



ADVANCED GCE
MATHEMATICS
Probability & Statistics 4

4735

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:

- Scientific or graphical calculator

Thursday 24 June 2010
Morning

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

- 1 For the variables A and B , it is given that $\text{Var}(A) = 9$, $\text{Var}(B) = 6$ and $\text{Var}(2A - 3B) = 18$.
- (i) Find $\text{Cov}(A, B)$. [3]
- (ii) State with a reason whether A and B are independent. [1]

- 2 The probability generating function of the discrete random variable X is $\frac{e^{4t}}{e^4}$. Find
- (i) $E(X)$, [3]
- (ii) $P(X = 2)$. [3]

- 3 X_1 and X_2 are continuous random variables. Random samples of 5 observations of X_1 and 6 observations of X_2 are taken. No two observations are equal. The 11 observations are ranked, lowest first, and the sum of the ranks of the observations of X_1 is denoted by R .
- (i) Assuming that all rankings are equally likely, show that $P(R \leq 17) = \frac{2}{231}$. [5]

The marks of 5 randomly chosen students from School A and 6 randomly chosen students from School B , who took the same examination, achieving different marks, were ranked. The rankings are shown in the table.

Rank	1	2	3	4	5	6	7	8	9	10	11
School	A	A	A	B	A	A	B	B	B	B	B

- (ii) For a Wilcoxon rank-sum test, obtain the exact smallest significance level for which there is evidence of a difference in performance at the two schools. [2]
- 4 The moment generating function of a continuous random variable Y , which has a χ^2 distribution with n degrees of freedom, is $(1 - 2t)^{-\frac{1}{2}n}$, where $0 \leq t < \frac{1}{2}$.
- (i) Find $E(Y)$ and $\text{Var}(Y)$. [5]
- For the case $n = 1$, the sum of 60 independent observations of Y is denoted by S .
- (ii) Write down the moment generating function of S and hence identify the distribution of S . [2]
- (iii) Use a normal approximation to estimate $P(S \geq 70)$. [3]

- 5 In order to test whether the median salary of employees in a certain industry who had worked for three years was £19 500, the salaries x , in thousands of pounds, of 50 randomly chosen employees were obtained.
- (i) The values $|x - 19.5|$ were calculated and ranked. No two values of x were identical and none was equal to 19.5. The sum of the ranks corresponding to positive values of $(x - 19.5)$ was 867. Stating a required assumption, carry out a suitable test at the 5% significance level. [10]
- (ii) If the assumption you stated in part (i) does not hold, what test could have been used? [1]

- 6 Nuts and raisins occur in randomly chosen squares of a particular brand of chocolate. The number of nuts and raisins are denoted by N and R respectively and the joint probability distribution of N and R is given by

$$f(n, r) = \begin{cases} c(n + 2r) & n = 0, 1, 2 \text{ and } r = 0, 1, 2, \\ 0 & \text{otherwise,} \end{cases}$$

where c is a constant.

- (i) Find the value of c . [3]
- (ii) Find the probability that there is exactly one nut in a randomly chosen square. [2]
- (iii) Find the probability that the total number of nuts and raisins in a randomly chosen square is more than 2. [2]
- (iv) For squares in which there are 2 raisins, find the mean number of nuts. [4]
- (v) Determine whether N and R are independent. [2]
- 7 The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{x}{2\theta^2} & 0 \leq x \leq 2\theta, \\ 0 & \text{otherwise,} \end{cases}$$

where θ is an unknown positive constant.

- (i) Find $E(X^n)$, where $n \neq -2$, and hence write down the value of $E(X)$. [3]
- (ii) Find
- (a) $\text{Var}(X)$,
- (b) $\text{Var}(X^2)$. [4]
- (iii) Find $E(X_1 + X_2 + X_3)$ and $E(X_1^2 + X_2^2 + X_3^2)$, where X_1, X_2 and X_3 are independent observations of X . Hence construct unbiased estimators, T_1 and T_2 , of θ and $\text{Var}(X)$ respectively, which are based on X_1, X_2 and X_3 . [6]
- (iv) Find $\text{Var}(T_2)$. [2]
- 8 For the events L and M , $P(L | M) = 0.2$, $P(M | L) = 0.4$ and $P(M) = 0.6$.
- (i) Find $P(L)$ and $P(L' \cup M')$. [3]
- (ii) Given that, for the event N , $P(N | (L \cap M)) = 0.3$, find $P(L' \cup M' \cup N')$. [3]



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