AQA

A-LEVEL

Further Mathematics

Paper 3 - Mechanics

Mark scheme

Practice paper - Set 2

Version 1.0

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme has been prepared for practice papers and has not, therefore, been through the process of standardising that would take place for live papers.

Further copies of this mark scheme are available from allaboutmaths.aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the paper
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
‘their’	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles

No method shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

Q	Marking instructions	AO	Marks	Typical solution
1	Circles correct answer	AO1.1b	B1	$\left(\frac{7a}{6}, \frac{5a}{6}\right)$
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2	Circles correct answer	AO1.1b	B1	100 Ns
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3(a)	States correct dimensions of density.	AO1.2	B1	$[\rho] = ML^{-3}$
	Total		**1**	
3(b)	Sets up equation based on dimensions, with correct dimensions for at least two of the four quantities.	AO1.1a	M1	$[L] = [\rho]^a[A]^b[v]^c$ $MLT^{-2} = (ML^{-3})^a(L^2)^b(LT^{-1})^c$ $MLT^{-2} = M^aL^{-3a}L^{2b}L^cT^{-c}$ Equating powers: For M, a=1 For T, $-c$ = -2 For L, $-3a+2b+c$ = 1 Hence $a = 1$ $c = 2$ $b = 1$
	Obtains a correct dimensions equation	AO1.1b	A1	
	Equates powers of dimensions to form equations for each of M, L and T (PI)	AO1.1a	M1	
	States the correct values for a, b and c.	AO1.1b	A1	
	Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
4(a)	Forms an equation using $P = Fv$	AO3.3	M1	$P = Fv = kv^2$ $72000 = k \times 40^2$ $k = \dfrac{72000}{40^2} = 45$
	Completes a full argument to obtain the required value.	AO2.1	R1	
	Total		**2**	
4(b)	Resolves parallel to the slope to obtain expression for the driving force.	AO3.3	M1	$F = 1200 \times 9.8 \sin 3° + 45v$ $72000 = (1200 \times 9.8 \sin 3° + 45v)v$ $45v^2 + 11760 \sin 3°\, v - 72000 = 0$ $v = 33.7 \text{ or } -47.4$ 34 m s^{-1}
	Uses the power formula $P = Fv$	AO1.1a	M1	
	Obtains a correct quadratic equation in v. Need not be simplified.	AO1.1b	A1	
	Solves their quadratic equation.	AO1.1a	M1	
	Obtains the correct speed stating it correct to 2 sig figs.	AO1.1b	A1	
	Total		**5**	
4(c)	Explains that the answer to (a) would not change, because it is independent of the mass.	AO2.4	E1	Mass is not relevant to (a) so the value of k would not change. However, in (b) the driving force would increase because of the increase in the component of weight parallel to the plane, so the speed would decrease.
	Explains that the answer to (b) will decrease because the driving force increases.	AO3.5a	E1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)	Forms a correct expression for initial GPE	AO3.3	B1	$GPE = m \times 9.81 \times 6\sin 40° = 37.8m$ $KE = \frac{1}{2} \times m \times 8.70^2 = 37.8m$ Tom is correct if energy is conserved. Assumptions: No air resistance. Slide is smooth.
	Forms a correct expression for final KE	AO1.1b	B1	
	Shows GPE = KE and explains that Tom is correct if energy is conserved.	AO2.1	R1	
	States two correct assumptions.	AO3.5a	A1	
	Total		**4**	
5(b)	Uses correct friction force in their solution.	AO3.3	B1	$F = \mu \times m \times 9.81\cos 40°$ $m \times 9.81 \times 6\sin 40° - \frac{1}{2} \times m \times 5^2$ $= 6 \times \mu \times m \times 9.81\cos 40°$ $\mu = \frac{9.81\times 6\sin 40° - 12.5}{6\times 9.81\cos 40°} = 0.562$
	Considers the energy lost by the swimmer on the slide.	AO1.1a	M1	
	Forms an equation to find the coefficient of friction using energy considerations.	AO3.4	M1	
	Obtains the correct coefficient of friction (3 sig figs not required)	AO1.1b	A1	
	Total		**4**	
