

**ADVANCED GCE**  
**MATHEMATICS**  
Mechanics 3

**4730**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**

- 8 page Answer Booklet
- List of Formulae (MF1)

**Other Materials Required:**

None

**Monday 19 January 2009**  
**Afternoon**

**Duration:** 1 hour 30 minutes



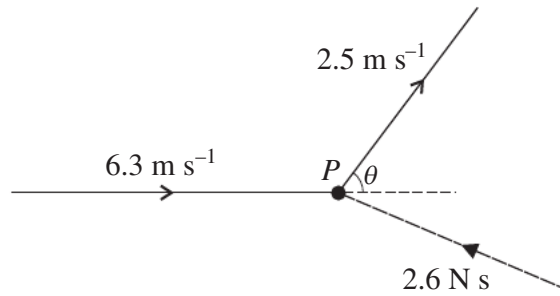
**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .
- You are permitted to use a graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1



A particle  $P$  of mass  $0.5 \text{ kg}$  is moving in a straight line with speed  $6.3 \text{ m s}^{-1}$ . An impulse of magnitude  $2.6 \text{ N s}$  applied to  $P$  deflects its direction of motion through an angle  $\theta$ , and reduces its speed to  $2.5 \text{ m s}^{-1}$  (see diagram). By considering an impulse-momentum triangle, or otherwise,

(i) show that  $\cos \theta = 0.6$ , [4]

(ii) find the angle that the impulse makes with the original direction of motion of  $P$ . [4]

2

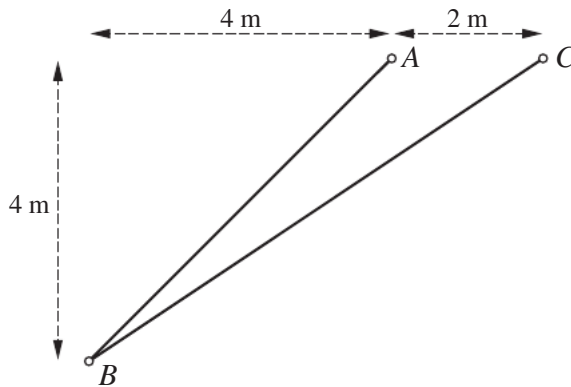


Fig. 1

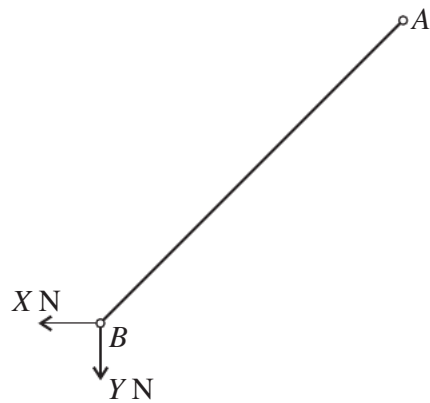


Fig. 2

Two uniform rods  $AB$  and  $BC$ , of weights  $70 \text{ N}$  and  $110 \text{ N}$  respectively, are freely jointed at  $B$ . The rods are in equilibrium in a vertical plane with  $A$  and  $C$  at the same horizontal level and  $AC = 2 \text{ m}$ . The rod  $AB$  is freely jointed to a fixed point at  $A$  and the rod  $BC$  is freely jointed to a fixed point at  $C$ . The horizontal distance between  $B$  and  $A$  is  $4 \text{ m}$  and  $B$  is  $4 \text{ m}$  below  $AC$ ; angle  $BAC$  is obtuse (see Fig. 1). The force exerted on the rod  $AB$  at  $B$ , by the rod  $BC$ , has horizontal and vertical components as shown in Fig. 2.

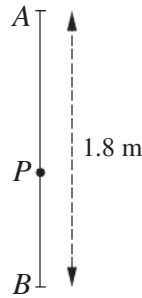
(i) By taking moments about  $A$  for the rod  $AB$  find the value of  $X - Y$ . [2]

(ii) By taking moments about  $C$  for the rod  $BC$  show that  $2X - 3Y + 165 = 0$ . [2]

(iii) Find the magnitude of the force acting between  $AB$  and  $BC$  at  $B$ . [4]

3

3



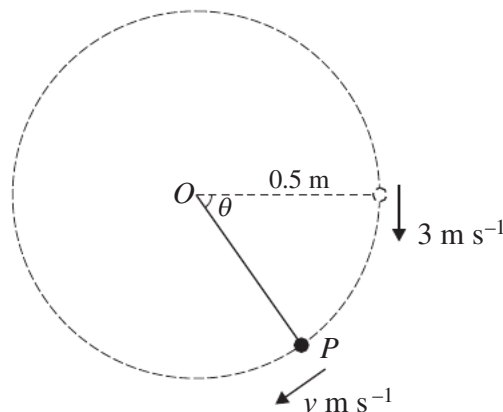
$A$  and  $B$  are fixed points with  $B$  at a distance of 1.8 m vertically below  $A$ . One end of a light elastic string of natural length 0.6 m and modulus of elasticity 24 N is attached to  $A$ , and one end of an identical elastic string is attached to  $B$ . A particle  $P$  of weight 12 N is attached to the other ends of the strings (see diagram).

