A-LEVEL

Further Mathematics

Paper 1

Mark scheme

Practice paper – Set 2

Version 1.0

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme has been prepared for practice papers and has not, therefore, been through the process of standardising that would take place for live papers.

Further copies of this mark scheme are available from allaboutmaths.aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the paper
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
‘their’	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles

No method shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

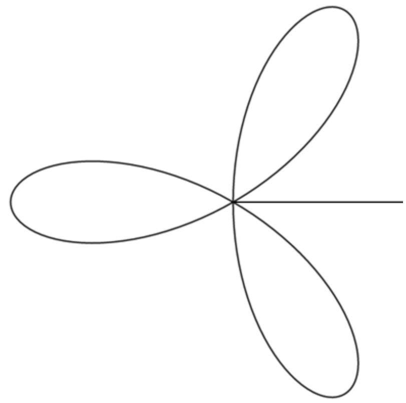
When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

Q	Marking instructions	AO	Marks	Typical solution
1	Circles correct answer	1.1b	B1	$y = 2x$
	Total		**1**	

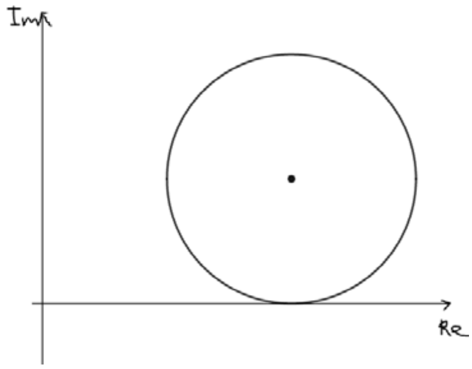
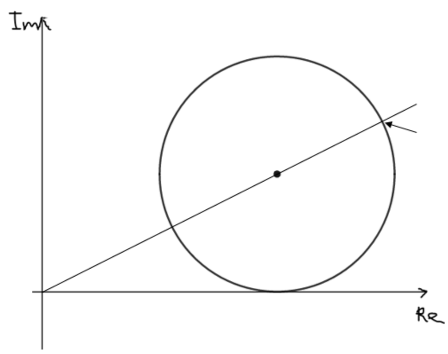
Q	Marking instructions	AO	Marks	Typical solution
2	Circles correct answer	1.1b	B1	$\frac{\sqrt{3}}{2}$
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3	Circles correct answer	1.1b	B1	$\frac{2\pi}{3}$
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
4(a)	Uses de Moivre's theorem to establish a relationship between $\cos 3\theta$ and $\cos\theta$	3.1a	M1	$\cos 3\theta = \text{Re}(\cos\theta + i\sin\theta)^3$ $= \text{Re}\left(\cos^3\theta + 3\cos^2\theta(i\sin\theta) + 3\cos\theta(i^2\sin^2\theta) + i^3\sin^3\theta\right)$ $= \text{Re}\left(+3\cos^2\theta(i\sin\theta) - 3\cos\theta\sin^2\theta - i\sin^3\theta\right)$ $= \cos^3\theta - 3\cos\theta\sin^2\theta$ $= \cos^3\theta - 3\cos\theta(1-\cos^2\theta)$ $= 4\cos^3\theta - 3\cos\theta$
	Obtains binomial expansion of $(\cos\theta + i\sin\theta)^3$	1.1b	A1	
	Uses $i^2 = -1$ and collects real terms.	1.1a	M1	
	Completes rigorous argument to prove the required result. AG	2.1	R1	
	Total		**4**	
4(b)	Selects a method to eliminate x and y by substituting $x = r\cos\theta$ and $y = r\sin\theta$	3.1a	M1	$x = r\cos\theta$ $y = r\sin\theta$ $(r^2\cos^2\theta + r^2\sin^2\theta)(6ar\cos\theta - r^2\cos^2\theta - r^2\sin^2\theta)$ $= 8ar^3\cos^3\theta$ $r^2(6ar\cos\theta - r^2) = 8ar^3\cos^3\theta$ $6ar^3\cos\theta - r^4 = 8ar^3\cos^3\theta$ $r^4 = 6ar^3\cos\theta - 8ar^3\cos^3\theta$ $r^4 = 2ar^3(3\cos\theta - 4\cos^3\theta)$ $r = -2a\cos 3\theta$
	Simplifies $r^2\cos^2\theta + r^2\sin^2\theta$ correctly, at least once	1.1b	B1	
	Rearranges and factorises so that the identity given in part (a) can be used	1.1a	M1	
	Completes rigorous argument to show the required result.	2.1	R1	
	Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
4(c)(i)	Deduces equation of at least one correct tangent	2.2a	M1	$\theta = \frac{\pi}{6}$
	Obtains all three correct tangents	1.1b	A1	$\theta = \frac{5\pi}{6}$ $\theta = \frac{\pi}{2}$
4(c)(ii)	Deduces the curve has three equally spaced loops centred at the pole	2.2a	M1	
	Draws loops with correct shape and orientation	1.1b	A1	
4(c)(iii)	Forms integral for area enclosed by a polar curve of form $\int kr^2 \mathrm{d}\theta$	1.1a	M1	Area of half a loop $\int_0^{\frac{\pi}{6}} \frac{1}{2}(-2a\cos 3\theta)^2 \mathrm{d}\theta = 2a^2 \int_0^{\frac{\pi}{6}} \cos^2 3\theta \,\mathrm{d}\theta$
	Deduces appropriate limits	2.2a	R1	$= 2a^2 \times \frac{\pi}{12}$
	Evaluates an appropriate integral	1.1b	A1	
