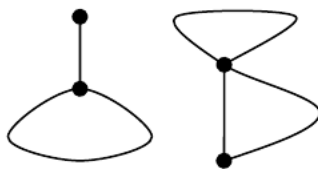
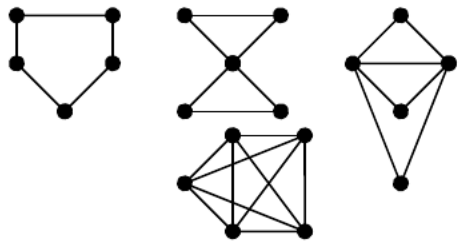


1(i)(a)	<p>31 75 87 42 43 70 56 61 95 28 (may be shown vertically or as separate swaps)</p> <p>9 comparisons and 8 swaps</p> <p>The smallest (final) mark, 28</p>	<p>M1 A1</p> <p>B1</p> <p>B1</p>	<p>[4]</p>	<p>28 moved to the end of the list, no other values moved Correct list at end of first pass (cao)</p> <p>9 and 8 (written, not tallies) (cao) - if not specified, assume the larger value is comparisons (their) 28 or smallest/least or final/last/end</p> <p>If sorted into increasing order: 28 31 75 42 43 70 56 61 87 95 M0 A0, then 9 and 6 = B1 and (their) 95 or largest/greatest/biggest or final/last/end = B1</p>
(b)	<p>75 87 42 43 70 56 61 95 31 28</p>	<p>B1</p>	<p>[1]</p>	<p>Correct list at end of second pass</p> <p>If sorted into increasing order and already penalised in (i)(a) then condone here: 28 31 42 43 70 56 61 75 87 95</p>
(c)	<p>7 more passes</p>	<p>B1</p>	<p>[1]</p>	<p>7 (cao)</p>
(ii)	<p>31 28 75 87 42 43 70 56 61 95 75 31 28 87 42 43 70 56 61 95</p> <p>1 comparison and 0 swaps in first pass 2 comparisons and 2 swaps in second pass</p>	<p>M1 A1</p> <p>B1 B1</p>	<p>[4]</p>	<p>31 28 75 or 31 28 75 ... Correct list, in full, at end of second pass Lists must be easily found, not picked out from working, if the candidate has labelled passes use them as labelled 1 and 0 (written)(cao) may appear next to list 2 and 2 (written)(cao) may appear next to list</p> <p>If sorted into increasing order: 28 31 75 ... M0, A0, then 1 and 1 = B1; 1 and 0 = B1</p>
(iii)	<p>Bubble sort does not terminate early, since it takes 9 passes to get 95 to the front of the list, so it uses $9+8+\dots+1$ or 45 comparisons</p> <p>Shuttle sort takes fewer than $1+2+\dots+9$ comparisons, since, for example, in the fourth pass 42 will be compared with 28, 31 and 75 but not with 87.</p>	<p>B1</p> <p>B1</p>	<p>[2]</p>	<p>Identifying that bubble sort <u>does not terminate early</u> (Just stating $9+8+\dots+1$ or $45 = B0$) Allow 'the largest number is at the end of the list' or '95 at end' A good explanation of why shuttle sort requires fewer comparisons <u>in this particular case</u> Do not accept 'because the list is not in reverse order'</p>
(iv)	<p>$20 \times \left(\frac{50}{10}\right)^2$ = 500 seconds</p>	<p>M1 A1</p>	<p>[2]</p>	<p>Correct method</p> <p>500 seconds or 8 mins 20 sec (without wrong working)</p>

2(i)	Cannot have an odd number of odd nodes Odd vertices come in pairs	B1	[1]	Sum of orders must be even Sum of orders is 9 so 4.5 arcs (which is impossible)
(ii)	eg  Many other correct possibilities	M1 A1	[2]	A diagram showing a graph with four vertices that is <u>not connected</u> and <u>not simple</u> Vertices have orders 1, 2, 3, 4
(iii)	The vertex of order 4 needs to connect to four other vertices, but there are only three other vertices available, so <u>one vertex must be joined twice or the vertex of order 4 is connected to itself</u> . Hence the graph cannot be simple	M1 A1	[2]	Specifically identifying that the problem is with the vertex of <u>order 4</u> <u>Explaining</u> why the graph cannot be simple (either reason) <u>and</u> stating that simple cannot be achieved Ignore any claims about whether or not the graph is connected
