OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE **4736/01**

MATHEMATICS

Decision Mathematics 1

THURSDAY 12 JUNE 2008 Morning

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
Insert for Question 4
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- There is an **insert** for use in Question **4**.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **7** printed pages, **1** blank page and an insert.

1 This question is about using bubble sort to sort a list of numbers into **increasing** order.

(i) Which numbers, if any, can be guaranteed to be in their correct final position after the first pass? [1]

Suppose now that the original, unsorted list was 3, 2, 1, 5, 4.

(ii) Write down the list that results after one pass through bubble sort. How many comparisons and how many swaps were used in this pass? [2]

(iii) Write down the list that results after a second pass through bubble sort. How many more passes will be required until the algorithm terminates? [2]

Bubble sort is a quadratic order algorithm.

(iv) A computer takes 0.2 seconds to sort a list of 500 numbers using bubble sort. Approximately how long will it take to sort a list of 3000 numbers? [2]

2 A *simple* graph is one in which any two vertices are directly connected by at most one arc and no vertex is directly connected to itself.

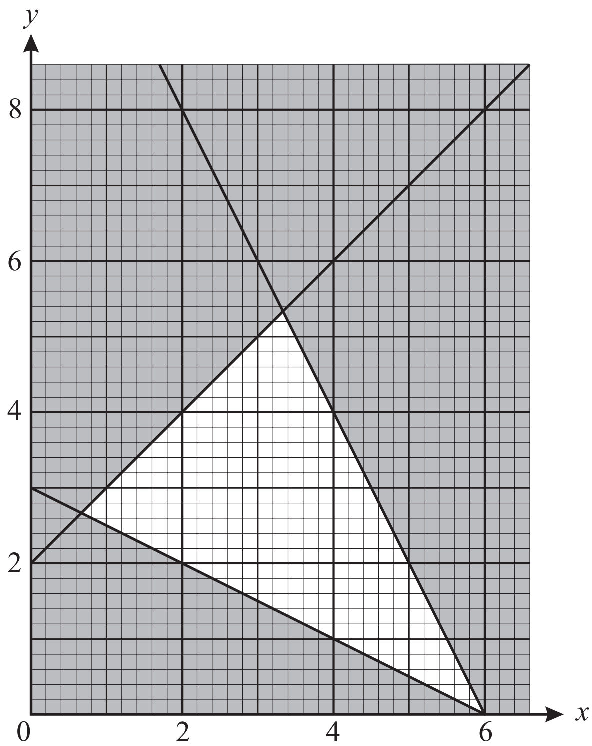
A *connected* graph is one in which every vertex is joined, directly or indirectly, to every other vertex.

A *simply connected* graph is one that is both simple and connected.

(i) Draw an Eulerian graph with four vertices, of orders 2, 2, 4 and 4, and no others. Explain why your graph is not simply connected. [3]

(ii) Draw a non-Eulerian graph with four vertices, of orders 2, 2, 4 and 4, and no others. Explain why your graph is non-Eulerian even though its vertices are all of even order. [3]

3 The constraints of a linear programming problem are represented by the graph below. The feasible region is the unshaded region, including its boundaries.



(i) Write down the inequalities that define the feasible region. [4]

(ii) Calculate the coordinates of the three vertices of the feasible region. [4]

The objective is to maximise $5x + 3y$.