	Obtains correct total area in terms of a FT their evaluated integral	1.1b	A1F	Area of all three loops $= \pi a^2$
	Total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)(i)	Deduces that n = 1,2,3,4,5 or 6 is an appropriate counterexample	1.1a	M1	When n = 1, $3^1 = 3$ and $1! = 1$ so $3^1 > 1!$ and the result does not hold
	Explains that $3^n > n!$ so Henry's proof cannot be correct	2.4	E1	
5(a)(ii)	Explains that the argument in step 3 is only true for values of $k \geq 2$	2.3	E1	Step 3 requires $k \geqslant 2$, but the initial value of n in step 1 is 0. Therefore this argument does not work correctly.
	Explains that the value of n checked in step 4 is less than 2 so the conclusion in step 5 is invalid	2.3	E1	
5(a)(iii)	Deduces N = 7	2.2a	B1	N = 7
	Total		**5**	
5(b)	Shows statement is true for n = 0 or 1	1.1b	B1	Consider n = 0 112 + 12 = 133 so true for n = 0 Assume true for $n = k$ Consider for $n = k + 1$ $11^{k+3} + 12^{2k+3} = 11\left(133m - 12^{2k+1}\right)$ $= 11m \times 133 + 12^{2k+1}\left(12^2 - 11\right)$ $= 11m \times 133 + 12^{2k+1} \times 133$ $= 133M$ Therefore, since true for n = 0 and true for $n = k$ implies true for $n = k + 1$ then by induction $11^{n+2} + 12^{2n+1}$ is a multiple of 133 for $n \geq 0$
	States assumption that statement is true for $n = k$	3.1a	M1	
	Considers statement for $n = k + 1$	1.1a	M1	
	Substitutes statement for $n = k$ in statement for $n = k + 1$	1.1a	M1	
	Completes rigorous algebraic argument to show statement for $n = k + 1$	2.1	R1	
	Concludes rigorous argument that statement is true for $n \geq 0$	2.1	R1	
	Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Sketches a circle	1.2	B1	
	Deduces the position of the centre of the circle at (10, 5)	2.2a	B1	
	Uses correct radius with circle just touching the real axis and approximately halfway between imaginary axis and centre.	1.1b	B1	
	Total		**3**	
6(b)	Selects a method to identify the range of values such as forming an isosceles triangle with sides of length 10 or identifying arg of centre of circle.	3.1a	M1	$\tan\theta = \frac{5}{10}$ $0 \le \arg z \le 2\arctan\frac{1}{2}$
	Deduces correct range of values	2.2a	A1	
	Total		**2**	
6(c)	Explains that the maximum value of $\lvert z \rvert$ will correspond to the point on a line through the centre of the circle, possibly implied by diagram	2.4	E1	
	Calculates the distance from the origin to the centre of the circle	3.1a	M1	
	Completes argument to show the required result.	2.1	A1	$\sqrt{10^2+5^2} = \sqrt{125}$ $5+5\sqrt{5} = 5\left(1+\sqrt{5}\right)$
	Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)	Uses correct common denominator and expands one numerator correctly	1.1a	B1	$\frac{1}{(r-1)^2}-\frac{1}{(r+1)^2}$ $=\frac{(r+1)^2}{(r-1)^2(r+1)^2}-\frac{(r-1)^2}{(r-1)^2(r+1)^2}$ $=\frac{r^2+2r+1-(r^2-2r+1)}{(r-1)^2(r+1)^2}$ $=\frac{4r}{(r-1)^2(r+1)^2}$
	Completes argument to show the given result	2.1	R1	
	Total		**2**	
7(b)	Uses the result from part (a) to write fractions as the sum of partial fractions.	3.1a	M1	required sum $\frac{1}{4}\sum_{2}^{n+1}\frac{4r}{(r-1)^2(r+1)^2}$ $=\frac{1}{4}\sum_{2}^{n+1}\frac{1}{(r-1)^2}-\frac{1}{(r+1)^2}$ $=\frac{1}{1}-\frac{1}{9}$ $+\frac{1}{4}-\frac{1}{16}$ $+\frac{1}{9}-\frac{1}{25}$ $+\frac{1}{16}-\frac{1}{36}$ . . . $+\frac{1}{(n-1)^2}-\frac{1}{(n+1)^2}$ $+\frac{1}{(n)^2}-\frac{1}{(n+2)^2}$ $=\frac{1}{4}\left(1+\frac{1}{4}-\frac{1}{(n+1)^2}-\frac{1}{(n+2)^2}\right)$
