

CL, LYGB, PAPER IV

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1. a)  $3^y = \frac{\sqrt{3}}{9}$

$3^y = \frac{3^{\frac{1}{2}}}{3^2}$

$3^y = 3^{-\frac{3}{2}}$

$y = -\frac{3}{2}$

b)  $\sqrt{525} = \sqrt{25 \times 21} = 5\sqrt{21} = 5\sqrt{7} \times \sqrt{3}$

2. a)  $\frac{y_2 - y_1}{x_2 - x_1} = -2$

$\Rightarrow \frac{-11 - k}{-2 - (-7)} = -2$

$\Rightarrow \frac{-11 - k}{5} = -2$

$\Rightarrow -11 - k = -10$

$\Rightarrow -1 = k$

$\Rightarrow k = -1$

b)  $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = (a, b)$

$\left(\frac{a+4}{2}, \frac{-3+1}{2}\right) = (a, b)$

$\therefore \frac{a+4}{2} = a$

$a+4 = 2a$

$a = 4$

$b = -1$

c)  $\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2} = \sqrt{17}$

$\Rightarrow \sqrt{(c-7)^2 + (-3 - (-1))^2} = \sqrt{17}$

$\Rightarrow \sqrt{(c-7)^2 + 16} = \sqrt{17}$

$\Rightarrow (c-7)^2 + 16 = 17$

$\Rightarrow (c-7)^2 = 1$

$c-7 = \begin{matrix} 1 \\ -1 \end{matrix}$

$c = \begin{matrix} 8 \\ 6 \end{matrix}$

3. a)

$f'(x) = (x-3)(3x-1)$

$f'(x) = 3x^2 - x - 9x + 3$

$f'(x) = 3x^2 - 10x + 3$

$f(x) = \int 3x^2 - 10x + 3 dx$

$f(x) = x^3 - 5x^2 + 3x + C$

$(2, 3) \Rightarrow 3 = 2^3 - 5 \times 2^2 + 3 \times 2 + C$

$\Rightarrow 3 = 8 - 20 + 6 + C$

$\Rightarrow 9 = C$

$\therefore f(x) = x^3 - 5x^2 + 3x + 9$

b) BY INSPECTION

$(x+k)(x-3)^2$

$= (x+k)(x^2 - 6x + 9)$

$\therefore k = 1$

$\therefore (x+1)(x^2 - 6x + 9)$

$= x^3 - 6x^2 + 9x + x^2 - 6x + 9$

$x^3 - 5x^2 + 3x + 9$

$\therefore f(x) = (x+1)(x-3)^2$

9



4.

a) 
$$\sum_{r=1}^n u_r = 7 + 3n^2$$

$$\sum_{r=1}^4 u_r = 7 + 3 \times 4^2 = 7 + (3 \times 16) = 7 + 48 = 55$$

b) 
$$\sum_{r=1}^5 u_r = 7 + 3 \times 5^2 = 7 + (3 \times 25) = 7 + 75 = 82$$

$$\therefore u_5 = \sum_{r=1}^5 u_r - \sum_{r=1}^4 u_r = 82 - 55 = 17$$

5.

$$y = k(2x^2 - x + 1) - 5x^2 + x - 2$$

$$\Rightarrow y = 2kx^2 - kx + k - 5x^2 + x - 2$$

$$\Rightarrow y = (2k - 5)x^2 + (1 - k)x + (k - 2)$$

Below THE x AXIS  $\Rightarrow$

$$\therefore 2k - 5 < 0$$
  
AND  
$$b^2 - 4ac < 0$$

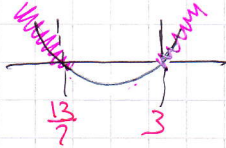
Thus  $(1 - k)^2 - 4(2k - 5)(k - 2) < 0$   
 $1 - 2k + k^2 - 4(2k^2 - 9k + 10) < 0$   
 $1 - 2k + k^2 - 8k^2 + 36k - 40 < 0$   
 $-7k^2 + 34k - 39 < 0$   
 $7k^2 - 34k + 39 > 0$   
 $(7k - 13)(k - 3) > 0$

C.V =  $\begin{cases} 13/7 \\ 3 \end{cases}$



C1, NGB, PAPER W

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$k < \frac{13}{7}$  OR  $k > 3$

BUT  $2k - 5 < 0$   
 $2k < 5$   
 $k < \frac{5}{2}$

$\therefore k < \frac{13}{7}$

6.

$t_{n+1} = at_n + b$

$t_1 = 2$   
 $t_2 = 3$

$t_2 = at_1 + b$   
 $3 = a \times 2 + b$

$2a + b = 3$



$3a + b = 7$   
 $2a + b = 3$

$t_1 + t_2 + t_3 = 12$

$2 + 3 + [at_2 + b] = 12$

$5 + [3a + b] = 12$

$3a + b = 7$



$\therefore a = 4$

$b = -5$

7.