- (i) Verify that  $P$  is in equilibrium when it is at a distance of 1.05 m vertically below  $A$ . [2]

$P$  is released from rest at the point 1.2 m vertically below  $A$  and begins to move.

- (ii) Show that, when  $P$  is  $x$  m below its equilibrium position, the tensions in  $PA$  and  $PB$  are  $(18 + 40x)$  N and  $(6 - 40x)$  N respectively. [2]
- (iii) Show that  $P$  moves with simple harmonic motion of period 0.777 s, correct to 3 significant figures. [3]
- (iv) Find the speed with which  $P$  passes through the equilibrium position. [2]

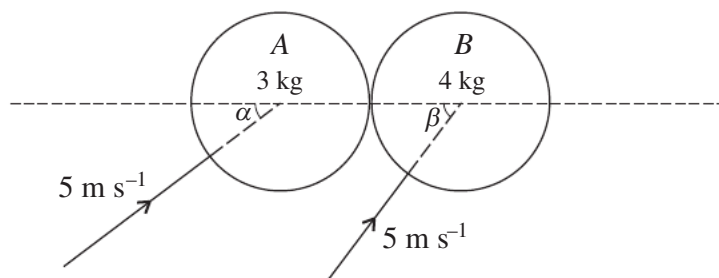
4



One end of a light inextensible string of length 0.5 m is attached to a fixed point  $O$ . A particle  $P$  of mass 0.2 kg is attached to the other end of the string. With the string taut and horizontal,  $P$  is projected with a velocity of  $3 \text{ m s}^{-1}$  vertically downward.  $P$  begins to move in a vertical circle with centre  $O$ . While the string remains taut the angular displacement of  $OP$  is  $\theta$  radians from its initial position, and the speed of  $P$  is  $v \text{ m s}^{-1}$  (see diagram).

- (i) Show that  $v^2 = 9 + 9.8 \sin \theta$ . [3]
- (ii) Find, in terms of  $\theta$ , the radial and tangential components of the acceleration of  $P$ . [3]
- (iii) Show that the tension in the string is  $(3.6 + 5.88 \sin \theta)$  N and hence find the value of  $\theta$  at the instant when the string becomes slack, giving your answer correct to 1 decimal place. [4]

5



Two smooth uniform spheres  $A$  and  $B$ , of equal radius, have masses  $3\text{ kg}$  and  $4\text{ kg}$  respectively. They are moving on a horizontal surface, each with speed  $5\text{ m s}^{-1}$ , when they collide. The directions of motion of  $A$  and  $B$  make angles  $\alpha$  and  $\beta$  respectively with the line of centres of the spheres, where  $\sin \alpha = \cos \beta = 0.6$  (see diagram). The coefficient of restitution between the spheres is  $0.75$ . Find the angle that the velocity of  $A$  makes, immediately after impact, with the line of centres of the spheres.

[10]

6 A stone of mass  $0.125\text{ kg}$  falls freely under gravity, from rest, until it has travelled a distance of  $10\text{ m}$ . The stone then continues to fall in a medium which exerts an upward resisting force of  $0.025v\text{ N}$ , where  $v\text{ m s}^{-1}$  is the speed of the stone  $t\text{ s}$  after the instant that it enters the resisting medium.

(i) Show by integration that  $v = 49 - 35e^{-0.2t}$ . [8]

(ii) Find how far the stone travels during the first 3 seconds in the medium. [4]

7 A particle of mass  $0.8\text{ kg}$  is attached to one end of a light elastic string of natural length  $2\text{ m}$  and modulus of elasticity  $20\text{ N}$ . The other end of the string is attached to a fixed point  $O$ . The particle is held at rest at  $O$  and then released. When the extension of the string is  $x\text{ m}$ , the particle is moving with speed  $v\text{ m s}^{-1}$ .

(i) By considering energy show that  $v^2 = 39.2 + 19.6x - 12.5x^2$ . [4]

(ii) Hence find

(a) the maximum extension of the string, [2]

(b) the maximum speed of the particle, [4]

(c) the maximum magnitude of the acceleration of the particle. [5]