	Cancels middle terms to obtain first two and last two fractions	1.1a	M1	
	Multiplies by $\frac{1}{4}$	1.1b	B1	
	Obtains correct expression ACF	1.1b	A1	
	Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
8(a)	Deduces the position of the minimum point	2.2a	M1	The curve will have its minimum point halfway between P and Q so
	Obtains a	1.1a	A1	h = 45 when $\cosh\left(\frac{x}{b}-c\right)=1$
	Deduces $\frac{175}{b}-c=0$	2.2a	R1	when x = 175
	Uses the model at x = 0, with their a to obtain c	3.4	M1	So a = 44 and $\frac{175}{b}-c=0$ When x = 0
	Obtains correct b	1.1b	A1	$44+\cosh(-c)=55$ $\Rightarrow c=\cosh^{-1}11=3.09$ $\frac{175}{\text{b}}-3.09=0\Rightarrow b=56.7$
	Total		**5**	
8(b)	Differentiates their h and uses arc length formula	3.1a	M1	$\int_0^{350}\sqrt{1+\left(\frac{1}{56.7}\sinh\left(\frac{x}{56.7}-3.09\right)\right)^2}\,\mathrm{d}x=351$
	Obtains correct integral using their a, b and c FT their h	1.1b	A1F	Length of cable 351 m
	Evaluates their integral and interprets result including units	3.2a	A1F	
	Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)	Explains that roots will be in conjugate pairs since coefficients are real	2.4	E1	As coefficients are real, roots will be in conjugate pairs. $\alpha = 1+i$
	Obtains 2nd complex root	1.1b	B1	$\beta = 1-i$
	Uses product of roots.	3.1a	M1	$\alpha\beta\gamma = 1 \Rightarrow \gamma = \frac{1}{(1+i)(1-i)} = \frac{1}{2}$
	Finds third root	1.1b	A1	
	Uses sums of roots or expands factors to find p and q	3.1a	M1	$\alpha+\beta+\gamma = -\frac{p}{2} \Rightarrow p = -5$
	Obtains p	1.1b	A1	$\alpha\beta+\alpha\gamma+\beta\gamma = \frac{q}{2} \Rightarrow q = 6$
	Obtains q	1.1b	A1	
	Total		**7**	
9(b)	Writes α and their complex root in the form $r(\cos\theta + i\sin\theta)$	3.1a	M1	$\alpha = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$
	Obtains correct α and β in modulus argument form – need not be exact	1.1b	A1	$\beta = \sqrt{2}\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)$
	Uses de Moivre's theorem on their α^n and β^n - need not be exact	3.1a	M1	$\alpha^n+\beta^n+\gamma^n$ $= 2^{\frac{n}{2}}\left(\cos\frac{n\pi}{4} + i\sin\frac{n\pi}{4}\right) + 2^{\frac{n}{2}}\left(\cos\frac{n\pi}{4} - i\sin\frac{n\pi}{4}\right) + \left(\frac{1}{2}\right)^n$
	Forms sum of $\alpha^n+\beta^n+\gamma^n$	1.1a	M1	$= 2^{\frac{n}{2}}\left(2\cos\frac{n\pi}{4}\right) + 2^{-n}$
	Deduces a correct expression with complex terms cancelled	2.2a	A1	$= 2^{\frac{n}{2}+1}\cos\frac{n\pi}{4} + 2^{-n}$
	Completes rigorous argument to show the required result. Must use exact values. AG	2.1	R1	
	Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
10(a)(i)	Forms equation of motion for carriage	3.3	M1	$m\ddot{x} = -mn^2 x$ $\ddot{x} = -n^2 x$
	Simplifies and explains that the equation shows SHM	2.4	E1	This is the equation for SHM
10(a)(ii)	Recalls the formula for the period	1.2	B1	
	Total		**3**	
10(b)(i)	Models the motion of the particle by forming a three-term second order differential equation. May include v and any linear expression for displacement.	3.3	M1	$m\ddot{x} = -mn^2 x - mkv$ $\ddot{x} + n^2 x + kv = 0$ $\ddot{x} + k\dot{x} + n^2 x = 0$
	Models resistance force correctly.	3.3	A1	
	Manipulates to obtain required differential equation.	2.1	R1	

