AQA

A-LEVEL

Further Mathematics

Paper 1

Mark scheme

Practice paper – Set 1

Version 1.0

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme has been prepared for practice papers and has not, therefore, been through the process of standardising that would take place for live papers.

Further copies of this mark scheme are available from allaboutmaths.aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the paper
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles:

No method shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

Q	Marking instructions	AO	Marks	Typical solution
1	Circles correct answer	AO1.2	B1	$\int_0^1 \frac{1}{x+1}\,\mathrm{d}x$
	Total		**1**	
2	Circles correct answer	AO2.2a	B1	$(z - z^*)^{2472}$
	Total		**1**	
3	Circles correct answer	AO1.1b	B1	1728
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
4 (a)	Writes as a fraction so that l'Hôpital's rule can be applied	AO1.1a	M1	$\lim\limits_{x\to\infty} xe^{-2x} = \lim\limits_{x\to\infty} \frac{x}{e^{2x}}$ Since both $x \to \infty$ and $e^{2x} \to \infty$ $\lim\limits_{x\to\infty} \frac{x}{e^{2x}} = \lim\limits_{x\to\infty} \frac{1}{2e^{2x}}$ $= 0$
	Explains why l'Hôpital's rule can be applied	AO2.4	E1	
	Deduces correct limit	AO2.2a	R1	
	Total		**3**	
4 (b)	Uses integration by parts, condone 1 error	AO3.1a	M1	$\int xe^{-2x}dx = -\frac{x}{2}e^{-2x} + \frac{1}{2}\int e^{-2x}dx$ $= -\frac{x}{2}e^{-2x} - \frac{1}{4}e^{-2x} \qquad (+c)$
	Obtains correct integral	AO1.1b	A1	
	Substitutes finite limits into 'their' integral	AO1.1b	A1F	$\int_0^{\infty} xe^{-2x}dx = \lim\limits_{a\to\infty} \int_0^{a} xe^{-2x}dx$
	Obtains the correct integral clearly using a limiting process CAO	AO2.1	R1	$\int_0^{a} xe^{-2x}dx = \left[-\frac{x}{2}e^{-2x} - \frac{1}{4}e^{-2x}\right]_0^a$ $= \left(\left(-\frac{a}{2}e^{-2a} - \frac{1}{4}e^{-2a}\right) - \left(-\frac{0}{2}e^{0} - \frac{1}{4}e^{0}\right)\right)$ $= \frac{1}{4} - \frac{a}{2}e^{-2a} - \frac{1}{4}e^{-2a}$
	Completes rigorous argument to show that the value is ¼	AO2.5	R1	$\therefore$ $\int_0^{\infty} xe^{-2x}dx = \lim\limits_{a\to\infty}\left(\frac{1}{4} - \frac{a}{2}e^{-2a} - \frac{1}{4}e^{-2a}\right)$ $= \frac{1}{4}$
	Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
5	Uses proof by induction and investigates formula for $n = 1$ and $n = k$ (must see evidence of both $n = 1$ and $n = k$ being considered)	AO3.1a	M1	Let $P(n)$ be the statement $\begin{pmatrix} 3 & 4i \\ i & -1 \end{pmatrix}^n = \begin{pmatrix} 2n+1 & 4ni \\ ni & 1-2n \end{pmatrix}$ When $n=1$
	Demonstrates that formula is true for $n = 1$	AO1.1b	B1	$\begin{pmatrix} 2n+1 & 4ni \\ ni & 1-2n \end{pmatrix} = \begin{pmatrix} 3 & 4i \\ i & -1 \end{pmatrix}$ $= \begin{pmatrix} 3 & 4i \\ i & -1 \end{pmatrix}^1$ $\therefore P(1)$ is true
