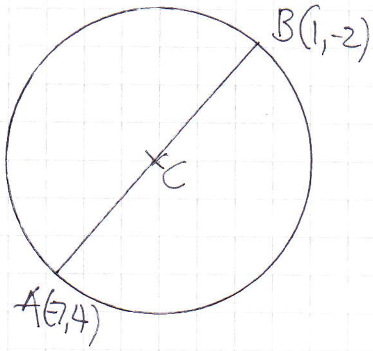
C2, IYGB, PAPER K

1. a)



C IS THE MIDPOINT OF AB $= \left(\frac{1-7}{2}, \frac{-2+4}{2}\right)$

$\therefore C(-3,1)$

$r = \sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$ A(-7,4), C(-3,1)

$r = \sqrt{(1-4)^2+(-3+7)^2}$

$r = \sqrt{9+16} = 5$

$\therefore$ RADIUS IS 5

b) EQUATION OF CIRCLE $(x+3)^2+(y-1)^2 = 25$

with $x=0$ $(0+3)^2+(y-1)^2=25$

$9+(y-1)^2=25$

$(y-1)^2=16$

$y-1 = 4$ or -4

$y = 5$ or -3 $\therefore a = 5$ or -3

2.

$$\boxed{\begin{aligned} f(x) &= x+10+\frac{25}{x} = x+10+25x^{-1} \\ f'(x) &= 1-25x^{-2} = 1-\frac{25}{x^2} \\ f''(x) &= 50x^{-3} = \frac{50}{x^3} \end{aligned}}$$

- $f'(x)=0$

$\Rightarrow 1-\frac{25}{x^2}=0$

$\Rightarrow 1 = \frac{25}{x^2}$

$\Rightarrow x^2=25$

$\Rightarrow x = 5$ or -5 $y = 5+10+\frac{25}{5}=20$ or $-5+10+\frac{25}{-5}=0$

- $f''(5) = \frac{50}{5^3} = \frac{2}{5} > 0$

$\therefore (5,20)$ IS A LOCAL MIN

- $f''(-5) = \frac{50}{(-5)^3} = -\frac{2}{5} < 0$

$\therefore (-5,0)$ IS A LOCAL MAX

3. a) $\boxed{u_n = ar^{n-1}}$

$$\left.\begin{aligned} u_3 &= 4 \\ u_6 &= 6.912 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} ar^2 &= 4 \\ ar^5 &= 6.912 \end{aligned}\right\} \Rightarrow \frac{\cancel{a}r^5}{\cancel{a}r^2} = \frac{6.912}{4}$$

$$\Rightarrow r^3 = 1.728$$

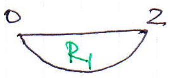
$$\Rightarrow r = 1.2 = \frac{6}{5}$$

$$ar^2 = 4$$
$$a \times (1.2)^2 = 4$$
$$a = \frac{25}{9}$$

b) $\boxed{S_n = \dfrac{a(r^n - 1)}{r - 1}} \Rightarrow S_{10} = \dfrac{\frac{25}{9}(1.2^{10} - 1)}{1.2 - 1} \simeq 72.107\ldots$

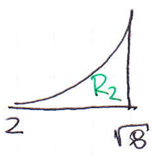
$$\simeq 72.1$$

4.



$$= \int_0^2 x^3 - 4x \, dx = \left[\frac{1}{4}x^4 - 2x^2\right]_0^2 = (4-8) - (0) = -4$$

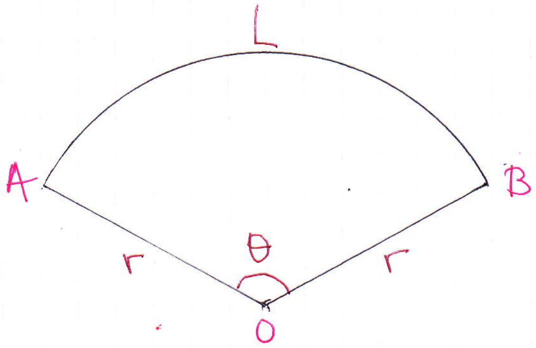
$\therefore$ AREA OF R_1 IS 4



$$= \int_2^{\sqrt{8}} x^3 - 4x \, dx = \left[\frac{1}{4}x^4 - 2x^2\right]_2^{\sqrt{8}} = (\cancel{16 - 16}) - (4 - 8) = 4$$

$\therefore$ AREA OF R_2 IS ALSO 4

5.



