RECOGNISING ACHIEVEMENT

**ADVANCED GCE UNIT** **4730/01**

## MATHEMATICS

Mechanics 3

**WEDNESDAY 10 JANUARY 2007** Afternoon

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

### INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

### INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

### ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

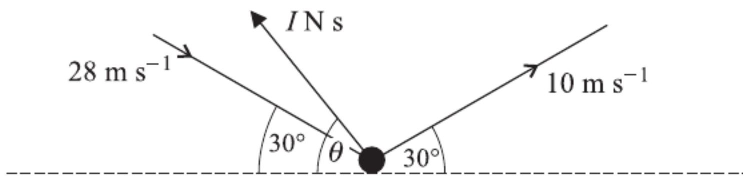
This document consists of **4** printed pages.



**[Turn over**

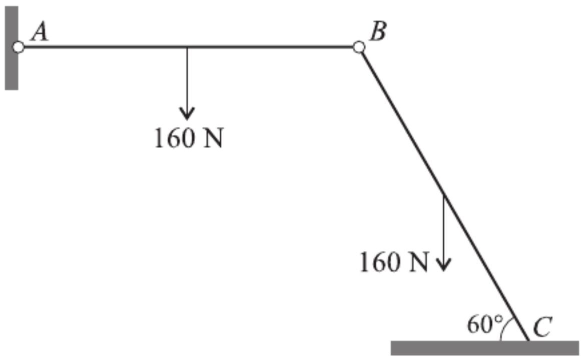
1 A particle $P$ of mass 0.6 kg is attached to a fixed point $O$ by a light inextensible string of length 0.4 m. While hanging at a distance 0.4 m vertically below $O$, $P$ is projected horizontally with speed $5\text{ m s}^{-1}$ and moves in a complete vertical circle. Calculate the tension in the string when $P$ is vertically above $O$. [6]

2




When a tennis ball of mass 0.057 kg bounces it receives an impulse of magnitude $I$ N s at an angle of $\theta$ to the horizontal. Immediately before the ball bounces it has speed $28\text{ m s}^{-1}$ in a direction of 30° to the horizontal. Immediately after the ball bounces it has speed $10\text{ m s}^{-1}$ in a direction of 30° to the horizontal (see diagram). Find $I$ and $\theta$. [7]

3




Two identical uniform rods, $AB$ and $BC$, are freely jointed to each other at $B$, and $A$ is freely jointed to a fixed point. The rods are in limiting equilibrium in a vertical plane, with $C$ resting on a rough horizontal surface. $AB$ is horizontal, and $BC$ is inclined at 60° to the horizontal. The weight of each rod is 160 N (see diagram).

**(i)** By taking moments for $AB$ about $A$, find the vertical component of the force on $AB$ at $B$. Hence or otherwise find the magnitude of the vertical component of the contact force on $BC$ at $C$. [3]

**(ii)** Calculate the magnitude of the frictional force on $BC$ at $C$ and state its direction. [4]

**(iii)** Calculate the value of the coefficient of friction at $C$. [2]

4 A particle $P$ of mass 0.2 kg is suspended from a fixed point $O$ by a light elastic string of natural length 0.7 m and modulus of elasticity 3.5 N. $P$ is at the equilibrium position when it is projected vertically downwards with speed $1.6\text{ m s}^{-1}$. At time $t$ s after being set in motion $P$ is $x$ m below the equilibrium position and has velocity $v\text{ m s}^{-1}$.

**(i)** Show that the equilibrium position of $P$ is 1.092 m below $O$. [3]

**(ii)** Prove that $P$ moves with simple harmonic motion, and calculate the amplitude. [5]

**(iii)** Calculate $x$ and $v$ when $t = 0.4$. [5]

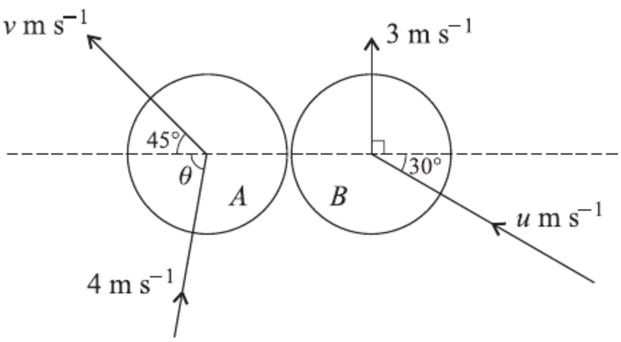
**5** The pilot of a hot air balloon keeps it at a fixed altitude by dropping sand from the balloon. Each grain of sand has mass $m$ kg and is released from rest. When a grain has fallen a distance $x$ m, it has speed $v\,\text{m s}^{-1}$. Each grain falls vertically and the only forces acting on it are its weight and air resistance of magnitude $mkv^2$ N, where $k$ is a positive constant.