	States assumption that formula true for $n = k$ and uses $\begin{pmatrix} 3 & 4i \\ i & -1 \end{pmatrix}^{k+1} = \begin{pmatrix} 3 & 4i \\ i & -1 \end{pmatrix}^k \begin{pmatrix} 3 & 4i \\ i & -1 \end{pmatrix}$	AO2.1	B1	Assume $P(k)$ is true $\begin{pmatrix} 3 & 4i \\ i & -1 \end{pmatrix}^{k+1} = \begin{pmatrix} 3 & 4i \\ i & -1 \end{pmatrix}^k \begin{pmatrix} 3 & 4i \\ i & -1 \end{pmatrix}$ $= \begin{pmatrix} 2k+1 & 4ki \\ ki & 1-2k \end{pmatrix} \begin{pmatrix} 3 & 4i \\ i & -1 \end{pmatrix}$
	Correct use of $i^2 = -1$	AO1.2	B1	Since $P(k)$ is true $= \begin{pmatrix} 6k+3+4ki^2 & 8ki+4i-4ki \\ 3ki+i-2ki & 4ki^2-1+2k \end{pmatrix}$
	Deduces formula also true for $n = k + 1$	AO2.2a	R1	$= \begin{pmatrix} 6k+3-4k & 4ki+4i \\ ki+i & -4k-1+2k \end{pmatrix}$ $= \begin{pmatrix} 2k+3 & 4(k+1)i \\ (k+1)i & -1-2k \end{pmatrix}$ $= \begin{pmatrix} 2(k+1)+1 & 4(k+1)i \\ (k+1)i & 1-2(k+1) \end{pmatrix}$ $\therefore P(k+1)$ is true
	Completes a rigorous argument with explanation of how their argument proves the required result AG	AO2.1	R1	Since $P(1)$ is true and $P(k) \Rightarrow P(k+1)$, hence, by induction, $P(n)$ is true for all $n \in \mathbb{N}$
	Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
6 (a)	Forms $\lvert \mathbf{M} - \lambda \mathbf{I} \rvert = 0$	AO1.2	B1	$\begin{vmatrix} 4+a-\lambda & -2a-2 & -2 \\ 2+a & -2a-1-\lambda & -1 \\ 2+a & -2a-2 & -\lambda \end{vmatrix} = 0$
	Expands determinant or Performs row/column operations	AO1.1a	M1	$\begin{vmatrix} 2-\lambda & 0 & \lambda-2 \\ 0 & 1-\lambda & \lambda-1 \\ 2+a & -2a-2 & -\lambda \end{vmatrix} = 0$ $\begin{vmatrix} 2-\lambda & 0 & 0 \\ 0 & 1-\lambda & \lambda-1 \\ 2+a & -2a-2 & 2+a-\lambda \end{vmatrix} = 0$
	Obtains correct factorised determinant or writes determinant in a form which allows deduction of eigenvalue	AO1.1b	A1	$(2-\lambda)\big((1-\lambda)(2+a-\lambda)-(-2a-2)(\lambda-1)\big)=0$
	Obtains correct eigenvalue	AO1.1b	A1	$(2-\lambda)(1-\lambda)\big((2+a-\lambda)+(-2a-2)\big)=0$ $(2-\lambda)(1-\lambda)(-\lambda-a)=0$ $\lambda = -a, 1, 2$
	Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
6 (b)	Forms appropriate equation for their eigenvalue	AO1.1a	M1	$\lambda = -a$ $\begin{pmatrix} 4+2a & -2a-2 & -2 \\ 2+a & -a-1 & -1 \\ 2+a & -2a-2 & a \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$
	Eliminates one variable	AO1.1a	M1	$R2 - R3 \Rightarrow (a+1)y - (a+1)z = 0$
	Deduces eigenvector CAO	AO2.2a	A1	$\Rightarrow y = z$ $\Rightarrow (2+a)x - (a+1)y - y = 0$ $\Rightarrow x = y$ $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$
	Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
6 (c)	Expresses M in the form $\mathbf{UDU}^{-1}$	AO1.1b	M1	$\mathbf{M}=\mathbf{UDU}^{-1}$ $=\begin{pmatrix}1&2&2\\1&1&1\\1&1&2\end{pmatrix}\begin{pmatrix}-a&0&0\\0&2&0\\0&0&1\end{pmatrix}\begin{pmatrix}-1&2&0\\1&0&-1\\0&-1&1\end{pmatrix}$ $\mathbf{M}^{2n}=\mathbf{UD}^{2n}\mathbf{U}^{-1}$ $=\begin{pmatrix}1&2&2\\1&1&1\\1&1&2\end{pmatrix}\begin{pmatrix}(-a)^{2n}&0&0\\0&2^{2n}&0\\0&0&1\end{pmatrix}\begin{pmatrix}-1&2&0\\1&0&-1\\0&-1&1\end{pmatrix}$ $=\begin{pmatrix}(-a)^{2n}&2^{2n+1}&\dots\\\dots&\dots&\dots\\\dots&\dots&\dots\end{pmatrix}\begin{pmatrix}-1&2&0\\1&0&-1\\0&-1&1\end{pmatrix}$ $=\begin{pmatrix}a^{2n}&2^{2n+1}&\dots\\\dots&\dots&\dots\\\dots&\dots&\dots\end{pmatrix}\begin{pmatrix}-1&2&0\\1&0&-1\\0&-1&1\end{pmatrix}$ $=\begin{pmatrix}2^{2n+1}-a^{2n}&\dots&\dots\\\dots&\dots&\dots\\\dots&\dots&\dots\end{pmatrix}$
