C2, IYGB, PAPER E

1.

x	0	0.25	0.5	0.75	1
y	1	$\frac{2}{3}$	$2-\sqrt{2}$	$4-2\sqrt{3}$	$\frac{1}{2}$

$\text{AREA} \approx \frac{\text{THICKNESS}}{2}\left[\text{FIRST} + \text{LAST} + 2 \times \text{REST}\right]$

$\text{AREA} \approx \frac{0.25}{2}\left[1 + \frac{1}{2} + 2\left(\frac{2}{3} + 2 - \sqrt{2} + 4 - 2\sqrt{3}\right)\right]$

$\text{AREA} \simeq 0.635$ 3 d.p.

2.

$\boxed{a = 32, \quad u_5 = 162}$

USING $u_n = ar^{n-1}$

$\Longrightarrow 162 = 32 \times r^4$

$\Longrightarrow \frac{81}{16} = r^4$

$\Longrightarrow r = \sqrt[4]{\frac{81}{16}}$

$\Longrightarrow \boxed{r = \frac{3}{2}}$

THUS

$u_3 = ar^2$

$u_3 = 32 \times \left(\frac{3}{2}\right)^2$

$u_3 = 72$

I.E 72 mph

3.

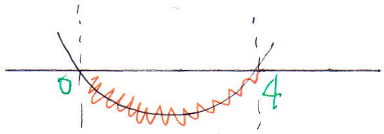
$y = x^3 - 6x^2 + 12$

$\frac{dy}{dx} < 0$ (IF DECREASING) $\rightarrow$

$3x^2 - 12x < 0$

$3x(x-4) < 0$

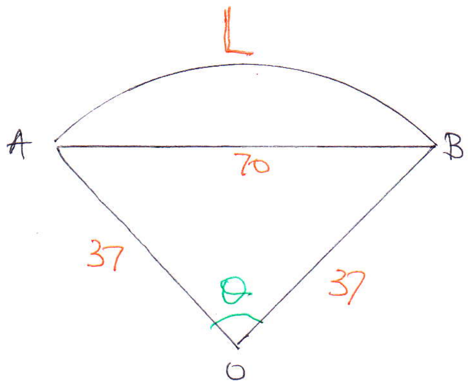
$\text{C.V} = \genfrac{}{}{0pt}{}{0}{4}$



$0 < x < 4$

C2, IYGB, PAPER E

4. a)



• BY THE COSINE RULE

$|AB|^2 = |OA|^2 + |OB|^2 - 2|OA||OB|\cos\theta$

$70^2 = 37^2 + 37^2 - 2\times37\times37\cos\theta$

$4900 = 1369 + 1369 - 2738\cos\theta$

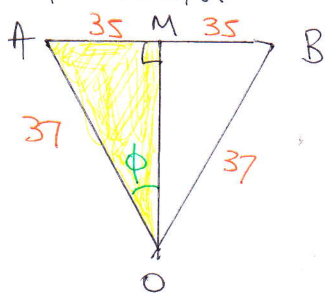
$2738\cos\theta = -2162$

$\cos\theta = -\frac{2162}{2738}$

$\theta = 2.480997944\ldots$

$\theta \simeq 2.481^c$

ALTERNATIVE



$\sin\phi = \frac{35}{37}$

$\phi \simeq 1.24049\ldots$

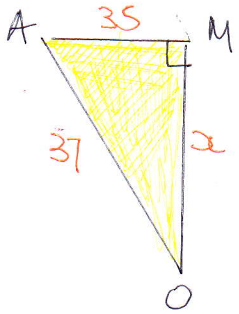
$\therefore \hat{AOB} = 2\phi \simeq 2.481^c$ AS BEFORE

b) (I) $L = r\theta^c$

$L = 37 \times 2.481$

$L \simeq 91.80$ m

(II)



BY PYTHAGORAS

$x^2 + 35^2 = 37^2$

$x^2 + 1225 = 1369$

$x^2 = 144$

$x = 12$ m

(III) AREA OF SECTOR $= \frac{1}{2}r^2\theta^c = \frac{1}{2}\times37^2\times2.481\ldots \simeq 1698.243\ldots$

AREA OF TRIANGLE $= \frac{1}{2}\times B\times H = \frac{1}{2}\times70\times12 = 420$

$\therefore$ AREA OF SEGMENT $= 1698.243 - 420 \approx 1278$ m^2

C2, IYGB, PAPER E

5. $(2+kx)^6 = \binom{6}{0}(2)^6(kx)^0 + \binom{6}{1}(2)^5(kx)^1 + \binom{6}{2}(2)^4(kx)^2 + \binom{6}{3}(2)^3(kx)^3 + \dots$

$= (1\times64\times1) + (6\times32\times kx) + (15\times16\times k^2x^2) + (20\times8\times k^3x^3) + \dots$

$= \underbrace{64}_{a} + \underbrace{192k}_{b}x + \underbrace{240k^2}_{b}x^2 + \underbrace{160k^3}_{c}x^3 + \dots$

