AQA

A-LEVEL
Further Mathematics

Paper 3 – Discrete
Mark scheme

Practice paper – Set 1

Version 1.0

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme has been prepared for practice papers and has not, therefore, been through the process of standardising that would take place for live papers.

Further copies of this mark scheme are available from allaboutmaths.aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make a judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
‘their’	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles:

No method shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

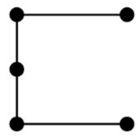
Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

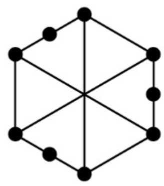
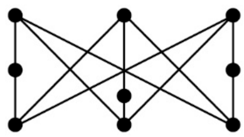
Q	Marking instructions	AO	Marks	Typical solution
1	Circles correct answer	AO1.1b	B1	15
	Total		**1**	
2	Circles correct answer	AO1.1b	B1	
	Total		**1**	
3 (a)	Determines the first four nodes from the nearest neighbour algorithm starting from *S*	AO3.1a	M1	*S*–*A*–*F*–*E*–*D*–*C*–*B*–*S* 76 miles
	Determines all correct nodes from the nearest neighbour algorithm starting from *S*	AO1.1b	A1	
	States the correct upper bound for the minimum distance for the journey, including units	AO3.2a	B1	
	Total		**3**	
3 (b)	Infers that the minimum distance travelled by the minibus is less than or equal to 'their' value to **(a)**	AO2.2b	B1	The minimum distance travelled by the minibus is not greater than 76 miles
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
4 (a)	Shows correctly that the result of combination of a pair of elements of *G* is not equal to the result when the same elements are combined in the reverse order	AO1.1b	B1	$q \bullet r = r^3q \neq r \bullet q$ As $q \bullet r \neq r \bullet q$, then the operation of *G* is not commutative. Abelian groups have an operation which is commutative for all combinations of elements, so *G* is not an abelian group.
	Explains correctly that as one pair of elements do not have a commutative property under the operation of the group, then the group is not an abelian group, as abelian groups have an operation which is commutative	AO2.4	E1	
	Total		**2**	
4 (b)	Uses the result $r^3 \bullet q = q \bullet r$ once to simplify the left-hand side	AO1.1a	M1	$r^2 \bullet q \bullet r^2 = r^2 \bullet q \bullet r \bullet r$ $= r^2 \bullet r^3 \bullet q \bullet r$ $= r^2 \bullet r^3 \bullet r^3 \bullet q$ $[= e \bullet e \bullet q]$ $= q$ Hence $r^2 \bullet q \bullet r^2 = q$
	Completes a rigorous argument to show the required result. AG	AO1.1b	A1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
4 (c)	Uses Lagrange's theorem to deduce correctly that *G* cannot have subgroups of order 3, 5, 6 or 7	AO2.2a	B1	The order of *G* is 8, but 8 is not divisible by 3, 5, 6 or 7 Hence, by Lagrange's theorem, *G* cannot have subgroups of order 3, 5, 6 or 7
	Uses Lagrange's theorem to infer that *G* may have subgroups of order 2 or 4	AO2.2b	B1	The order of *G* is 8, and 8 is divisible by both 2 and 4 Hence, by Lagrange's theorem, *G* may have subgroups of order 2 or 4
	Recalls correctly that any group is a subgroup of itself and that any group has a trivial subgroup	AO1.2	B1	*G* has a subgroup of order 8, itself. *G* has a subgroup of order 1, the subgroup containing only the identity of *G*
	Assesses the validity of Jenny's claim, stating that she has incorrectly included orders of 3, 5, 6 and 7 as well as not accounting for the possibility of trivial and non-proper subgroups	AO2.3	E1	Jenny's claim is only partially correct. She is correct that *G* may have subgroups of order 2 or 4, but she is incorrect to claim that *G* may have subgroups of order 3, 5, 6 or 7. She has also not accounted for the fact that there is an order 1 and an order 8 subgroup of *G*
	Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
5 (a)	States that none of Harry's strategies or Tom's strategies are dominated	AO1.1b	B1	There are no dominated strategies
	Modifies pay-off matrix so that it only includes positive values by increasing each value by an integer greater than or equal to 4 (PI)	AO1.1b	M1	$\begin{array}{\|c\|c\|c\|} \hline 1 & 6 & 7 \\ \hline 2 & 7 & 6 \\ \hline 6 & 3 & 3 \\ \hline \end{array}$ $p_1 + 2p_2 + 6p_3 \geq v$
	Translates information in their modified pay-off matrix into an inequality involving the three probability variables	AO3.1a	A1F	$6p_1 + 7p_2 + 3p_3 \geq v$ $7p_1 + 6p_2 + 3p_3 \geq v$ $p_1 + p_2 + p_3 \leq 1$
	Finds two more correct inequalities (for their modified pay-off matrix) involving the three probability variables	AO1.1b	A1F	$p_1, p_2, p_3 \geq 0$ Maximise $v - 4$ Subject to $p_1 + 2p_2 + 6p_3 \geq v$
	Writes down the correct (trivial) conditions for the probability variables	AO1.1b	B1	$6p_1 + 7p_2 + 3p_3 \geq v$ $7p_1 + 6p_2 + 3p_3 \geq v$ $p_1 + p_2 + p_3 \leq 1$ $p_1, p_2, p_3 \geq 0$