10(b)(ii)	Selects the correct method to solve the differential equation by forming the auxiliary equation	3.1a	M1	$\lambda^2+\frac{5n}{2}\lambda+n^2=0$ $\Rightarrow 2\lambda^2+5n\lambda+2n^2$ $\Rightarrow (2\lambda+n)(\lambda+2n)=0$ $\lambda=-\frac{n}{2},\lambda=-2n$ when $t=0$, $x=0\Rightarrow A+B=0$ $\dot{x}=U\Rightarrow -2nA-\frac{n}{2}B=U$ $A=-\frac{2U}{3n}$ $B=\frac{2U}{3n}$ $x=-\frac{2U}{3n}e^{-2nt}+\frac{2U}{3n}e^{-\frac{n}{2}t}=\frac{2U}{3n}\left(e^{-\frac{n}{2}t}-e^{-2nt}\right)$
	Uses an appropriate method to solve auxiliary equation	1.1a	M1	
	Obtains correct values of λ	1.1b	A1	
	States correct form of general solution FT their values of λ	1.1b	B1F	
	Uses model by substituting in initial values	3.3	M1	
	Differentiates general solution to find expression for velocity and substitutes initial values	1.1a	M1	
	Obtains correct value for both constants	1.1b	A1	
	Completes argument to show required result AG	2.1a	R1	
10(b)(iii)	States that the motion is heavily damped.	1.2	B1	The motion of the carriage is heavily damped which means the carriage will compress the buffer and then slowly return to the original position where it struck the buffer.
	Explains that the carriage will compress the buffer to a maximum and then return towards the buffer's uncompressed position.	3.2a	E1	
10(b)(iv)	Uses the model with $t=1$ and $\dot{x}=0$	3.4	M1	$t=1,\dot{x}=0$ $\dot{x}=\frac{2U}{3n}\left(-\frac{n}{2}e^{-\frac{n}{2}t}+2ne^{-2nt}\right)=0$ $A=-B\Rightarrow 2e^{-2n}-\frac{1}{2}e^{-\frac{n}{2}}=0$ $\Rightarrow n=0.924$
	Obtains correct value of n	1.1b	A1	
	Total		**15**	