- $192k = 240k^2 \implies$ AS $k \neq 0$

$192 = 240k$

$\boxed{k = \frac{4}{5}}$

$\therefore\ b = 192k = 192 \times \frac{4}{5} = 153.6$

$c = 160k^3 = 160 \times \left(\frac{4}{5}\right)^3 = 81.92$

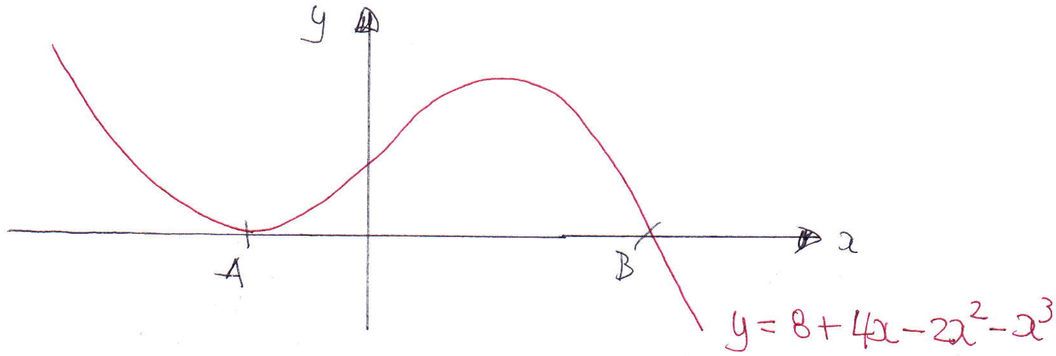
THUS

$a = 64$

$b = 153.6$

$c = 81.92$

6. a)



- BY VERIFICATION

IF $x = -2$

$y = 8 + 4(-2) - 2(-2)^2 - (-2)^3$

$y = 8 - 8 - 8 + 8$

$y = 0$

IE $A(-2, 0)$

- $$\begin{array}{r|l} & -x^2 + 4 \\ x+2 & -x^3 - 2x^2 + 4x + 8 \\ & \cancel{x^3} + \cancel{2x^2} \\ \hline & 4x + 8 \\ & -4x - 8 \\ \hline & 0 \end{array}$$

$\therefore\ y = (x+2)(-x^2+4)$

$y = (x+2)(4-x^2)$

$y = (x+2)(2-x)(2+x)$

$y = (x+2)^2(2-x)$

C2, IYGB, PAPER E

$\therefore$ $A(-2,0)$ $\longleftarrow$ TOUCHING POINT FROM $(x+2)^2$

$B(2,0)$ $\longleftarrow$ CROSSING POINT FROM $(2-x)$

b) $y = 8 + 4x - 2x^2 - x^3$

$\frac{dy}{dx} = 4 - 4x - 3x^2$

• SOLVE FOR ZERO

$4 - 4x - 3x^2 = 0$

$0 = 3x^2 + 4x - 4$

$0 = (x+2)(3x-2)$

$x = \begin{cases} -2 \leftarrow A \\ \frac{2}{3} \leftarrow C \end{cases}$

$y = 8 + 4\left(\frac{2}{3}\right) - 2\left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^3$

$y = \frac{256}{27}$

$\therefore C\left(\frac{2}{3}, \frac{256}{27}\right)$

c) $\text{AREA} = \int_{-2}^{2} 8 + 4x - 2x^2 - x^3 \, dx$

$= \left[8x + 2x^2 - \frac{2}{3}x^3 - \frac{1}{4}x^4\right]_{-2}^{2}$

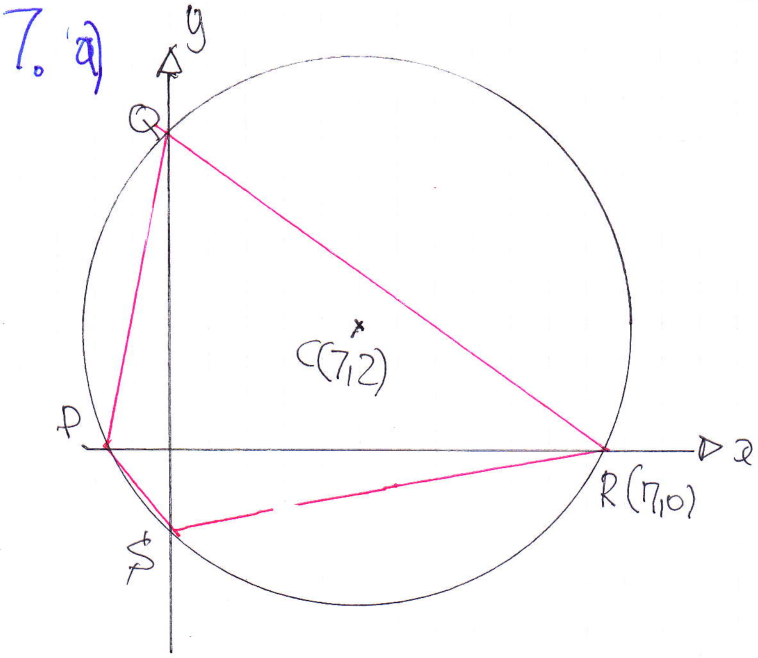
$= \left(16 + 8 - \frac{16}{3} - 4\right) - \left(-16 + 8 + \frac{16}{3} - 4\right)$