	Obtains a correct **D** for 'their' third eigenvalue	AO1.1b	B1F	
	Obtains a corresponding correct **U** FT their **D**	AO1.1b	B1F	
	Calculates $\mathbf{U}^{-1}$	AO1.1b	B1F	
	Uses $\mathbf{M}^{2n}=\mathbf{UD}^{2n}\mathbf{U}^{-1}$	AO3.1a	M1	
	Calculates required elements of left or right product of their $\mathbf{M}^{2n}=\mathbf{UD}^{2n}\mathbf{U}^{-1}$	AO1.1a	M1	
	Obtains correct required elements of left or right product.	AO1.1b	A1	
	Obtains correct top left element ACF	AO1.1b	A1	
	Total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
6 (d)	Deduces that the line is parallel to $\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$ and passes through the origin	AO2.2a	R1	The line passes through the origin in the direction of $\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$ The line is in the direction of the eigen vector with eigenvalue 1 so points are invariant under **M** *T* is the result of successive transformations using the matrix **M** so the points on the line will remain invariant.
	Explains that the line is in the direction of the eigenvector with eigenvalue 1 so points are invariant under **M**	AO2.4	E1	
	Explains that it does not matter how many times **M** is applied the points on the line will remain invariant hence the line is a line of invariant points under *T*	AO2.4	E1	
	Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
7 (a)	Selects an appropriate split of the integrand	AO3.1a	M1	$I_n = \int_0^{\frac{\pi}{4}} \tan^n x \, dx$ $= \int_0^{\frac{\pi}{4}} \tan^{n-2} x \tan^2 x \, dx$
	Uses $1 + \tan^2 x = \sec^2 x$	AO1.2	B1	$= \int_0^{\frac{\pi}{4}} \left(\tan^{n-2} x\right)\left(\sec^2 x - 1\right) dx$
	Separates into two integrals	AO1.1a	M1	$= \int_0^{\frac{\pi}{4}} \sec^2 x \tan^{n-2} x - \tan^{n-2} x \, dx$ $= \int_0^{\frac{\pi}{4}} \sec^2 x \tan^{n-2} x \, dx - \int_0^{\frac{\pi}{4}} \tan^{n-2} x \, dx$
	Uses integration by substitution or integrates directly from the form $\int f'(x)\left(f(x)\right)^n dx$	AO3.1a	M1	$= \left[\frac{\tan^{n-1} x}{n-1}\right]_0^{\frac{\pi}{4}} - I_{n-2}$
	Substitutes limits into definite integral	AO1.1a	M1	$= \frac{1}{n-1}\left(\tan^{n-1}\left(\frac{\pi}{4}\right) - \tan^{n-1}(0)\right) - I_{n-2}$
	Evaluates integral and completes rigorous argument to show result AG	AO2.1	R1	$= \frac{1}{n-1}\left(1^{n-1} - 0^{n-1}\right) - I_{n-2}$ $= \frac{1}{n-1} - I_{n-2}$
	Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