$y = 2x^3 - 5x^2 + a$

$\frac{dy}{dx} = 6x^2 - 10x$

$\frac{dy}{dx} \Big|_{x=2} = 6 \times 2^2 - 10 \times 2 = 24 - 20 = 4$

$y \Big|_{x=2} = 2 \times 2^3 - 5 \times 2^2 + a = a - 4$

$\frac{dy}{dx} \Big|_{x=1} = 6 \times 1^2 - 10 \times 1 = 6 - 10 = -4$

$y \Big|_{x=1} = 2 \times 1^3 - 5 \times 1^2 + a = a - 3$

CI, 1XGB, PAPER W

● EQUATION OF TANGENT AT  $(2, a-4)$

$$y - (a-4) = 4(x-2)$$

$$y - a + 4 = 4x - 8$$

$$\boxed{y = 4x + a - 12}$$

● EQUATION OF NORMAL AT  $(1, a-3)$

$$y - (a-3) = \frac{1}{4}(x-1)$$

$$y - a + 3 = \frac{1}{4}(x-1)$$

$$4y - 4a + 12 = x - 1$$

$$\boxed{4y = x + 4a - 13}$$

MEET ON THE x  
AXIS IMPLIES  $y=0$

W/ W  $x$  &  $a$  IS THE SAME ON BOTH EQUATIONS

$$4x + a = 12$$

$$\boxed{a = 12 - 4x}$$

$$x + 4a = 13$$

$$x + 4(12 - 4x) = 13$$

$$x + 48 - 16x = 13$$

$$35 = 15x$$

$$x = \frac{35}{15} = \frac{7}{3}$$

∴  $Q\left(\frac{7}{3}, 0\right)$

∴  $a = 12 - 4\left(\frac{7}{3}\right)$

$$a = 12 - \frac{28}{3} = \frac{36 - 28}{3}$$

$$a = \frac{8}{3}$$

8. a)

$a = 10$   
 $d = 2$   
 $n = 12$

●  $u_n = a + (n-1)d$

$$u_{12} = 10 + 11 \times 2$$

$$u_{12} = 32$$

●  $S_n = \frac{n}{2}[a + l]$

$$S_{12} = \frac{12}{2}[10 + 32]$$

$$S_{12} = 6 \times 42$$

$$S_{12} = 240 + 12 = 252$$

∴ TOTAL = 252

$$\begin{array}{r} 252 \\ + 125 \\ \hline 377 \end{array}$$



CI, IYBB, PAPER W — 5 —

b)

WEEK	IN	OUT	NET
1	10	-3	7
2	12	-3	9
3	14	-3	11
4	16	-3	13
			ETC

- New A.P with  $a=7$   
 $d=2$
- $600 - 125 = 475$  ← NEW TOTAL

•  $S_n = 475$

$$\frac{n}{2} [2 \times 7 + 2(n-1)] = 475$$

$$\frac{n}{2} [14 + 2n - 2] = 475$$

$$\frac{n}{2} [12 + 2n] = 475$$

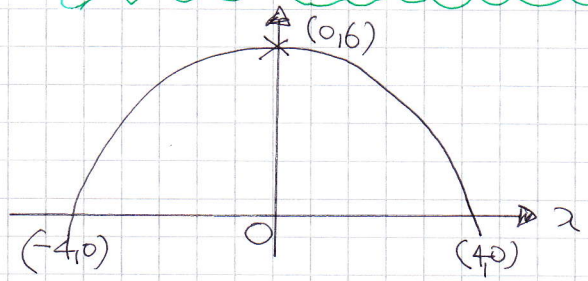
$$n(n+6) = 475$$

BY TRIAL

$n=5 \quad 5 \times 11 = 55$   
 $n=9 \quad 9 \times 15 = 135$   
 $n=15 \quad 15 \times 21 = 315$   
 $n=19 \quad 19 \times 25 = \underline{\underline{475}}$

∴  $n=19$

9. MODEL BY  $y = A - Bx^2$   $A, B > 0$



- BY INSPECTION  $A=6$
- $y = 6 - Bx^2$   
 $0 = 6 - B \times 4^2$   
 $0 = 6 - 16B$   
 $16B = 6$   
 $B = \frac{3}{8}$

∴  $y = 6 - \frac{3}{8}x^2$

THE LOAD MUST PASS THROUGH THE MIDDLE IN A SYMMETRICAL FASHION

with  $x=3$ ,  $y = 6 - \frac{3}{8} \times 3^2$   
 $y = 6 - \frac{27}{8} = \frac{21}{8} > 2$   
 ∴ it will PASS