(iv) (a)	Each vertex of <u>order 4</u> connects to each of the <u>others</u> , since graph is simple. Hence the other two vertices must have order (at least) 3. But <u>Eulerian</u> , so all must have order 4.	B1	[1]	Any reasonable explanation, but <u>not just a diagram</u> of a specific case ‘the other two must be odd but they can’t because Eulerian’ is not enough Note: the graph has five vertices
(b)	Graph is Eulerian - so each vertex order is even; simple - so no vertex has order more than 4; and connected - so no vertex has order 0. Hence <u>each vertex has order either 2 or 4</u> . But cannot have 3 or 4 vertices of order 4. So must have <u>0, 1, 2 or 5</u> vertices of order 4. 	B1 M1 A1	[3]	<u>Explaining</u> why there are only four such graphs Or list all the possibilities (eg 22222 42222 44222 44444) Any two correct (note: must be simply connected and Eulerian) All four correct and <u>no extras</u> (apart from topologically equivalent variations)

3(i)	$y \geq x$ $x \geq 0$ $y \leq 7 - \frac{2}{3}x$	M1 M1 A1	[3]	Boundaries $y = x$ and $x = 0$ in any form (may be shown as an equality or an inequality with inequality sign wrong) Boundary $2x + 3y = 21$ in any form <u>All</u> inequalities correct (and any extras do not affect the feasible region)
(ii)	$(0, 7) \Rightarrow 42$ $(4.2, 4.2) \Rightarrow 29.4$ or $(\frac{21}{5}, \frac{21}{5}) \Rightarrow \frac{147}{5}$ At optimum, $x = 0$ and $y = 7$ $P_1 = 42$	M1 A1 A1	[3]	Substantially correct attempt at testing vertices (at least one vertex apart from $(0, 0)$) or using a line of constant profit (may be implied) Accept $(0, 7)$ identified (cao) 42 (stated) (cao) NOT deduced from earlier working, unless identified
(iii)	$(4.2, 4.2)$ $P_k = 4.2(k+6)$ or $4.2k + 25.2$	B1 B1	[2]	cao cao
(iv)	Compare $kx + 6y$ with boundary $2x + 3y$ or algebraically, $4.2(k+6)$ with 42 or $-\frac{k}{6}$ with $-\frac{2}{3}$ $\Rightarrow k \leq 4$ $k \leq 4$ or $k < 4$ implies M1, A1	M1 A1	[2]	Algebraically or using line, <u>or implied</u> (allow = here) Accept $k < 4$ No need to say that $k > 0$, but candidates may also say $k > 0$ or $k \geq 0$ Note: k is continuous, so answers such as ' $k = 1, 2, 3, 4$ ' or ' $k = 1, 2, 3$ ', with no other working, would get M1, A0

<p>4(i)</p>	<p>Route: $A - B - D - F - G$</p>	<p>M1 A1 B1 B1 B1</p>	<p>[5]</p>	<p>1.7 shown as a temporary label at G</p> <p>All temporary labels correct with no extras (may not have written temporary label when it becomes permanent)</p> <p>All permanent labels correct (cao)</p> <p>Order of labelling correct (cao)</p> <p>This route written down (not reversed) (cao)</p>
<p>(ii)</p>	<p>Route Inspection problem</p>	<p>B1</p>	<p>[1]</p>	<p>Accept Chinese postman Allow 'postman', 'postman route', but not just 'inspection'</p>
<p>(iii)</p>	<p>$CD (CBD) = 0.3, DG (DFG) = 0.65,$ $CG (CBDFG) = 0.95$ $CD (CBD) \text{ and } FG = 0.75$ or $CD (CBD) \text{ and } EG (EFG) = 1.05$ Length = $3.7 + 0.5 + 0.3 + 0.75$ = 5.25 km</p>	<p>M1 A1 M1 M1 A1</p>	<p>[5]</p>	<p>Any one of these seen (explicitly or as part of a calculation)</p> <p>All three of these seen (explicitly or as parts of calculations)</p> <p>Or either of these with AB to give 1.25 or 1.55 respectively</p> <p>Adding their 0.75 to 3.7 or their 0.75 to $3.7 + 0.5 + 0.3$ (cao) units not needed 5.25 implies M1, M1 A1, irrespective of working</p>