	Formulates the situation as a linear programming problem, including use of the words 'maximise' and 'subject to', as well as using an objective function of the form v – 'their' integer CAO	AO2.5	A1	
	Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
5 (b)	Substitutes the three probability values into the left-hand side of their non-trivial inequalities (any one)	AO1.1a	M1	$p_1 + 2p_2 + 6p_3$ $= 0.1875 + 2 \times 0.1875 + 6 \times 0.625$ [= 4.3125] = 4.3125 – 4
	Finds the correct value for the value of the game for Harry by subtracting their integer from the value found from their inequality	AO1.1b	A1	= 0.3125
	Reasons correctly that as the game is a zero sum game, the value of the game for Harry + the value of the game for Tom equals zero, resulting in a value of the game for Tom	AO2.4	A1F	The value of the game for Tom is –0.3125 because Harry's winnings + Tom's winnings = 0 since they are playing a zero-sum game
	Total		**3**	
5 (c)	Deduces that Harry should play strategy **C**	AO2.2a	E1	Harry should only play strategy **C** each time, because this will result in Harry winning 2 each game.
	Explains that strategy **C** will result in the highest winnings per game for Harry by comparing **C** with **A** and **B**	AO2.4	E1	Playing strategy **A** or **B** will result in Harry losing 3 or 2 each game.
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
6	Identifies a correct subgraph of *N*	AO1.1a	M1	 This is a subgraph of *N* The above subgraph can be redrawn as  This is a subdivision of $K_{3,3}$ N contains a subgraph which is a subdivision of $K_{3,3}$ Therefore, by Kuratowski's theorem, *N* is a non-planar graph.
	Shows correctly that the subgraph is a subdivision of a graph that has $K_{3,3}$ as a subgraph	AO1.1b	A1	
	Correctly deduces that *N* is non-planar through explicit use of Kuratowski's theorem	AO2.2a	E1	
	Total		**3**	
6 (Alt)	States that they are assuming that *N* is planar and uses Euler's formula for connected planar graphs	AO1.1a	M1	Let us assume that *N* is planar. Therefore, we can use Euler's formula $v - e + f = 2$ The graph *N* has $v = 15$ and $e = 22$, giving $f = 9$ The shortest cycle in *N* has 5 edges, so each face must have at least 5 edges, and each edge forms a boundary between two adjacent faces, so $2e \geq 5f$. However, $2e = 44$ and $5f = 45$, so there is a contradiction. Therefore, *N* is non-planar
	Correctly determines the number of faces from Euler's formula	AO1.1b	A1	
	Correctly deduces that *N* is non-planar through a contradiction to the original assumption	AO2.2a	E1	
	Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
7	Uses closure condition by multiplying two different members of *M* together	AO1.1a	M1	$\begin{pmatrix}\cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1\end{pmatrix}\begin{pmatrix}\cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2\end{pmatrix}$ $= \begin{pmatrix}\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 & -\cos\theta_1\sin\theta_2 - \sin\theta_1\cos\theta_2 \\ \cos\theta_1\sin\theta_2 + \sin\theta_1\cos\theta_2 & \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2\end{pmatrix}$
	Finds the correct elements of the matrix from the closure condition	AO1.1b	A1	$= \begin{pmatrix}\cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2)\end{pmatrix}$ $\begin{pmatrix}\cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2)\end{pmatrix} \in M$ Thus the set is closed under matrix multiplication.
	Uses the compound angle formulae for both sine and cosine	AO1.1a	M1	The identity element of *M* is $\begin{pmatrix}1 & 0 \\ 0 & 1\end{pmatrix} = \begin{pmatrix}\cos 0 & -\sin 0 \\ \sin 0 & \cos 0\end{pmatrix}$
	Explains correctly that the form of the resulting matrix is also in *M* and concludes that the set is closed under matrix multiplication	AO2.5	E1	The inverse of $\begin{pmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{pmatrix}$ is $\frac{1}{\cos^2\theta + \sin^2\theta}\begin{pmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{pmatrix}$ $= \begin{pmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{pmatrix}$
	Explains that **I** is the identity element of *M*	AO2.4	B1	$= \begin{pmatrix}\cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta)\end{pmatrix} \in M$
	Determines the inverse of an element by finding the inverse of a 2 × 2 matrix or by direct multiplication of an element and its inverse	AO1.1a	M1	Matrix multiplication is an associative operation, so matrix multiplication for the set *M* will also be associative. Hence, the set *M* under matrix multiplication satisfies the associativity condition for groups. As *M* satisfies all four group conditions under matrix multiplication, then *M* forms a group under matrix multiplication.