5(c)	Attributes some energy loss to air resistance.	AO3.5a	M1	In reality, there would be air resistance. This would account for some of the lost energy and so the friction force would be less and hence the coefficient of friction would be less.
	Explains correctly that the coefficient of friction should be less.	AO2.4	A1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
6(a)	States correct mass of hemisphere. Condone missing ρ.	AO1.1b	B1	Mass of hemisphere $= \frac{2\pi\rho a^3}{3}$ Equation of circle to rotate to create hemisphere: $x^2 + y^2 = a^2$ $\frac{2\pi\rho a^3}{3} \times \bar{x} = \int_0^a \pi\rho x(a^2 - x^2)dx$ $\frac{2\pi\rho a^3}{3} \times \bar{x} = \pi\rho\left[\frac{1}{2}a^2x^2 - \frac{1}{4}x^4\right]_0^a$ $\frac{2\pi\rho a^3}{3} \times \bar{x} = \pi\rho \times \frac{1}{4}a^4$ $\bar{x} = \frac{3a}{8}$
	States correct equation of circle to rotate around the x-axis.	AO1.1b	B1	
	Forms equation to obtain the position of the centre of mass using their mass and equation for the circle.	AO1.1a	M1	
	Obtains correct integral	AO1.1b	A1F	
	Uses correct limits to obtain correct value of the integral.	AO1.1b	A1F	
	Obtains correct distance from fully correct working which clearly refers to mass. CAO	AO2.1	R1	
	Total		**6**	
6(b)	Obtains correct angle for sliding or its tangent.	AO1.1b	B1	On point of sliding: $mg\sin\theta = 0.95 \times mg\cos\theta$ $\theta = 43.5°$ On point of toppling: $\tan\theta = \frac{a}{\frac{3a}{8}} = \frac{8}{3}$ $\theta = 69.4°$ As 43.5 < 69.4 the hemisphere slides.
	Forms an equation to obtain the angle for toppling.	AO3.3	M1	
	Obtains correct angle for toppling or its tangent.	AO1.1b	A1	
	Deduces that hemisphere slides by comparing (tangents of) angles.	AO2.2a	R1	
	Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)	States total initial momentum is zero.	AO3.3	B1	Initial momentum $mu - km \times \frac{u}{k} = 0$
	Deduces that particles must be moving in opposite directions so that total zero momentum is conserved and therefore their directions are reversed.	AO2.2a	E1	$\therefore$ P and Q must move in opposite directions after the collision, which means they both reverse direction.
	Total		**2**	
7(b)	Forms an equation using conservation of momentum	AO1.1b	B1	Let speeds of P and Q after collision be v_P and v_Q.
	Forms an equation using Newton's Experimental Law	AO1.1b	B1	Conservation of momentum $kmv_Q - mv_P = 0 \Rightarrow v_P = kv_Q$
	Eliminates a variable to find one speed in terms of e, u (and k)	AO1.1a	M1	Newton's Experimental Law
	Forms an equation in e, u and k only	AO1.1a	M1	$v_Q + v_P = e\left(u + \frac{u}{k}\right)$ $v_Q + kv_Q = eu\left(\frac{k+1}{k}\right)$
	Deduces the correct range of values for k.	AO2.2a	R1	$v_Q = \frac{eu}{k}$ and $v_P = eu$ $v_P - v_Q = \frac{u}{2}$ $eu - \frac{eu}{k} = \frac{u}{2}$ Given $e = \frac{k}{2(k-1)} \le 1$ $\therefore k \le 2k - 2$ $\therefore k \ge 2$
	Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
8	States tension at maximum speed.	AO3.3	B1	At maximum speed $T = mg$ At lowest point $T = 5mg$ $5mg = \frac{\lambda x}{l}$ $x = \frac{5mgl}{\lambda}$ By energy $mg(x + l) = \frac{\lambda x^2}{2l}$ $mg\left(\frac{5mgl}{\lambda} + l\right) = \frac{\lambda}{2l}\left(\frac{5mgl}{\lambda}\right)^2$ $5mg + \lambda = 12.5mg$ $\lambda = 7.5mg$
	Forms equation using Hooke's Law for tension at lowest point.	AO1.1a	M1	
	Obtains correct extension at the lowest point.	AO1.1b	A1	
	Forms energy equation when particle is at the lowest point.	AO3.1b	M1	
	Substitutes their extension for the string at the lowest point into their energy equation.	AO1.1a	M1	
	Obtains a correct equation for the modulus of elasticity.	AO1.1b	A1	
	Obtains correct modulus of elasticity, from a rigorous argument.	AO2.1	R1	
	Total		**7**	