<p>(iv)</p>	<p>$B - D - F - G - C - B$ 1.9 km</p>	<p>B1 B1</p>	<p>[2]</p>	<p>cao 1.9 (cao) irrespective of method</p>
<p>(v)</p>	<p>[TREE] Vertices added in order $BDCF$ or $BDFC$ Arcs added in order BD, BC, DF or BD, DF, BC Two shortest arcs from G total $0.45 + 0.65 = 1.1$ Lower bound = $0.5 + 1.1 = 1.6$ km</p>	<p>B1 B1 M1 A1</p>	<p>[4]</p>	<p>Correct tree drawn A valid order of adding vertices or a valid order of adding arcs 0.45 and 0.65, or total 1.1 (may be implied from 1.6) 1.6 (cao) units not needed 1.6 implies M1, A1</p>

5(i)	$600x + 800y + 500z \leq 5000$ $\Rightarrow 6x + 8y + 5z \leq 50$ $120x + 80y + 120z \leq 800$ $\Rightarrow 3x + 2y + 3z \leq 20$ May use slack variables, provided they also specify slack variables non-negative eg $6x + 8y + 5z + t = 50, t \geq 0 = M1, A1$	M1 A1		Correct inequality, allow $<$ for M mark only Correct fully simplified form (cao)																																								
		M1 A1	[4]	Correct inequality, allow $<$ for M mark only Correct fully simplified form (cao)																																								
				If slack variable form used and fully simplified but without specifying that slack variables are non-negative, SC M1 A0 for each																																								
(ii)	<table border="1" data-bbox="135 582 662 761"> <thead> <tr> <th>P</th> <th>x</th> <th>y</th> <th>z</th> <th>s</th> <th>t</th> <th>u</th> <th>RHS</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>-100</td> <td>-40</td> <td>-120</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <td>0</td> <td>12</td> <td>20</td> <td>15</td> <td>1</td> <td>0</td> <td>0</td> <td>60</td> </tr> <tr> <td>0</td> <td>6</td> <td>8</td> <td>5</td> <td>0</td> <td>1</td> <td>0</td> <td>50</td> </tr> <tr> <td>0</td> <td>3</td> <td>2</td> <td>3</td> <td>0</td> <td>0</td> <td>1</td> <td>20</td> </tr> </tbody> </table>	P	x	y	z	s	t	u	RHS	1	-100	-40	-120	0	0	0	0	0	12	20	15	1	0	0	60	0	6	8	5	0	1	0	50	0	3	2	3	0	0	1	20	M1		Objective row correct <u>and</u> three slack variables used
P	x	y	z	s	t	u	RHS																																					
1	-100	-40	-120	0	0	0	0																																					
0	12	20	15	1	0	0	60																																					
0	6	8	5	0	1	0	50																																					
0	3	2	3	0	0	1	20																																					
		A1		Three constraint rows correct (ft (i), if reasonable) Accept variations in order of rows and columns Condone P column missing here																																								
(ii)	$60 \div 15 = 4, 50 \div 5 = 10, 20 \div 3 = 6\frac{2}{3}$ Pivot on the 15 in the z column New row 2 = row 2 \div 15 New row 1 = row 1 + $120 \times$ new row 2 New row 3 = row 3 - $5 \times$ new row 2 New row 4 = row 4 - $3 \times$ new row 2	B1		Correct pivot choice from <u>their z column</u>																																								
		M1		Correct method for <u>their</u> pivot row seen (or implied from <u>correct row</u> in tableau if no attempt seen)																																								
		A1		Correct method for their <u>three</u> other rows seen as a <u>formula</u>																																								