$= \frac{44}{3} - \left(-\frac{20}{3}\right)$

$= \frac{64}{3}$

C2, IYGB, PAPER E

7. a)



• $R(17,0)$ • $C(7,2)$

$|RC| = \sqrt{(17-7)^2+(0-2)^2}$

$|RC| = \sqrt{100+4}$

$|RC| = \sqrt{104}$

∴ EQUATION

$(x-7)^2+(y-2)^2 = \sqrt{104}^{\,2}$

$(x-7)^2+(y-2)^2 = 104$

b) WHEN $x=0$

$(-7)^2+(y-2)^2 = 104$

$\Rightarrow 49 + (y-2)^2 = 104$

$\Rightarrow (y-2)^2 = 55$

$\Rightarrow y-2 = \pm\sqrt{55}$

$\Rightarrow y = 2 \pm \sqrt{55}$

∴ $Q(0, 2+\sqrt{55})$

$S(0, 2-\sqrt{55})$

∴ DIFFERENCE OF $2\sqrt{55}$

OR $(2+\sqrt{55})-(2-\sqrt{55})$

$= 2\sqrt{55}$

c) WHEN $y=0$

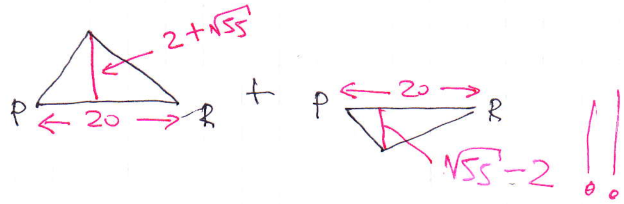
$(x-7)^2+(0-2)^2 = 104$

$(x-7)^2 + 4 = 104$

$(x-7)^2 = 100$

$x-7 = \begin{cases} 10 \\ -10 \end{cases}$

$x = \begin{cases} 17 \leftarrow R \\ -3 \leftarrow P \end{cases}$



$= \frac{1}{2}\times 20\times(2+\sqrt{55}) + \frac{1}{2}\times 20\times(2-\sqrt{55})$

$= 10(2+\sqrt{55}) + 10(\sqrt{55}-2)$

$= \cancel{20} + 10\sqrt{55} + 10\sqrt{55} - \cancel{20}$

$= 20\sqrt{55}$

ALTERNATIVE AREA $= \frac{1}{2}|PR||QS|$

$= \frac{1}{2}\times 20\times 2\sqrt{55}$

8. $\log_3 8 - 3\log_3 t = 3$

$\Rightarrow \log_3 8 - \log_3 t^3 = 3\log_3 3$

$\Rightarrow \log_3\left(\frac{8}{t^3}\right) = \log_3 27$

$\Rightarrow \frac{8}{t^3} = 27$

$\Rightarrow 27t^3 = 8$

$\Rightarrow t^3 = \frac{8}{27}$

$\Rightarrow t = \frac{2}{3}$

9. a) $f(x) = x^3 - 4x^2 - \frac{1}{2}x + 2$

$f(4) = 64 - 64 - 2 + 2 = 0$

$\therefore (x-4)$ IS A FACTOR OF $f(x)$

b) BY LONG DIVISION OR INSPECTION

$$\begin{array}{r|l} & x^2 - \frac{1}{2} \\ x-4 & x^3 - 4x^2 - \frac{1}{2}x + 2 \\ & \underline{-x^3 + 4x^2} \\ & 0 \quad 0 \quad - \frac{1}{2}x + 2 \\ & \underline{+\frac{1}{2}x - 2} \\ & 0 \end{array}$$

$\therefore f(x) = (x-4)(x^2 - \frac{1}{2})$

c) $\cos^3\theta - 4\cos^2\theta - \frac{1}{2}\cos\theta + 2 = 0$

$(\cos\theta - 4)(\cos^2\theta - \frac{1}{2}) = 0 \quad \leftarrow$ FROM PART (b)

EITHER $\cancel{\cos\theta = 4}$ OR $\cos^2\theta - \frac{1}{2} = 0$

$\cos^2\theta = \frac{1}{2}$

$\cos\theta = \pm\sqrt{\frac{1}{2}}$

$\cos\theta = \pm\frac{\sqrt{2}}{2}$

C2, IYGB, PAPER E

- $\arccos\left(\frac{\sqrt{2}}{2}\right) = 45°$

$$\begin{cases} \theta = 45 \pm 360n \\ \theta = 315 \pm 360n \end{cases}$$

- $\arccos\left(-\frac{\sqrt{2}}{2}\right) = 135$

$$\theta = 135 \pm 360n$$
$$\theta = 225 \pm 360n$$

$n = 0, 1, 2, 3 \ldots$

$\theta_1 = 45°$

$\theta_2 = 315°$

$\theta_3 = 135°$

$\theta_4 = 225°$