7 (b)	Uses symmetry of $\tan^6 x$ about the y-axis so $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^6 x \, dx = 2I_6$	AO3.1a	M1	$\tan^6 x$ is symmetrical about the y-axis so $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^6 x \, dx = 2I_6$ $\int_{0}^{\frac{\pi}{4}} \tan^6 x \, dx = I_6$ $I_6 = \frac{1}{5} - I_4$ $= \frac{1}{5} - \left(\frac{1}{3} - I_2\right)$ $= \frac{1}{5} - \frac{1}{3} + (1 - I_0)$ $= \frac{13}{15} - \int_{0}^{\frac{\pi}{4}} \tan^0 x \, dx$ $= \frac{13}{15} - \frac{\pi}{4}$ Hence $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^6 x \, dx = 2\left(\frac{13}{15} - \frac{\pi}{4}\right)$ $= \frac{26}{15} - \frac{\pi}{2}$
	Uses iterative formula to write I_6 in terms of I_4	AO1.1a	M1	
	Continues iteration to a correct expression in terms of I_0 or I_2 and $\int (\sec^2 x - 1) dx$	AO1.1b	A1	
	Correctly evaluates I_0	AO1.1b	A1	
	Obtains correct exact answer	AO1.1b	A1	
	Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
8 (a)	Obtains correct expansion in any correct form	AO1.1a	B1	$1+x+\frac{(x)^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}$
	Total		**1**	
8 (b)	Splits the series into two sums	AO3.1a	M1	$\sum_{r=1}^{\infty}\frac{3-r}{r!}=3\sum_{r=1}^{\infty}\frac{1}{r!}-\sum_{r=1}^{\infty}\frac{r}{r!}$ $=3\sum_{r=1}^{\infty}\frac{1}{r!}-\sum_{r=1}^{\infty}\frac{1}{(r-1)!}$ $=3\left(\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+...\right)-\left(\frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+...\right)$ $=3(e^1-1)-(e^1)$ $=2e-3$
	Expands the first few terms of each series (at least three terms) in any correct form	AO1.1a	M1	
	Compares either series with expansion of e	AO1.1b	A1	
	Obtains fully correct simplified answer	AO2.1	R1	
	Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
9 (a)	Uses proportionality model to form an equation for tension and weight	AO3.4	M1	$T \propto e \Rightarrow T = ke$ $2 \times 10 = k \times 0.4$ $k = \frac{20}{0.4} = 50$ $\therefore T = 50e$
	Correctly shows required result	AO1.1b	A1	
	Total		**2**	
9 (b)(i)	Models the motion of the particle by forming a four-term second order differential equation. May include v and any linear expression for displacement	AO3.3	M1	$F = ma \Rightarrow 2\frac{d^2x}{dt^2} = 2 \times 10 - 20\frac{dx}{dt} - 50(x - 0.3)$ $\frac{d^2x}{dt^2} = 10 - 10\frac{dx}{dt} - 25(x - 0.3)$ $\frac{d^2x}{dt^2} + 10\frac{dx}{dt} = 10 - 25x + 7.5$ $\frac{d^2x}{dt^2} + 10\frac{dx}{dt} + 25x = 17.5$
	Models tension and displacement correctly	AO3.3	A1	
	Obtains correct DE	AO1.1b	A1	
	Manipulates to show required form	AO2.1	R1	
	Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
9 (b)(ii)	Begins to solve given differential equation to find x in terms of t. Forms and solves auxiliary equation	AO3.1a	M1	$\lambda^2 + 10\lambda + 25 = 0$ $\lambda = -5$ $x = Ae^{-5t} + Bte^{-5t}$ PI $x = C \Rightarrow C = \frac{17.5}{25} = 0.7$ $x = Ae^{-5t} + Bte^{-5t} + 0.7$
	States correct form of PI to form general solution	AO1.1b	B1	
	States correct form of general solution	AO1.1b	A1	$x(0) = 0.9$ $\Rightarrow A + 0.7 = 0.9$ $\Rightarrow A = 0.2$ $\dot{x}(0) = 0$ $\Rightarrow -5Ae^{-5t} + Be^{-5t} - 5Bte^{-5t} = 0$ $\Rightarrow -5A + B = 0$ $B = 1$ $x(1.5) = 0.70094$ Particle is less than 1 millimetre below equilibrium position.