	<table border="1" data-bbox="135 1108 662 1332"> <thead> <tr> <th>P</th> <th>x</th> <th>y</th> <th>z</th> <th>s</th> <th>t</th> <th>u</th> <th>RHS</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>-4</td> <td>120</td> <td>0</td> <td>8</td> <td>0</td> <td>0</td> <td>480</td> </tr> <tr> <td>0</td> <td>$\frac{4}{5}$</td> <td>$1\frac{1}{3}$</td> <td>1</td> <td>$\frac{1}{15}$</td> <td>0</td> <td>0</td> <td>4</td> </tr> <tr> <td>0</td> <td>2</td> <td>$1\frac{1}{3}$</td> <td>0</td> <td>$-\frac{1}{3}$</td> <td>1</td> <td>0</td> <td>30</td> </tr> <tr> <td>0</td> <td>$\frac{3}{5}$</td> <td>-2</td> <td>0</td> <td>$-\frac{1}{5}$</td> <td>0</td> <td>1</td> <td>8</td> </tr> </tbody> </table>	P	x	y	z	s	t	u	RHS	1	-4	120	0	8	0	0	480	0	$\frac{4}{5}$	$1\frac{1}{3}$	1	$\frac{1}{15}$	0	0	4	0	2	$1\frac{1}{3}$	0	$-\frac{1}{3}$	1	0	30	0	$\frac{3}{5}$	-2	0	$-\frac{1}{5}$	0	1	8	M1		Iterate to get a tableau with exactly <u>four basis columns</u> and <u>non-negative entries in final column</u> , in which the value of the <u>objective has not decreased</u>
P	x	y	z	s	t	u	RHS																																					
1	-4	120	0	8	0	0	480																																					
0	$\frac{4}{5}$	$1\frac{1}{3}$	1	$\frac{1}{15}$	0	0	4																																					
0	2	$1\frac{1}{3}$	0	$-\frac{1}{3}$	1	0	30																																					
0	$\frac{3}{5}$	-2	0	$-\frac{1}{5}$	0	1	8																																					
		A1		Values in final column correct (follow through)																																								
	$4 \div \frac{4}{5} = 5, 30 \div 2 = 15, 8 \div \frac{3}{5} = 13\frac{1}{3}$ Pivot on the $\frac{4}{5}$ in the x column New row 2 = row 2 \div $\frac{4}{5}$ New row 1 = row 1 + $4 \times$ new row 2 New row 3 = row 3 - $2 \times$ new row 2 New row 4 = row 4 - $\frac{3}{5} \times$ new row 2	B1		Correct pivot choice for their second iteration																																								
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P	x	y	z	s	t	u	RHS																																					
1	0	$126\frac{2}{3}$	5	$8\frac{1}{3}$	0	0	500																																					
0	1	$1\frac{2}{3}$	$1\frac{1}{4}$	$\frac{1}{12}$	0	0	5																																					
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0	0	-3	$-\frac{3}{4}$	$-\frac{1}{4}$	0	1	5																																					
		A1		Values in final column correct (follow through)																																								

	Make 5 litres of <i>fruit salad</i> only	B1		Interpretation of <u>their</u> final (non-negative) x, y and z , in context (need 'only' or equivalent; '5 <i>fruit salads</i> ' is not enough) $x = 5, y = 0, z = 0$ gives B0
			[13]	
(iii)	$60 \div 12 = 5, 50 \div 6 = 8\frac{1}{3}, 20 \div 3 = 6\frac{2}{3}$ Pivot on the 12 in the x column New row 2 = row 2 \div 12 New row 1 = row 1 + 100 \times new row 2 Showing that there are no negative entries in objective row Saying that optimum has been achieved ('no negatives in top row')	B1 M1 A1 M1 A1		Correct pivot choice <u>from their x column</u> Correct method for <u>their</u> pivot row (seen or implied from correct row in tableau) Correct method for their <u>objective</u> row seen as a formula Showing that there are no negative entries in objective row Or achieving a final tableau, in one iteration, with exactly four basis columns and non-negative entries in final column, in which the value of the objective has not decreased
			[5]	