

# Mark Scheme 4737 June 2006

1	(i)	$4+4+8+7+6 = 29$ litres per second	B1 [1]	For 29
	(ii)	$4-1-2+3+3+5 = 12$ litres per second  $0 - 5 - 4 + 3 + 0 + 5 = -1$ So minimum flow across cut is <b>0</b>	M1 A1 M1 A1 [4]	For using upper and lower capacities correctly For showing how 12 (given) was worked out For a substantially correct calculation For 0, from an appropriate calculation
	(iii)	Flow in arc <b>CE</b> $\geq 2$ and flow in arc <b>CF</b> $\geq 3$ , so at least 5 litres per second must flow into C  At most 4 litres per second flow into A, of which at least 1 flows out to B and 2 flow out to E, so at most 1 litre per second can flow along AD	M1 A1  M1  A1 [4]	For any reasonable attempt (eg $CE = 2$ , $CF = 3$ ) For correct reasoning  For identifying $\leq 4$ in and $\geq 3$ out or equivalent  For a correct conclusion
	(iv)	Either a diagram or a description of a flow of 11 litres per second. Arcs AD, AE, BE, CE, CF must all be at their minimum capacities.	M1  A1 A1 [3]	For a flow of 11 litres per second from S to T  Flow satisfies all lower capacities Flow satisfies all upper capacities
	(v)	$11 \leq$ maximum flow $\leq 12$	B1 B1 [2]	11 as lower bound 12 as upper bound (max flow = 12 $\Rightarrow$ B0, B1)
				<b>14</b>

2	(i)	The route for which the minimum weight on the route is greatest				B1 B1 [2]	For identifying <b>route minima</b> For identifying <b>what has been maximised</b> (‘maximises the minimum’ ⇒ B0 B1)																																						
	(ii)	<table border="1"> <thead> <tr> <th>Stage</th> <th>State</th> <th>Action</th> <th>Working</th> <th>Maximin</th> </tr> </thead> <tbody> <tr> <td rowspan="3">1</td> <td>0</td> <td>0</td> <td></td> <td>18</td> </tr> <tr> <td>1</td> <td>0</td> <td></td> <td>15</td> </tr> <tr> <td>2</td> <td>0</td> <td></td> <td>15</td> </tr> <tr> <td rowspan="4">2</td> <td rowspan="2">0</td> <td>0</td> <td>min(16,18) =16</td> <td rowspan="2">16</td> </tr> <tr> <td>1</td> <td>min(13,15) =13</td> </tr> <tr> <td rowspan="2">1</td> <td>0</td> <td>min(14,15) =14</td> <td rowspan="2">18</td> </tr> <tr> <td>1</td> <td>min(19,18) =18</td> </tr> <tr> <td rowspan="2">3</td> <td rowspan="2">0</td> <td>0</td> <td>min(20,16) =16</td> <td rowspan="2">16</td> </tr> <tr> <td>1</td> <td>min(16,18) =16</td> </tr> </tbody> </table>				Stage	State	Action	Working	Maximin	1	0	0		18	1	0		15	2	0		15	2	0	0	min(16,18) =16	16	1	min(13,15) =13	1	0	min(14,15) =14	18	1	min(19,18) =18	3	0	0	min(20,16) =16	16	1	min(16,18) =16	B1 B1 [2]	Stage and state columns completed correctly Action column completed correctly
		Stage	State	Action	Working	Maximin																																							
1		0	0		18																																								
		1	0		15																																								
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3	0	0	min(20,16) =16	16																																									
		1	min(16,18) =16																																										
<p>Maximin routes: (3; 0) – (2; 0) – (1; 0) – (0; 0)                      (3; 0) – (2; 1) – (1; 0) – (0; 0)                      Maximum load = <b>16 tonnes</b></p>				- M1 A1 M1 A1 M1 A1 [6]	----- For calculating minima for stage 2 state 0 For maximin values identified (may be implied from working seen for stage 3) For calculating minima for stage 2 state 1 For maximin values identified (may be implied from working seen for stage 3) For calculating minima for stage 3 For maximin value identified (Forwards working scores M0, M0, M0)																																								
(iii)	<p>18 tonnes                      (3; 0) – (2; 0) – (2; 1) – (1; 0) – (0; 0)</p>				B1 B1 [2]	For 18 For this route																																							