**(i)** Show that $\left(\dfrac{v}{g - kv^2}\right)\dfrac{dv}{dx} = 1$. [2]

**(ii)** Find $v^2$ in terms of $k$, $g$ and $x$. Hence show that, as $x$ becomes large, the limiting value of $v$ is $\sqrt{\dfrac{g}{k}}$. [7]

**(iii)** Given that the altitude of the balloon is 300 m and that each grain strikes the ground at 90% of its limiting velocity, find $k$. [3]

**6**




Two uniform smooth spheres $A$ and $B$ of equal radius are moving on a horizontal surface when they collide. $A$ has mass 0.4 kg, and $B$ has mass $m$ kg. Immediately before the collision, $A$ is moving with speed $4\,\text{m s}^{-1}$ at an acute angle $\theta$ to the line of centres, and $B$ is moving with speed $u\,\text{m s}^{-1}$ at 30° to the line of centres. Immediately after the collision $A$ is moving with speed $v\,\text{m s}^{-1}$ at 45° to the line of centres, and $B$ is moving with speed $3\,\text{m s}^{-1}$ perpendicular to the line of centres (see diagram).

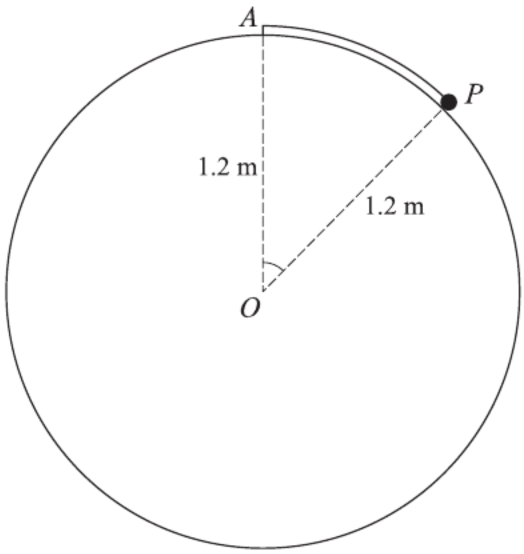
**(i)** Find $u$. [2]

**(ii)** Given that $\theta = 88.1°$ correct to 1 decimal place, calculate the approximate values of $v$ and $m$. [5]

**(iii)** The coefficient of restitution is 0.75. Show that the exact value of $\theta$ is a root of the equation $8\sin\theta - 6\cos\theta = 9\cos 30°$. [5]

**[Question 7 is printed overleaf.]**

**7**




The diagram shows a particle $P$ of mass 0.5 kg attached to the highest point $A$ of a fixed smooth sphere by a light elastic string. The sphere has centre $O$ and radius 1.2 m. The string has natural length 0.6 m and modulus of elasticity 6.86 N. $P$ is released from rest at a point on the surface of the sphere where the acute angle $AOP$ is at least 0.5 radians.

**(i) (a)** For the case angle $AOP = \alpha$, $P$ remains at rest. Show that $\sin\alpha = 2.8\alpha - 1.4$. [4]

**(b)** Use the iterative formula

$$\alpha_{n+1} = \frac{\sin\alpha_n}{2.8} + 0.5,$$

with $\alpha_1 = 0.8$, to find $\alpha$ correct to 2 significant figures. [2]

**(ii)** Given instead that angle $AOP = 0.5$ radians when $P$ is released, find the speed of $P$ when angle $AOP = 0.8$ radians, given that $P$ is at all times in contact with the surface of the sphere. State whether the speed of $P$ is increasing or decreasing when angle $AOP = 0.8$ radians. [7]