

Mathematics

Advanced GCE 4735

Mark Scheme for June 2010

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1(i)	$\text{Var}(2A - 3B) = 4\text{Var}(A) + 9\text{Var}(B) - 12\text{Cov}(A, B)$ $\Rightarrow 18 = 36 + 54 - 12\text{Cov}(A, B)$ $\Rightarrow \text{Cov}(A, B) = 6$	M1 A1 A1	3	Correct formula. Allow one error Substitute relevant values CAO
(ii)	Since $\text{Cov}(A, B) \neq 0$, A and B are not independent	B1 ft	1 (4)	Must have a reason. ft $\text{Cov} \neq 0$
2(i)	$G'(t) = 8te^{4t^2} / e^4$ $E(X) = G'(1)$ $= 8$	M1A1 A1	3	M1 for ct^2/e^4
(ii)	EITHER: $G(t) = e^{-4}(1 + 4t^2 + \dots)$ $P(X=2) = \text{coefficient of } t^2 = 4e^{-4}$ or $4/e^4$ or 0.0733 OR $G''(t) = (8+64t^2)e^{4t^2-4}$ $P(X=2) = \frac{1}{2}G''(0) = 4e^{-4}$ or $4/e^4$ or 0.0733	M1A1 A1 M1A1 A1	3 (6)	Expand in powers of t M1 for reasonable attempt at $M''(t)$
3(i)	Number of different rankings ${}^{11}C_5$ $= 462$ For $R \leq 17$: $1+2+3+4+5 = 15$ $1+2+3+4+6 = 16$ $1+2+3+5+6 = 17$ $1+2+3+4+7 = 17$ $P(R \leq 17) = 4/462 = 2/231$ AG	M1 A1 B2 A1	5	Number of selections of 5 from 11 B1 for 2 or 3 correct
(ii)	$W = 17$ $P(W \leq 17) = \frac{2}{231}$ Smallest SL = $\frac{400}{231} \%$	M1 A1ft	2 (7)	Allow $\frac{4}{231}$; ft $\frac{2}{231}$, but must be exact
4(i)	EITHER: (a) $M'(t) = n(1 - 2t)^{-\frac{1}{2}n - 1}$ $E(Y) = M'(0) = n$ $M''(t) = n(n+2)(1 - 2t)^{-\frac{1}{2}n - 2}$ $\text{Var}(Y) = n(n+2) - n^2 = 2n$ OR: $M(t) = 1 + nt + \frac{1}{2}n(n+2)t^2$ $E(Y) = n$ $\text{Var}(Y) = n(n+2) - n^2 = 2n$	M1 A1 A1 M1 A1 M1A1A1 A1 A1	5 5	Correct form for M1 Ft similar $M'(t)$ $M''(0) - (M'(0))^2$
(ii)	$\text{MGF} = (1 - 2t)^{-30}$ χ^2 distribution with 60 d.f.	B1 B1	2	From $[(1 - 2t)^{-1/2}]^{60}$
(iii)	$E(S) = 60$, $\text{Var}(S) = 120$ Using CLT, Probability $= 1 - \Phi(10/\sqrt{120})$ $= 0.181$	B1ft M1 A1	3 (10)	From (i) Correct tail: allow cc

