OCR
Oxford Cambridge and RSA

Wednesday 24 May 2017 – Morning

A2 GCE MATHEMATICS

4737/01 Decision Mathematics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4737/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Turn over

Answer **all** the questions.

1 Five workers, A, B, C, D and E, are to be allocated to four tasks, W, X, Y and Z. Each worker is to do one task. Task W needs two workers, tasks X, Y and Z need one worker each.

Worker A can do W or X, worker B can do W or Y, worker C can do X or Y, worker D can do X or Z and worker E can do Y or Z.

(i) Represent this information as a bipartite graph with five vertices in each set. **[1]**

Initially worker A is allocated to task X, worker B to task Y and worker E to task Z.

(ii) Construct the shortest possible alternating path, **starting with the arc from *C* to *X***, and use this to write down a suitable way of allocating four of the workers. Write down a shortest possible alternating path on this improved matching and use this to write down a complete matching. **[4]**

(iii) How many ways are there to allocate the five workers to the four tasks? Explain how you know that there are no more than this. **[3]**

2 Tom is playing a game using six cards numbered 1, 2, 3, 4, 5 and 6. Three cards have been placed on the table face down, so that their numbers cannot be seen. Tom is holding the other three cards in his hand.

Tom chooses one card from the three in his hand, let its value be A. He then chooses one card from the three on the table, let its value be B. His score is A – B if this is positive. Otherwise his score is $-(\mathrm{A}^2)$.

(i) Suppose that the cards in Tom's hand have the values 2, 3 and 5 and the cards on the table have the values 1, 4 and 6 (but Tom does not know which card has which number). Construct a table to show Tom's score for each combination of cards. Hence find Tom's play-safe strategy. **[4]**

(ii) Construct a table to show Tom's score for each combination of cards if, instead, Tom had held the cards with values 1, 4 and 6 in his hand with the other three cards on the table. What is Tom's play-safe strategy in this case? **[3]**

(iii) Describe Tom's play-safe strategy:

(a) when he holds the card with value 6; **[1]**

(b) when he does not hold the card with value 6. **[1]**

(iv) With a certain hand of cards, Tom's play-safe strategy is to play the card with the value 3. Which cards were in Tom's hand in this case? **[1]**

3 The tabulation below shows a partially completed dynamic programming formulation for finding a maximin route through a directed network. The weights on the arcs are shown (for example, the arc from (3; 0) to (4; 0) has weight 5).

Stage	State	Action	Weight	Working	Maximin
3	0	0	5		
	1	0	8		
2	0	0	7		
	1	0	4		
	2	0	4		
		1	6		
1	0	0	3		
		1	5		
	1	1	3		
		2	2		
0	0	0	6		
		1	4		

(i) Explain how the table shows that the weight 6 in row 6 of the table represents the weight on the arc from (2; 2) to (3; 1). **[2]**

(ii) Complete the tabulation in your answer book. State the maximin value and write down the maximin route from (0; 0) to (4; 0). **[9]**

4 A radio station is going to broadcast an event being held in four locations at the same time. The producer needs to choose a presenter to be the host at each location.

The table shows the cost (in units of £1000) of using each presenter at each location, the producer wants to minimise the total cost.

		Location			
		Belfast	Cardiff	Glasgow	London
Presenter	Jai	9	2	3	1
	Karen	9	9	2	4
	Mike	9	8	3	6
	Nina	4	5	4	1

(i) Use the Hungarian algorithm to find a suitable allocation. For each table that you use state what it represents (e.g. rows reduced). **[7]**

Suppose that Nina is not able to host the broadcast from Belfast, but otherwise the table is unchanged.

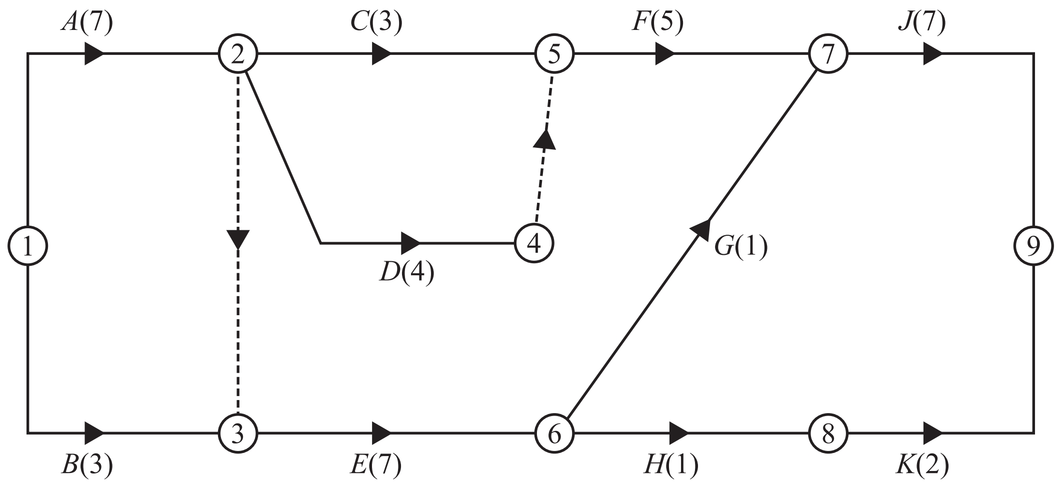
(ii) Without using the Hungarian algorithm again, explain how you know that the best solution is now to choose Jai for Cardiff, Karen for Glasgow, Nina for London and Mike for Belfast. **[1]**