- $P = 33$

$$r + r + L = 33$$
$$\boxed{2r + r\theta = 33}$$
$$\downarrow \times r$$

- $A = 67.5$

$$\frac{1}{2}r^2\theta = 67.5$$
$$\boxed{r^2\theta = 135}$$

$$2r^2 + r^2\theta = 33r$$
$$2r^2 + 135 = 33r$$
$$2r^2 - 33r + 135 = 0$$

C2, IYGB, PAPER K

BY FACTORIZATION

$(2r-15)(r-9)=0$

$r = \begin{cases} 9 \\ 15/2 = 7.5 \end{cases}$

OR QUADRATIC FORMULA

$r = \dfrac{-(-33) \pm \sqrt{(-33)^2 - 4\times2\times135}}{2\times2}$

$r = \dfrac{33 \pm \sqrt{9}}{4} = \begin{cases} 9 \\ \frac{15}{2} \end{cases}$

& $\theta = \dfrac{135}{r^2}$

$\theta = \begin{cases} \frac{135}{9^2} = \frac{5}{3} \\ \frac{135}{(15/2)^2} = \frac{12}{5} = 2.4^c \end{cases}$

$\therefore$ EITHER $r=9,\ \theta=\frac{5}{3}$

OR $r=\frac{15}{2},\ \theta=\frac{12}{5}$

6. a) $\boxed{\frac{\pi}{3} \div 4 = \frac{\pi}{12}}$

	(0°)	(15°)	(30°)	(45°)	(60°)
x	0	$\frac{\pi}{12}$	$\frac{2\pi}{12}=\frac{\pi}{6}$	$\frac{3\pi}{12}=\frac{\pi}{4}$	$\frac{4\pi}{12}=\frac{\pi}{3}$
y	1	$\frac{2+\sqrt{3}}{4}$ (0.9330)	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$$\int_0^{\frac{\pi}{3}} \cos^2 x \, dx \approx \frac{\text{THICKNESS}}{2}\left[\text{FIRST} + \text{LAST} + 2\times\text{REST}\right]$$

$$\approx \frac{\pi/12}{2}\left[1 + \frac{1}{4} + 2\left(\frac{2+\sqrt{3}}{4} + \frac{3}{4} + \frac{1}{2}\right)\right]$$

$$\approx 0.735 \quad (3 \text{ s.f.})$$

b) $$\int_0^{\frac{\pi}{3}} \sin^2 x\,dx = \int_0^{\frac{\pi}{3}} 1 - \cos^2 x\,dx = \boxed{\int_0^{\frac{\pi}{3}} 1\,dx} - \boxed{\int_0^{\frac{\pi}{3}} \cos^2 x\,dx}$$

FOUND IN (a)

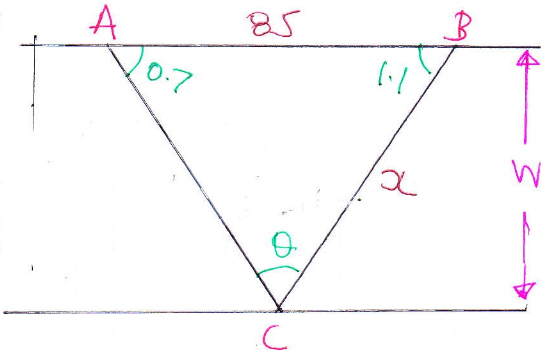
$$\approx \boxed{\left[x\right]_0^{\frac{\pi}{3}}} - \boxed{0.735\ldots\ldots}$$

$$\approx \left(\frac{\pi}{3} - 0\right) - 0.735$$

$$\approx 0.312$$

C2, IYGB, PAPER K

7.

