.05^c \pm 2\pi n \end{cases} \quad n = 0, 1, 2, 3, \ldots$

$\therefore x_1 = 1.23^c \quad \therefore C(1.23, 0)$

$x_2 = 5.05^c \quad \therefore D(5.05, 0)$

7. a) EITHER

250×0.9^2

$= 202.5$

OR

$a = 250$

$r = 0.9$

$n = 3$

$u_n = ar^{n-1}$

$u_3 = 250 \times 0.9^2$

$u_3 = 202.5$

b)

START	END OF 1	END OF 2	.....	END OF 12
250				
a	u_2	u_3		u_{13}

$u_n = ar^{n-1}$

$u_{13} = 250 \times 0.9^{12} = 60.71$ kg

c) AT THE END OF MONTH...

1	250×0.9
2	$250 \times 0.9^2 + 250 \times 0.9$
3	$250 \times 0.9^3 + 250 \times 0.9^2 + 250 \times 0.9$
4	$250 \times 0.9^4 + 250 \times 0.9^3 + 250 \times 0.9^2 + 250 \times 0.9$
⋮	
12	$250 \times 0.9^{12} + 250 \times 0.9^{11} + 250 \times 0.9^{10} + \cdots + 250 \times 0.9^1$

WE REQUIRE $= 250 \times 0.9 + 250 \times 0.9^2 + 250 \times 0.9^3 + \cdots + 250 \times 0.9^{12}$

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$$= 250\left[0.9 + 0.9^2 + 0.9^3 + \ldots + 0.9^{12}\right]$$

G.P

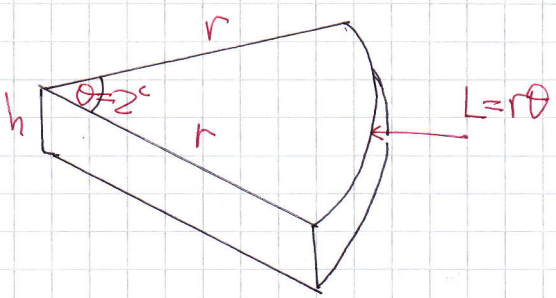
first term $= 0.9$
$r = 0.9$
$n = 12$

USING $S_n = \dfrac{a(1-r^n)}{1-r}$

$$S_{12} = \frac{0.9(1-0.9^{12})}{1-0.9} = 6.458\ldots$$

$\therefore$ TOTAL $= 250 \times 6.458 \simeq 1615$ lts

8. a)



$V = 1000$

$$\left(\frac{1}{2}r^2\theta^c\right) \times h = 1000$$

$$\frac{1}{2}r^2 \times 2 \times h = 1000$$

$$r^2h = 1000$$

$$rh = \frac{1000}{r}$$

$$4rh = \frac{4000}{r}$$

$$S = 2\left(\frac{1}{2}r^2\theta\right) + 2(rh) + hL$$

$$S = r^2\theta + 2rh + h(r\theta)$$

$$S = r^2\theta + 2rh + hr\theta$$

$$S = 2r^2 + 2rh + 2rh$$

$$S = 2r^2 + 4rh$$

$$S = 2r^2 + \frac{4000}{r}$$

b) $S = 2r^2 + 4000r^{-1}$

$$\frac{dS}{dr} = 4r - 4000r^{-2}$$

SOLVE FOR ZERO

$\Rightarrow 4r = \frac{4000}{r^2}$

$\Rightarrow 4r^3 = 4000$

$\Rightarrow r^3 = 1000$

$r = 10$ //

& $r^2 h = 1000$

$100h = 1000$

$h = 10$ //

TO SHOW THAT WHEN $r = h = 10$, S IS LEAST

$\frac{d^2S}{dr^2} = 4 + 8000r^{-3} = 4 + \frac{8000}{r^3}$

$\left.\frac{d^2S}{dr^2}\right|_{r=10} = 4 + \frac{8000}{10^3} = 12 > 0$

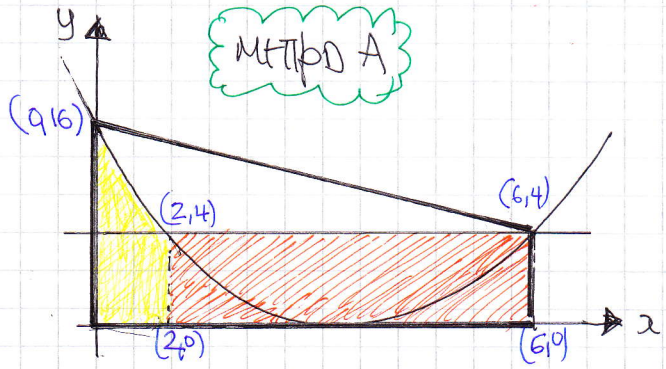
INDEED S IS MINIMISED

9. $y = (x-4)^2$

- BY INSPECTION $C(0,16)$ CURVE TOUCHES THE x AXIS AT $(4,0)$
- $(x-4)^2 = 4$

  $x - 4 = \lessgtr^{2}_{-2}$

  $x = \lessgtr^{6}_{2}$ $\therefore$ $A(2,4)$, $B(6,4)$



- AREA OF TRAPEZIUM $= \frac{16+4}{2} \times 6 = 60$
- AREA OF "ORANGE RECTANGLE" $= 4 \times 4 = 16$
- "YELLOW AREA" $= \int_0^2 (x-4)^2 \, dx = \int_0^2 x^2 - 8x + 16 \, dx$

$= \left[\frac{1}{3}x^3 - 4x^2 + 16x\right]_0^2 = \left(\frac{8}{3} - 16 + 32\right) - 0$

$= \frac{56}{3}$

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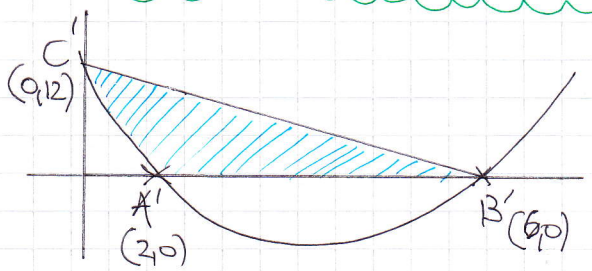
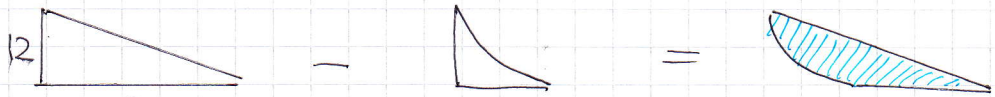
REQUIRED AREA = $60 - 16 - \frac{56}{3}$

$= \frac{76}{3}$

METHOD B — TRANSLATE "WHOLE PICTURE" 4 UNITS DOWN



$y = (x-4)^2 - 4$



$\frac{1}{2} \times 6 \times 12 = 36$

$\int_0^2 (x-4)^2 - 4 \, dx = \int_0^2 x^2 - 8x + 12 \, dx$

$= \left[\frac{1}{3}x^3 - 4x^2 + 12x\right]_0^2 = \left(\frac{8}{3} - 16 + 24\right) - 0$

$= \frac{32}{3}$

REQUIRED AREA = $36 - \frac{32}{3} = \frac{76}{3}$