

**Mark Scheme 4730
June 2007**

1	(i) $[\omega = 2\pi/6.1 = 1.03]$	M1	For using $T = 2\pi/\omega$
	Speed is 3.09ms^{-1}	M1 A1	For using $v_{\max} = a\omega$
	(ii)	M1	For using $v^2 = \omega^2(A^2 - x^2)$ or for using $v = A\omega \cos \omega t$ and $x = A\sin \omega t$
	$2.5^2 = 1.03^2(3^2 - x^2)$ or $x = 3\sin(1.03 \times 0.60996\dots)$	A1ft	ft incorrect ω
	Distance is 1.76m	A1	3
2	[Magnitudes 0.6, 0.057×7 , 0.057×10]	M1	For triangle with magnitudes shown
	For magnitudes of 2 sides correctly marked	A1	
	For magnitudes of all 3 sides correctly marked	A1	
		M1	For attempting to find angle (α) opposite to the side of magnitude 0.057×7
		M1	For correct use of the cosine rule or equivalent
	$0.399^2 = 0.57^2 + 0.6^2 - 2 \times 0.57 \times 0.6 \cos \alpha$	A1ft	
	Angle is 140°	A1	7 $(180 - 39.8)^\circ$
2	ALTERNATIVE METHOD		
		M1	For using $I = \Delta mv$ parallel to the initial direction of motion or parallel to the impulse
	$-0.6 \cos \alpha = 0.057 \times 7 \cos \beta - 0.057 \times 10$ or $0.6 = 0.057 \times 10 \cos \alpha + 0.057 \times 7 \cos \gamma$	A1	
		M1	For using $I = \Delta mv$ perpendicular to the initial direction of motion or perpendicular to the impulse
	$0.6 \sin \alpha = 0.057 \times 7 \sin \beta$ or $0.057 \times 10 \sin \alpha = 0.057 \times 7 \sin \gamma$	A1	
		M1	For eliminating β *or γ
	$0.399^2 = (0.57 - 0.6 \cos \alpha)^2 + (0.6 \sin \alpha)^2$ or $0.399^2 = (0.6 - 0.57 \cos \alpha)^2 + (0.057 \sin \alpha)^2$	A1ft	
Angle is 140°	A1	7 $(180 - 39.8)^\circ$	

3	(i) $[0.2v \, dv/dx = -0.4v^2]$	M1		For using Newton's second law with $a = v \, dv/dx$
	$(1/v) \, dv/dx = -2$	A1	2	AG
	(ii) $[\int (1/v) \, dv = \int -2 \, dx]$	M1		For separating variables and attempting to integrate
	$\ln v = -2x \quad (+C)$	A1		
	$[\ln v = -2x + \ln u]$	M1		For using $v(0) = u$
	$v = ue^{-2x}$	A1	4	AG
	(iii) $[\int e^{2x} \, dx = \int u \, dt]$	M1		For using $v = dx/dt$ and separating variables
	$e^{2x}/2 = ut \quad (+C)$	A1		
	$[e^{2x}/2 = ut + 1/2]$	M1		For using $x(0) = 0$
	$u = 6.70$	A1	4	Accept $(e^4 - 1)/8$
ALTERNATIVE METHOD FOR PART (iii)				
	$[\int \frac{1}{v^2} \, dv = -2 \int dt \rightarrow -1/v = -2t + A, \text{ and}$	M1		For using $a = dv/dt$, separating variables, attempting to integrate and using $v(0) = u$
	$A = -1/u]$			
	$-e^{2x}/u = -2t - 1/u$	M1		For substituting $v = ue^{-2x}$
	$u = 6.70$	A1	4	Accept $(e^4 - 1)/8$
4	$y = 15 \sin \alpha \quad (=12)$	B1		
	$[4(15 \cos \alpha) - 3 \times 12 = 4a + 3b]$	M1		For using principle of conservation of momentum in the direction of l.o.c.
	Equation complete with not more than one error	A1		
	$4a + 3b = 0$	A1		
		M1		For using NEL in the direction of l.o.c.
	$0.5(15 \cos \alpha + 12) = b - a$	A1		
	$[a = -4.5, b = 6]$	M1		For solving for a and b
	$[\text{Speed} = \sqrt{(-4.5)^2 + 12^2},$	M1		For correct method for speed or direction of A
	$\text{Direction } \tan^{-1}(12/(-4.50))]$			
	Speed of A is 12.8ms^{-1} and direction is 111° anticlockwise from 'i' direction	A1		Direction may be stated in any form, including $\theta = 69^\circ$ with θ clearly and appropriately indicated
Speed of B is 6ms^{-1} to the right	A1	10	Depends on first three M marks	

5	(i)	M1	For taking moments of forces on BC about B
	$80 \times 0.7 \cos 60^\circ = 1.4T$	A1	
	Tension is 20N	A1	
	$[X = 20 \cos 30^\circ]$	M1	For resolving forces horizontally
	Horizontal component is 17.3N	A1ft	ft $X = T \cos 30^\circ$
	$[Y = 80 - 20 \sin 30^\circ]$	M1	For resolving forces vertically
	Vertical component is 70N	A1ft	ft $Y = 80 - T \sin 30^\circ$
	(ii)	M1	For taking moments of forces on AB, or on ABC, about A
	$17.3 \times 1.4 \sin \alpha = (80 \times 0.7 + 70 \times 1.4) \cos \alpha$ or	A1ft	
	$80 \times 0.7 \cos \alpha + 80(1.4 \cos \alpha + 0.7 \cos 60^\circ) =$		
	$20 \cos 60^\circ(1.4 \cos \alpha + 1.4 \cos 60^\circ) +$		
	$20 \sin 60^\circ(1.4 \sin \alpha + 1.4 \sin 60^\circ)$		
	$[\tan \alpha = (\frac{1}{2} 80 + 70)/17.3 = 11/\sqrt{3}]$	M1	For obtaining a numerical expression for $\tan \alpha$
	$\alpha = 81.1^\circ$	A1	4