<p>5(i)</p>	<p>Assumes salaries symmetrically distributed $H_0: m(\text{edian}) = 19.5, H_1: m(\text{edian}) \neq 19.5$ $P = 867$ (or 408) Using normal approximation $\mu = \frac{1}{4} \times 50 \times 51 (= 637.5)$ $\sigma^2 = 50 \times 51 \times 101/24 (= 10731.25)$ $z = (a - 637.5) / \sqrt{10731.25}$ Use $a = 866.5$ $= 2.211$, or 2.215 or 2.220 (– from 408) Compare their z with 1.96 and reject H_0 There is sufficient evidence at the 5% SL that the median salary differs from £19 500</p>	<p>B1 B1 M1 A1 A1 M1 A1 A1 M1 A1 ft 10</p>	<p>In context For both ; not μ ; accept words $a=866.5, 867, 867.5$ (or $408.5, 408, 407.5$) Or p-value rounding to 0.026 or 0.027 Compare with 0.05 or equivalent ft z Or find critical region</p>																												
<p>(ii)</p>	<p>Use sign test when salary distribution is skewed</p>	<p>B1 1 (11)</p>																													
<p>6(i)</p>	<table border="0" style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td colspan="3" style="text-align: center;">N</td> </tr> <tr> <td></td> <td style="text-align: center;">0</td> <td style="text-align: center;">1</td> <td style="text-align: center;">2</td> </tr> <tr> <td style="text-align: center;">0</td> <td style="text-align: center;">0</td> <td style="text-align: center;">c</td> <td style="text-align: center;">$2c$</td> </tr> <tr> <td style="text-align: center;">R 1</td> <td style="text-align: center;">$2c$</td> <td style="text-align: center;">$3c$</td> <td style="text-align: center;">$4c$</td> </tr> <tr> <td style="text-align: center;">2</td> <td style="text-align: center;">$4c$</td> <td style="text-align: center;">$5c$</td> <td style="text-align: center;">$6c$</td> </tr> <tr> <td></td> <td colspan="3" style="text-align: center;">Total $27c = 1$</td> </tr> <tr> <td></td> <td colspan="3" style="text-align: center;">$c = \frac{1}{27}$</td> </tr> </table>		N				0	1	2	0	0	c	$2c$	R 1	$2c$	$3c$	$4c$	2	$4c$	$5c$	$6c$		Total $27c = 1$				$c = \frac{1}{27}$			<p>B1 M1 A1 3</p>	<p>Calculate 9 probs in terms of c</p>
	N																														
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<p>(ii)</p>	<p>$9c/27c$ $= \frac{1}{3}$</p>	<p>M1 A1 ft 2</p>	<p>Marginal probability AEF; ft c</p>																												
<p>(iii)</p>	<p>$P(N + R > 2)$ $= 15c/27c = \frac{5}{9}$</p>	<p>M1 A1 ft 2</p>	<p>AEF; ft c</p>																												
<p>(iv)</p>	<p>$P(R=2) = \frac{15}{27}$ $P(N R=2): p_0 = \frac{4}{15}, p_1 = \frac{1}{3}, p_2 = \frac{2}{5}$ $E(N R=2) = 1 \times \frac{1}{3} + 2 \times \frac{2}{5}$ $= \frac{17}{15}$</p>	<p>M1 A1 ft A1 ft A1 4</p>	<p>Using conditional probabilities One value; ft values in (i) All values Or 1.13</p>																												
<p>(v)</p>	<p>Eg $P(N = 0 \text{ and } R = 0) = 0$ $P(N=0) \times P(R=0) = \frac{6}{27} \times \frac{3}{27} \neq 0$ So N and R are not independent</p>	<p>M1 A1 2 (13)</p>	<p>Or from conditional probs M0 from $N=1$ with $R=1$ or 2 All correct</p>																												

<p>7(i)</p> $\int_0^{2\theta} \frac{x^{n+1}}{2\theta^2} dx = \left[\frac{x^{n+2}}{2(n+2)\theta^2} \right]$ $= 2^{n+1} \theta^n / (n+2)$ <p>$E(X) = 4\theta/3$</p> <hr/> <p>(ii)</p> $\text{Var}(X) = 2\theta^2 - (4\theta/3)^2 = 2\theta^2/9$ $\text{Var}(X^2) = E(X^4) - (E(X))^2$ $= 16\theta^4/3 - 4\theta^4 = 4\theta^4/3$ <hr/> <p>(iii)</p> $E(\sum X_i) = 3 \times 4\theta/3$ $= 4\theta$ $T_1 = \frac{1}{4} \sum X_i$ $E(\sum X_i^2) = 3 \times 2\theta^2$ $= 6\theta^2$ $T_2 = (\sum X_i^2) / 27$ <hr/> <p>(iv)</p> $\text{Var}(T_2) = 1/27^2 \times 3 \times \text{Var}(X^2)$ $= 4\theta^4/729$		<p>M1</p> <p>A1</p> <p>B1 ft 3</p> <hr/> <p>M1A1ft</p> <p>M1A1ft 4</p> <hr/> <p>M1</p> <p>A1 ft</p> <p>A1 ft</p> <p>M1</p> <p>A1 ft</p> <p>A1 ft 6</p> <hr/> <p>M1</p> <p>A1 2</p> <p>(15)</p>	<p>Correct integral</p> <p>AEF</p> <p>B0 if not 'deduced'</p> <hr/> <p>--</p> <p>ft (i) with no n</p> <p>ft (i) with no n</p> <hr/> <p>---</p> <p>ft with no n</p> <p>ft with no n or θ</p> <p>ft with no n</p> <p>ft with no n or θ</p> <hr/> <p>---</p> <p>CAO</p>
<p>8(i)</p> <p>$P(L \cap M) = P(L M)P(M) = 0.12$ and</p> <p>$P(L) = P(M \cap L) / P(M L) = 0.12/0.4 = 0.3$</p> $P(L' \cup M') = P[(L \cap M)']$ $= 1 - P(L \cap M)$ $= 1 - 0.2 \times 0.6 = 0.88$ <hr/> <p>-</p>		<p>A1 M1</p> <p>A1</p> <hr/> <p>B1 3</p> <hr/> <p>M1</p> <p>A1</p> <p>A1 3</p> <p>[6]</p>	

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