	Models initial displacement and velocity to find constants	AO3.3	M1	
	Obtains correct value for one constant	AO1.1b	A1	
	Obtains correct value for both constants	AO1.1b	A1	
	Uses the model to find displacement when $t = 1.5$	AO3.4	A1	
	Interprets value of x	AO3.2a	A1	
	Total		**8**	
9 (b)(iii)	States that the motion is critically damped	AO1.2	B1	The motion of the sphere is critically damped which means that the particle will get ever closer to its equilibrium position. Velocity will become negligible and movement will become imperceptible
	Explains that the particle will get ever closer to its equilibrium position	AO3.2a	E1	
	Total		**2**	

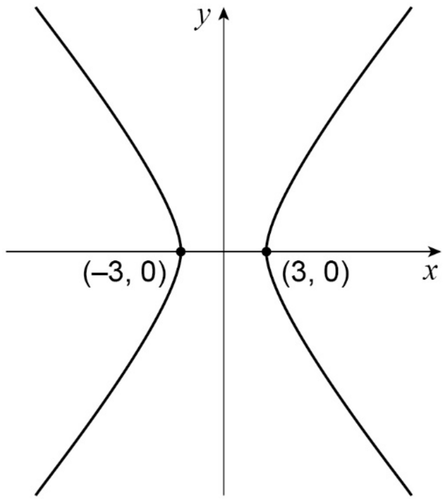
Q	Marking instructions	AO	Marks	Typical solution
9 (b)(iv)	States that the object has been modelled as a particle	AO3.5b	B1	The object has been modelled as a particle. The object having size would mean that the initial extension would be less, but damping would still be critical.
	Explains that the object having size would mean that the initial extension would be less but damping would still be critical.	AO3.2b	E1	
	Total		**2**	
9 (b)(v)	Explains that the resistive force would be different	AO3.5c	E1	The resistive force would be different, resulting in the $20v$ term changing in the model. The likely effect on the motion would be light-damping with the particle oscillating about its equilibrium position.
	Describes the likely effect on the motion	AO2.2b	E1	
	Total		**2**	

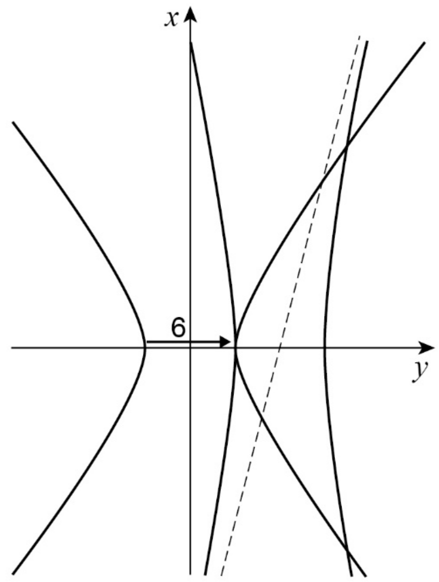
Q	Marking instructions	AO	Marks	Typical solution
10	Uses de Moivre's theorem and expands $(\cos\theta+i\sin\theta)^4$	AO1.1a	M1	$\cos(4\theta)+i\sin(4\theta)\equiv(\cos\theta+i\sin\theta)^4$ $\equiv\cos^4\theta+4\cos^3\theta i\sin\theta+6\cos^2\theta i^2\sin^2\theta+4\cos\theta i^3\sin^3\theta+\sin^4\theta$ $\equiv\cos^4\theta+4i\cos^3\theta\sin\theta-6\cos^2\theta\sin^2\theta-4i\cos\theta\sin^3\theta+\sin^4\theta$
	Obtains correct binomial expansion	AO1.1b	A1	Comparing real and imaginary parts $\cos(4\theta)\equiv\cos^4\theta-6\cos^2\theta\sin^2\theta+\sin^4\theta$ $\sin(4\theta)\equiv4\cos^3\theta\sin\theta-4\cos\theta\sin^3\theta$