Now suppose that, in addition to Nina not being able to host the broadcast from Belfast, Jai is keen to host the broadcast from London and offers to do it at no cost.

(iii) Explain why having Jai host the broadcast from London would still not give a minimum cost allocation. **[2]**

(iv) What would the cost of using Jai at Cardiff need to be for there to be a minimum cost allocation in which Jai hosts the broadcast from London? **[2]**

5 A project is represented by the activity network below. The durations of the activities are shown in brackets and represent times in hours. The events are numbered (1) to (9).



(i) Explain the purpose of each of the dummy activities. **[3]**

(ii) Carry out a forward pass and a backward pass through the activity network to find the early event time and the late event time at each event. **[3]**

(iii) State the minimum project completion time, and list the critical activities. **[2]**

(iv) Draw a cascade chart for the project. Put the critical activities together in one row and use a separate row for each non-critical activity. Show the float for the non-critical activities. **[3]**

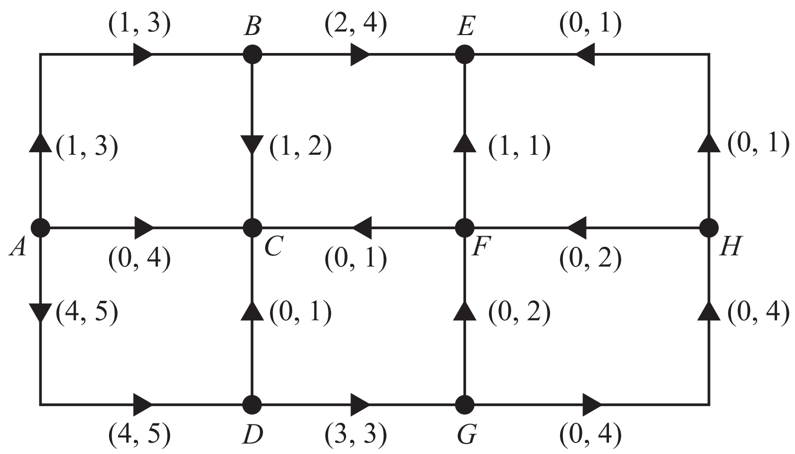
Activities *A*, *B*, *C* and *D* each need one worker. Activity *H* does not require any workers. The other activities each need two workers, with both workers starting at the same time. Once a worker has started an activity they must work on that activity until it is completed.

(v) What is the shortest time in which the project can be completed when only three workers are available? Explain your reasoning. **[2]**

(vi) What is the shortest time in which the project can be completed when only two workers are available? **[2]**

6 The network below models a system of pipes through which fluid can flow. Valves restrict the direction of flow in each pipe. Pipes meet at junctions, labelled *A*, *B*, …, *H*.

For each pipe, the minimum and maximum rate of flow, in $\text{cm}^3\,\text{s}^{-1}$, is shown as a pair of weights on the corresponding arc.



(i) Which vertices are the source and sink for the flow? **[2]**

(ii) Explain what (3, 3) on arc *DG* tells you about the flow in pipe *DG*. **[1]**

(iii) By considering the amount that can flow through junction *D*, what can be deduced about the flow in pipes *AD* and *DC*? **[2]**

(iv) By considering the amount that can flow through junction *B*, what can be deduced about the flow in pipes *AB*, *BC* and *BE*? **[2]**

In a particular flow, the flow in pipe *GF* is $1\,\text{cm}^3\,\text{s}^{-1}$ and the total flow through the system of pipes is $9\,\text{cm}^3\,\text{s}^{-1}$.

(v) By considering the flow through *G*, *H* and *F* show that the flow in *FC* must be $1\,\text{cm}^3\,\text{s}^{-1}$. **[3]**

(vi) Draw a feasible flow of $9\,\text{cm}^3\,\text{s}^{-1}$, with $1\,\text{cm}^3\,\text{s}^{-1}$ flowing in *GF*. Which pipes are at their maximum capacity in your flow? **[3]**

(vii) Describe how this flow can be increased by $2\,\text{cm}^3\,\text{s}^{-1}$. Write down a cut, by giving the set of vertices on the source side and the set of vertices on the sink side, to show that $11\,\text{cm}^3\,\text{s}^{-1}$ is the maximum flow. **[3]**

END OF QUESTION PAPER

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