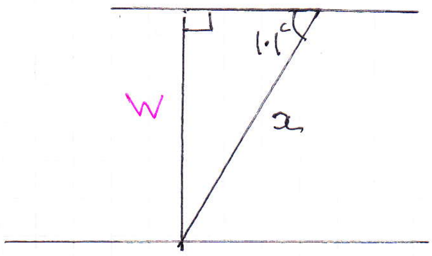


- FIRSTLY $\pi-(1.1+0.7)=1.3416$
- BY THE SINE RULE

$$\frac{x}{\sin(0.7^c)}=\frac{85}{\sin(1.3416)}$$

$$x=\frac{85\sin(0.7)}{\sin(1.3416^c)}$$

$$x=56.22902565\ldots$$



- $\frac{W}{x}=\sin(1.1^c)$

$$W=x\sin(1.1^c)$$

$$W=56.229\ldots\times\sin(1.1)$$

$$W=50.1112\ldots$$

$$\therefore\ W\simeq 50$$

8 a) $f(x)=x^3-x^2-3x+3$

$f(1)=1^3-1^2-3\times1+3=1-1-3+3=0$ $\quad\therefore (x-1)$ IS A FACTOR OF $f(x)$

b)

$$\begin{array}{r|l} & x^2-3 \\ \hline x-1 & x^3-x^2-3x+3 \\ & -x^3+x^2 \\ \hline & -3x+3 \\ & 3x-3 \\ \hline & 0 \end{array}$$

$\therefore f(x)=(x-1)(x^2-3)$ ← DIFFERENCE OF SQUARES

$f(x)=(x-1)(x-\sqrt{3})(x+\sqrt{3})$

c) $\tan^3\theta-\tan^2\theta-3\tan\theta+3=0$

LET $x=\tan\theta$ GIVES part (a)

SO BY part (b)

$$x=\begin{cases}\sqrt{3}\\-\sqrt{3}\\1\end{cases}\qquad \tan\theta=\begin{cases}\sqrt{3}\\-\sqrt{3}\\1\end{cases}$$

Thus

- $\tan\theta = \sqrt{3}$
- $\arctan\sqrt{3} = 60$
- $\theta = 60 \pm 180n$

$n = 0, 1, 2, 3 \ldots$

- $\tan\theta = -\sqrt{3}$
- $\arctan(-\sqrt{3}) = -60$
- $\theta = -60 \pm 180n$

$n = 0, 1, 2, 3, \ldots$

- $\tan\theta = 1$
- $\arctan 1 = 45$
- $\theta = 45 \pm 180n$

$n = 0, 1, 2, 3 \ldots$

$60°, 240°, 120°, 300°, 45°, 225°$

9.

- $\log_y x = 5$

$\log_y x = 5\log_y y$

$\log_y x = \log_y y^5$

$\boxed{x = y^5}$

- $\log_2 x = 2 + \log_2 y$

$\log_2 x - \log_2 y = 2\log_2 2$

$\log_2\left(\frac{x}{y}\right) = \log_2 4$

$\boxed{\frac{x}{y} = 4}$

(OR $4y = x$)

$y^5 = 4y$

$y^4 = 4 \quad (x, y > 0$

$y^2 = \begin{cases} 2 \\ \cancel{-2} \end{cases}$

$y = \begin{cases} \sqrt{2} \\ \cancel{-\sqrt{2}} \end{cases} \quad (x, y > 0)$

$\therefore \ x = (\sqrt{2})^5$

$x = \sqrt{2}\sqrt{2}\sqrt{2}\sqrt{2}\sqrt{2} = 2 \times 2 \times \sqrt{2}$

$x = 4\sqrt{2}$

$\therefore (4\sqrt{2}, \sqrt{2})$

10.

- IN BOTH PART THE ONLY NUMBERS THAT WILL BE PRESENT ARE THE BINOMIAL COEFFICIENTS
- BINOMIAL COEFFICIENTS ARE SYMMETRICAL

a) IF $n=13$ <u>JOINT</u> "HIGHEST" POWERS WILL BE THE COEFFICIENTS

$$\binom{13}{7} = \binom{13}{6} = \frac{13\times12\times11\times10\times9\times8}{1\times2\times3\times4\times5\times6}$$

$$= 1716$$

b) IF $n=14$ THE $\binom{14}{7}$ IS THE HIGHEST

$$\therefore \binom{14}{6} = \binom{14}{8} \text{ IS THE SECOND HIGHEST}$$

I.E 3003