	Continues argument equating real or imaginary parts to find expressions for $\cos(4\theta)$ or $\sin(4\theta)$ then	AO2.1	R1	$\tan(4\theta)\equiv\dfrac{\sin(4\theta)}{\cos(4\theta)}$ $\equiv\dfrac{4\cos^3\theta\sin\theta-4\cos\theta\sin^3\theta}{\cos^4\theta-6\cos^2\theta\sin^2\theta+\sin^4\theta}$
	Expresses $\tan(4\theta)$ in terms of $\cos\theta$ and $\sin\theta$	AO1.1a	M1	Dividing numerator and denominator by $\cos^4\theta$ gives: $\equiv\dfrac{\left(4\dfrac{\cos^3\theta\sin\theta}{\cos^4\theta}-4\dfrac{\cos\theta\sin^3\theta}{\cos^4\theta}\right)}{\left(\dfrac{\cos^4\theta}{\cos^4\theta}-6\dfrac{\cos^2\theta\sin^2\theta}{\cos^4\theta}+\dfrac{\sin^4\theta}{\cos^4\theta}\right)}$
	Completes rigorous argument to show given result	AO2.1	R1	$\equiv\dfrac{\left(4\dfrac{\sin\theta}{\cos\theta}-4\dfrac{\sin^3\theta}{\cos^3\theta}\right)}{\left(1-6\dfrac{\sin^2\theta}{\cos^2\theta}+\dfrac{\sin^4\theta}{\cos^4\theta}\right)}$ $\equiv\dfrac{4(\tan x-\tan^3x)}{1-6\tan^2x+\tan^4x}$
	Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
11	Explains meaning of commutaivity for either operation PI			$\begin{pmatrix} a & b \\ c & d \end{pmatrix}+\begin{pmatrix} e & f \\ g & h \end{pmatrix}=\begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix}$ $=\begin{pmatrix} e+a & f+b \\ g+c & h+d \end{pmatrix}$ $=\begin{pmatrix} e & f \\ g & h \end{pmatrix}+\begin{pmatrix} a & b \\ c & d \end{pmatrix}$
	Begins argument by forming the addition of two generalised 2×2 matrices, **A** and **B**, say	AO1.1a	M1	
	Completes argument to show matrix addition is commutative	AO2.1	R1	Therefore addition of 2×2 matrices is commutative.
	Begins argument that matrix multiplication is not commutative by finding a counterexample	AO1.1a	B1	$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}=\begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$ $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}=\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$ Therefore multiplication of matrices is not commutative as this example does not work.