15

3 (i)	3 Y	M1 A1 [2]	For 3 (allow -3) For Y (cao)																							
(ii)	<p><math>5 &gt; 3, -2 &gt; -4, 5 &gt; -1</math> and <math>6 &gt; 0</math> or using signs of differences +2, +2, +6, +6</p> <p><math>3 &gt; -2, -5 &gt; -6, 1 &gt; 0, 4 &gt; 2</math> or equivalent, or using differences</p> <p>Reduced matrix:</p> <table style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th colspan="2"></th> <th colspan="3">Colin's strategy</th> </tr> <tr> <th colspan="2"></th> <th>W</th> <th>X</th> <th>Y</th> </tr> </thead> <tbody> <tr> <th rowspan="3">Rose's strategy</th> <th>A</th> <td>-1</td> <td>4</td> <td>-3</td> </tr> <tr> <th>B</th> <td>5</td> <td>-2</td> <td>5</td> </tr> <tr> <th>D</th> <td>-5</td> <td>6</td> <td>-4</td> </tr> </tbody> </table>			Colin's strategy					W	X	Y	Rose's strategy	A	-1	4	-3	B	5	-2	5	D	-5	6	-4	<p>M1 A1</p> <p>M1 A1</p> <p>B1 [5]</p>	<p>For an appropriate comparison, or implied For all four comparisons seen</p> <p>For an appropriate comparison, or implied For all four comparisons seen</p> <p>For correct reduced matrix, with rows and columns labelled A, B, D and W, X, Y Cao</p>
		Colin's strategy																								
		W	X	Y																						
Rose's strategy	A	-1	4	-3																						
	B	5	-2	5																						
	D	-5	6	-4																						
(iii)	<p>Row minima are -3, -2, -5 Play-safe for Rose is <b>B</b></p> <p>Column maxima are 5, 6, 5 Play-safes for Colin are <b>W and Y</b></p> <p><b>Not stable</b></p>	<p>M1</p> <p>M1</p> <p>A1 [3]</p>	<p>Follow through their 3×3 reduced matrix For identifying row B</p> <p>For identifying columns W and Y</p> <p>For 'no' or 'not stable'</p>																							
(iv)	<p><b>5 is added</b> throughout the matrix to make the entries non-negative. In this augmented reduced matrix, <math>9p_1 + 3p_2 + 11p_3</math> is the expected number of points won by Rose when Colin plays <b>strategy X</b></p>	<p>M1</p> <p>A1 [2]</p>	<p>For 'add 5' or equivalent</p> <p>For identifying that this is when Colin plays strategy X</p>																							
(v)	<p><math>p_1 = \frac{7}{48}, p_2 = \frac{27}{48}, p_3 = \frac{14}{48}</math>  <math>\Rightarrow m \leq \frac{298}{48}</math> (or <math>6\frac{5}{24}, 6.2083, 6.21</math>)          in all three cases  <math>\Rightarrow M = \frac{58}{48}</math> (or <math>\frac{29}{24}, 1\frac{5}{24}, 1.2083, 1.21</math>)</p>	<p>M1</p> <p>A1 [2]</p>	<p>For attempting to evaluate <math>m</math> cao (in any appropriate form)</p> <p style="text-align: right;"><b>14</b></p>																							

4

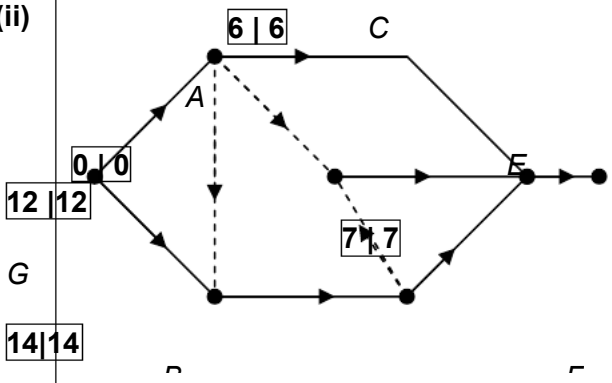
(i)