	Explains correctly that the inverse of any element of *M* is also a member of *M*, so the inverse condition for groups is satisfied	AO2.4	E1	
	Deduces that as matrix multiplication is an associative operation, then the set satisfies the associativity condition for groups	AO2.2a	B1	
	Completes a rigorous proofreferring to all four group conditions and making a concluding statement.	AO2.1	R1	
	Total		**9**	

Q	Marking instructions	AO	Marks	Typical solution
8 (a)	Introduces three variables and defines at least one of them as 'number of'	AO3.3	B1	x = number of badminton racquets y = number of squash racquets z = number of tennis racquets
	Finds an inequality for the model by considering the time required for machine *A*, machine *B* or machine *C*	AO3.1a	B1	$8x + 20y \leq 1200$ $16x + 12z \leq 1600$ $16y + 20z \leq 2400$
	Finds two more inequalities for the model by considering the time required for the machines	AO1.1b	B1	
	Uses 'their' model to set up a simplex tableau with three slack variables OE	AO3.4	M1	
	Identifies a correct pivot from 'their' tableau	AO3.1a	A1	
	Uses correctly the simplex algorithm to modify 'their' three non-pivot rows	AO1.1a	M1	
	Identifies the correct pivot in 'their' modified tableau	AO1.1b	A1F	
	Uses correctly the simplex algorithm to modify 'their' three non-pivot rows	AO1.1a	M1	
	States no further use of the simplex algorithm is required and explains why	AO2.4	E1	
	Makes a correct interpretation of 'their' final tableau in the context of the problem, provided the objective row is non-negative	AO3.2a	E1	

P	x	y	z	r	s	t	value
1	–12	–30	-60	0	0	0	0
0	8	20	0	1	0	0	1200
0	16	0	12	0	1	0	1600
0	0	16	**<u>20</u>**	0	0	1	24 00
1	–12	18	0	0	0	3	72 00
0	8	20	0	1	0	0	1200
0	**<u>16</u>**	–9.6	0	0	1	–0.6	160
0	0	0.8	1	0	0	0.005	120
1	0	10.8	0	0	0.75	2.55	7320
0	0	24.8	0	1	–0.5	0.3	1120
0	1	–0.6	0	0	0.0625	–0.0375	10
0	0	0.8	1	0	0	0.05	120

Objective row is all non-negative, so no need for another pass of the simplex algorithm

To maximise the profit, the business should make 10 badminton racquets, 0 squash racquets and 120 tennis racquets each week.

	Total		**10**	
8 (b)	Explains that only squash racquets can be made in this week	AO3.5c	B1	Machine B not being in use means that badminton and tennis racquets cannot be made, leaving only squash racquets being able to be made. $20y \leq 1200$ and $16y \leq 2400$ which results in $y = 60$ $30 \times 60 = £1800$
	Determines the number of squash racquets that can be made in the week	AO3.1b	M1	
	Determines correctly the business’s profit in this week	AO3.2a	A1	
	Total		**3**	
	TOTAL		**50**	