	Explains why this shows that matrix multiplication is not commutative	AO2.4	E1	
	Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
12 (a)(i)	Obtains direction vector parallel to plane	AO1.1a	M1	$\begin{pmatrix}9\\-8\\5\end{pmatrix}-\begin{pmatrix}5\\4\\-5\end{pmatrix}=\begin{pmatrix}4\\-12\\10\end{pmatrix}$
	Uses vector product to find perpendicular to the plane	AO1.1a	M1	$\begin{pmatrix}-4\\3\\-1\end{pmatrix}\times\begin{pmatrix}2\\-6\\5\end{pmatrix}=\begin{pmatrix}9\\18\\18\end{pmatrix}$
	States a cartesian equation of the plane	AO1.1a	A1	$\begin{pmatrix}5\\4\\-5\end{pmatrix}\bullet\begin{pmatrix}1\\2\\2\end{pmatrix}=3$ $x+2y+2z=3$
	Total		**3**	
12 (a)(ii)	Forms an equation in terms of a	AO1.1a	M1	$\mathbf{r}=\begin{pmatrix}5\\4\\-5\end{pmatrix}+\mu\begin{pmatrix}2\\3\\a\end{pmatrix}$
	Finds value of a	AO1.1b a	A1	$\mu=1\Rightarrow\mathbf{r}=\begin{pmatrix}7\\7\\a-5\end{pmatrix}$ $7+14+2(a-5)=3$ $a=-4$
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
12 (b)	Forms equation of line through A perpendicular to their plane	AO3.1a	M1	$\mathbf{r}=\begin{pmatrix}8\\20\\0\end{pmatrix}+t\begin{pmatrix}1\\2\\2\end{pmatrix}=\begin{pmatrix}8+t\\20+2t\\2t\end{pmatrix}$ $8+t+2(20+2t)+2\times 2t=3$ $48+9t=3$ $t=-5$ A and A' will be equidistant from the plane so t will need to be doubled $\mathbf{r}=\begin{pmatrix}8\\20\\0\end{pmatrix}-10\begin{pmatrix}1\\2\\2\end{pmatrix}=\begin{pmatrix}-2\\0\\-20\end{pmatrix}$ Hence A' has coordinates $(-2, 0, -20)$
	Doubles the parameter	AO1.1b	A1	
	Completes rigorous argument to show that A' has given coordinates	AO2.1	E1	
	Total		**3**	
12 (b) (Alt)	Obtains $\overrightarrow{A'A}$ and midpoint M	AO3.1a	M1	$\overrightarrow{A'A}=\begin{pmatrix}10\\20\\20\end{pmatrix}$ $M=\begin{pmatrix}3\\10\\-10\end{pmatrix}$ $\begin{pmatrix}10\\20\\20\end{pmatrix}=10\begin{pmatrix}1\\2\\2\end{pmatrix}$ so $A'A$ is perpendicular to the plane $3+2\times 10+2\times(-10)=3$ Therefore, M lies on the plane Hence, since the points lie on a line which is perpendicular to the plane and they are equidistant from the plane, A' must have coordinates $(-2, 0, -20)$
	Begins to construct argument showing that $\overrightarrow{A'A}$ is perpendicular to plane or showing that M lies on the plane	AO1.1b	A1	
	Completes argument explaining their reasoning.	AO2.1	E1	
	Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
13 (a)	States "Hyperbola"	AO1.2	B1	Hyperbola
	Total		**1**	
13 (b)	Sketches the graph with the correct shape Intersections with the x-axis correctly labelled	AO1.1b AO1.1b	B1 B1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
13 (c)(i)	States asymptotes of PI	AO2.2a	M1	$y = \pm\frac{4}{3}x \to y = \pm(4x-4)$
	Compares asymptotes of C_1 with C_2 and deduces one transformation PI	AO2.2a	A1	$y \to \frac{y}{3},\ x \to x-1$ C_1 is mapped onto C_2 by a stretch in the y-direction SF 3 and a translation $\begin{pmatrix}1\\0\end{pmatrix}$
	Deduces both transformations PI	AO1.1a	M1	$\frac{(x-1)^2}{9} - \frac{\left(\frac{y}{3}\right)^2}{16} = 1$
	Writes down equation of C_2 ACF	AO1.1a	M1	$\frac{(x-1)^2}{9} - \frac{y^2}{144} = 1$
	Total		**4**	
13 (c)(ii)	Explains that the left section of C_2 should intersect the right section of C_1 on the x-axis (PI by a sketch)	AO2.4	E1	
	Deduces that a translation of $\begin{pmatrix}6\\0\end{pmatrix}$ is required, or the asymptotes should intersect at (6, 0)	AO2.2a	R1	
	Uses $4x - a = 0$ PI by correct a	AO1.1a	M1	
	Obtains value of a	AO1.1b	A1	For the curves to meet in 3 points the translation required is $\begin{pmatrix}6\\0\end{pmatrix}$ So the asymptotes will cross the x-axis at (6, 0) $4x - a = 0$ $x = 6 \Rightarrow a = 24$
	Total		**4**	
	TOTAL		**100**	