ALTERNATIVE METHOD FOR PART (i)			
		M1	For taking moments of forces on BC about B
	$H \times 1.4 \sin 60^\circ + V \times 1.4 \cos 60^\circ = 80 \times 0.7 \cos 60^\circ$	A1	Where H and V are components of T
		M1	For using $H = V\sqrt{3}$ and solving simultaneous equations
	Tension is 20N	A1	
	Horizontal component is 17.3N	B1ft	ft value of H used to find T
	$[Y = 80 - V]$	M1	For resolving forces vertically
	Vertical component is 70N	A1ft	ft value of V used to find T

6	(i) [T = 2058x/5.25] 2058x/5.25 = 80 x 9.8 (x = 2) OP = 7.25m	M1 A1 A1	3	For using T = λ x/L AG From 5.25 + 2
	(ii) Initial PE = (80 + 80)g(5) (= 7840) or (80 + 80)gX used in energy equation Initial KE = ½ (80 + 80)3.5 ² (= 980) [Initial EE = 2058x ² /(2x5.25) (= 784), Final EE = 2058x ⁷ /(2x5.25) (= 9604), or 2058(X + 2) ² /(2x5.25)] [Initial energy = 7840 + 980 + 784, final energy = 9604 or 1568X + 980 + 784 = 196(X ² + 4X + 4) → 196X ² - 784X - 980 = 0]	B1 B1 M1 M1		For using EE = λ x ² /2L For attempting to verify compatibility with the principle of conservation of energy, or using the principle and solving for X
	Initial energy = final energy or X = 5 → P&Q just reach the net	A1	5	AG
	(iii) [PE gain = 80g(7.25 + 5)] PE gain = 9604 PE gain = EE at net level → P just reaches O	M1 A1 A1	3	For finding PE gain from net level to O AG
(iv) For any one of 'light rope', 'no air resistance', 'no energy lost in rope' For any other of the above	B1 B1	2		

FIRST ALTERNATIVE METHOD FOR PART (ii)				
[160g - 2058x/5.25 = 160v dv/dx]	M1			For using Newton's second law with a = v dv/dx, separating the variables and attempting to integrate
v ² /2 = gx - 1.225x ² (+ C)	A1			Any correct form
C = -8.575	M1			For using v(2) = 3.5
[v(7) ²]/2 = 68.6 - 60.025 - 8.575 = 0 → P&Q just reach the net	A1	5	AG	

SECOND ALTERNATIVE METHOD FOR PART (ii)				
$\ddot{x} = g - 2.45x$ (= -2.45(x - 4))	B1			
	M1			For using n ² = 2.45 and v ² = n ² (A ² - (x - 4) ²)
3.5 ² = 2.45(A ² - (-2) ²) (A = 3)	A1			
[(4 - 2) + 3]	M1			For using 'distance travelled downwards by P and Q = distance to new equilibrium position + A
distance travelled downwards by P and Q = 5 → P&Q just reach the net	A1	5	AG	

7	(i) [a = 0.7 ² /0.4]	M1	For using a = v ² /r
	For not more than one error in	A1	
	$T - 0.8g\cos 60^\circ = 0.8 \times 0.7^2 / 0.4$	A1	
	Above equation complete and correct Tension is 4.9N	A1	4
(ii)	M1	For using the principle of conservation of energy	
$\frac{1}{2} 0.8v^2 =$	A1	(v = 2.1)	
$\frac{1}{2} 0.8(0.7)^2 + 0.8g0.4 - 0.8g0.4 \cos 60^\circ$	M1	For using NEL	
$(2.1 - 0)/7 = 2u$	A1	4 AG	
Q's initial speed is 0.15ms ⁻¹			
(iii)	M1	For using Newton's second law transversely	
$(m)0.4 \ddot{\theta} = -(m)g \sin \theta$	A1	*Allow m = 0.8 (or any other numerical value)	
$[0.4 \ddot{\theta} \approx -g\theta]$	M1	For using $\sin \theta \approx \theta$	
$[\frac{1}{2} m0.15^2 = mg0.4(1 - \cos \theta_{\max})$	M1	For using the principle of conservation of energy to find	
$\rightarrow \theta_{\max} = 4.34^\circ (0.0758\text{rad})]$		θ_{\max}	
θ_{\max} small justifies $0.4 \ddot{\theta} \approx -g\theta$, and this implies SHM	A1	5	
(iv) [T = 2π / √24.5 = 1.269..]	M1	For using T = 2π/n	
[√24.5 t = π]		or	
		for solving either sin nt = 0 (non-zero t) (considering displacement) or cos nt = -1 (considering velocity)	
Time interval is 0.635s	A1ft	2	From t = ½ T