Q	Marking instructions	AO	Marks	Typical solution
11(a)	Uses the standard expansion of $\cos x$ with a clear attempt to substitute $3x$	1.1a	M1	$\cos 3x \approx 1 - \frac{(3x)^2}{2!} + \frac{(3x)^4}{4!}$ $\approx 1 - \frac{9x^2}{2} + \frac{27x^4}{8}$
	Obtains correct answer	1.1b	A1	
	Total		**2**	
11(b)	Selects a method to find the expansion by differentiating $\ln(3-2e^x)$ using an appropriate technique.	3.1a	M1	$f(x) = \ln(3-2e^x)$ $f'(x) = \frac{-2e^x}{3-2e^x}$ $f''(x) = \frac{-2e^x(3-2e^x)-(-2e^x)(-2e^x)}{(3-2e^x)^2}$ $= \frac{-6e^x}{(3-2e^x)^2}$ Maclaurin series needs values of derivatives at zero. $f(0) = 0$ $f'(0) = -2$ $f''(0) = -6$ $f'''(0) = -30$ $\ln(3-2e^x) \approx -2x - \frac{6x^2}{2!} - \frac{30x^3}{3!}$ $\approx -2x - 3x^2 - 5x^3$
	Obtains correct first derivative	1.1b	A1	
	Explains need to evaluate at 0 to form the Maclaurin series	2.4	E1	
	Obtains $f(0) = 0$ (PI) and $f'(0) = -2$	1.1b	B1	
	Differentiates a second time to find second derivative at zero.	1.1a	M1	
	Obtains $f'''(0) = -30$	1.1b	A1	
	Uses their $f'(0), f''(0)$ and $f'''(0)$ to form the expansion of $\ln(3-2e^x)$	1.1a	M1	
	Obtains the correct expansion	2.1	A1	
	Total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
11(c)	Uses their (a) and (b) to write $\dfrac{x\ln(3-2e^x)}{1-\cos 3x}$ as a fraction using power series or Attempts to use L'Hôpital's rule	3.1a	M1	$\dfrac{x\ln(3-2e^x)}{1-\cos 3x} \approx \dfrac{x\left(-2x-3x^2-5x^3\right)}{1-\left(1-\dfrac{9x^2}{2}+\dfrac{27x^4}{8}\right)}$ $\approx \dfrac{-2x^2-3x^3-5x^4}{\dfrac{9x^2}{2}-\dfrac{27x^4}{8}}$ $\approx \dfrac{-2-3x^1-5x^2}{\dfrac{9}{2}-\dfrac{27x^2}{8}}$
	Simplifies their fraction or begins second application of L'Hôpital's rule	1.1a	M1	
	Completes rigorous argument to show the given result.	2.1a	R1	$\lim_{x\to 0}\left[\dfrac{x\ln(3-2e^x)}{1-\cos 3x}\right] = \lim_{x\to 0}\left[\dfrac{-2-3x-5x^2}{\dfrac{9}{2}-\dfrac{27x^2}{8}}\right]$ $= \dfrac{-2}{\dfrac{9}{2}} = -\dfrac{4}{9}$
	Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
12(a)	Selects a method to find k by calculating the determinant of the matrix	3.1a	M1	det = 18 $k = \sqrt{18} = 3\sqrt{2}$
	Finds the correct value of $k = 3\sqrt{2}$	1.1b	A1	
	Total		**2**	
12(b)	Selects a method to identify the correct transformation e.g. sketching the image of the unit square.	3.1a	M1	Rotation 45 degrees clockwise about the origin
	Fully describes the correct transformation	2.5	A1	
	Total		**2**	