Activity	Duration	Immediate predecessors
A	6	-
B	4	-
C	5	A
D	1	A, B
E	5	A, D
F	4	D
G	2	C, E, F

B1  
B1  
B1  
[3]

**ANSWERED ON INSERT**  
For predecessors for activities A, B and C correct  
For predecessors for activities D, F and G correct  
For predecessors for activity E correct

(ii)



Minimum completion time = 14 hours  
Critical activities: A, D, E, G

M1  
A1  
M1  
A1

For carrying out forward pass (no more than one independent error)  
For all early event times correct  
For carrying out backwards pass (no more than one independent error)  
For all late event times correct

B1  
B1  
[6]

For 14 cao  
For A, D, E, G only cao

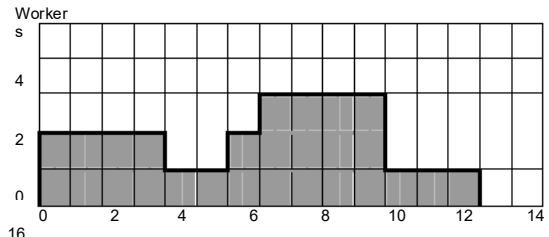
(iii)

Increased by 2 (hours)  
Becomes 16 (hours)

B1  
[1]

For stating that time increases by 2, or equivalent

(iv)



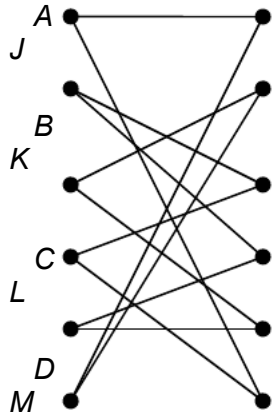
Number of workers required = 3

B1  
M1  
A1  
B1  
[4]

For a resource histogram with no overhanging cells  
For a reasonable attempt, ft their start times if possible  
For a completely correct histogram (cao)  
For 3 or follow through their histogram if possible

14

5 (i)



M1

**ANSWERED ON INSERT**

For a substantially correct attempt

A1  
[2]

For a completely correct bipartite graph

(ii)

$C-N \quad E-M \quad F-K$   
 $A-J \quad B-L \quad D-O$

M1

For pairing  $F-K, C-N, E-M$

A1  
[2]

For all correct (Diagram only  $\Rightarrow$  M1, A0)

(iii)

	J	K	L	M	N	O
A	2	5	2	2	5	2
B	2	5	2	0	5	5
C	5	0	5	5	2	2
D	2	5	0	5	5	2
E	5	2	5	2	0	5
F	2	2	5	5	2	2

B1

For '5' in all the entries that should be 5

B1

For '2' in all the entries that should be 2

B1  
[3]

For '0' in all the entries that should be 0

(iv)

Reduce rows

0	3	0	0	3	0
2	5	2	0	5	5
5	0	5	5	2	2
2	5	0	5	5	2
5	2	5	2	0	5
0	0	3	3	0	0

M1

For a substantially correct attempt from their matrix

A1

For a correct reduction of rows and columns (or columns and rows) for their matrix

Columns are already reduced

Or, reduce columns

0	5	2	2	5	0
0	5	2	0	5	3
3	0	5	5	2	0
0	5	0	5	5	0
3	2	5	2	0	3
0	2	5	5	2	0

Rows are already reduced

M1

Cannot cross out 0's using fewer than 6 lines so matching is complete

A1

For achieving a reduced cost matrix with a complete matching of zero cost (without unnecessary augmenting)

$A-J \quad B-M \quad C-K \quad D-L \quad E-N$   
 $F-O$

B1

0's in correct cells (not ft)

$A-O \quad B-M \quad C-K \quad D-L \quad E-N$   
 $F-J$

B1

For this matching or ft their reduced cost matrix

B1

For this matching or ft their reduced cost matrix

[8]

First matching: Fred and Jenny  
Second matching: Jenny and Olivia

For the names for their first matching  
For the names for their second matching