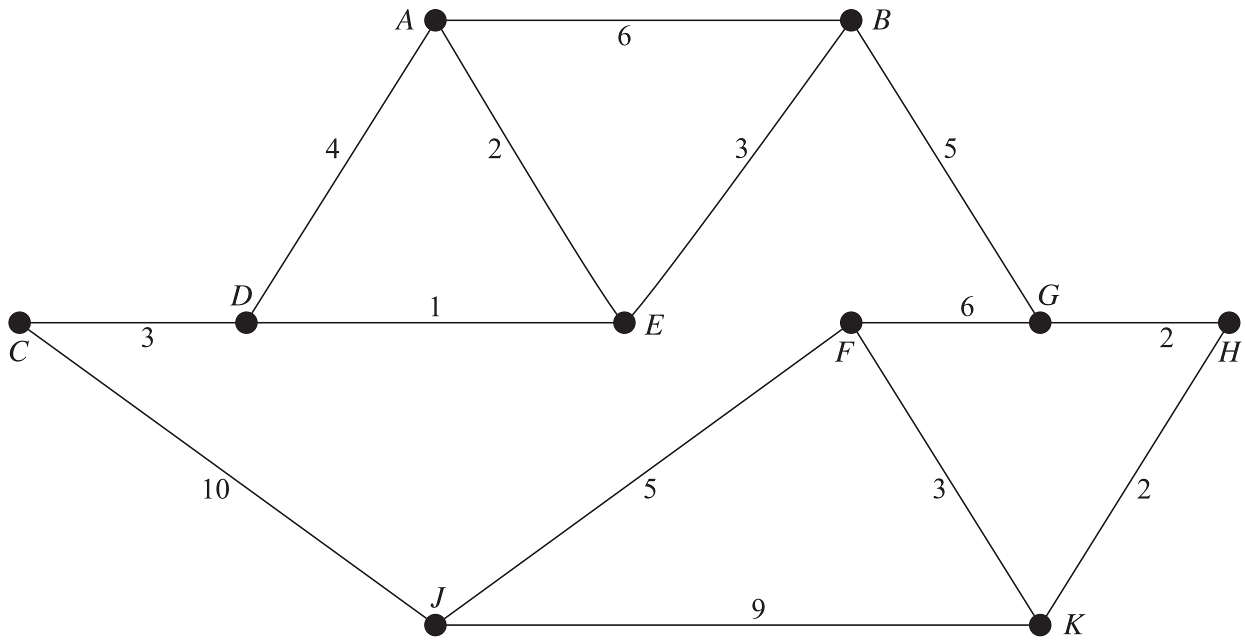
(iii) Find the values of x and y at the optimal point, and the corresponding maximum value of $5x + 3y$. [3]

The objective is changed to maximise $5x + ky$, where k is positive.

(iv) Find the range of values of k for which the optimal point is the same as in part **(iii)**. [3]

4 **Answer this question on the insert provided.**

The vertices in the network below represent the rooms in a house. The arcs represent routes between rooms, and the weights on the arcs represent distances in metres.



(i) On the diagram in the insert, use Dijkstra's algorithm to find the shortest path from A to K. You must show your working, including temporary labels, permanent labels and the order in which permanent labels are assigned. Write down the route of the shortest path from A to K and give its length in metres. [7]

A locked door blocks the route CJ, so this arc cannot be used.

(ii) Use your answer to part **(i)** to find the route of the shortest path from A to J and its length in metres. [2]

(iii) Alterations mean that the length of route FJ changes from its current value of 5 metres. By how much would it have to change if the route of the shortest path from A to J, not using CJ, changes from that found in part **(ii)**? [2]

5 Laura is booking buses to transport students home from a college party. She wants to book four buses to travel to Easton and five buses to travel to Weston. She contacts the local bus companies to ask about availability and cost. This information is summarised in the table below.

Company	Number of buses available	Cost per bus to Easton	Cost per bus to Weston
Anywhere Autos (A)	3	£250	£250
Busy Buses (B)	3	£200	£140
County Coaches (C)	3	£300	£280

Suppose that Laura books x buses to travel to Easton from company A and y buses to travel to Easton from company B.

(i) **Copy and complete** the following table to show, in terms of x and y, how many buses Laura books from each company to each town and show that the total cost is £$(2090 - 20x + 40y)$. [5]

	E	W
A	x	
B	y	
C		$x + y - 1$

(ii) Laura wants to spend no more than £2150 on the buses.

Show that this leads to the constraint $-x + 2y \leqslant 3$. [1]

When Laura looks at the times that the companies could run the buses, she realises that she will need at least one bus from C to E. This leads to the constraint $x + y \leqslant 3$.

Each bus from A can carry 50 students, each bus from B can carry 40 students and each bus from C can carry 60 students. Laura wants to maximise the number of students who can travel to W.

(iii) Show that this leads to needing to maximise the objective function $x + 2y$. [2]

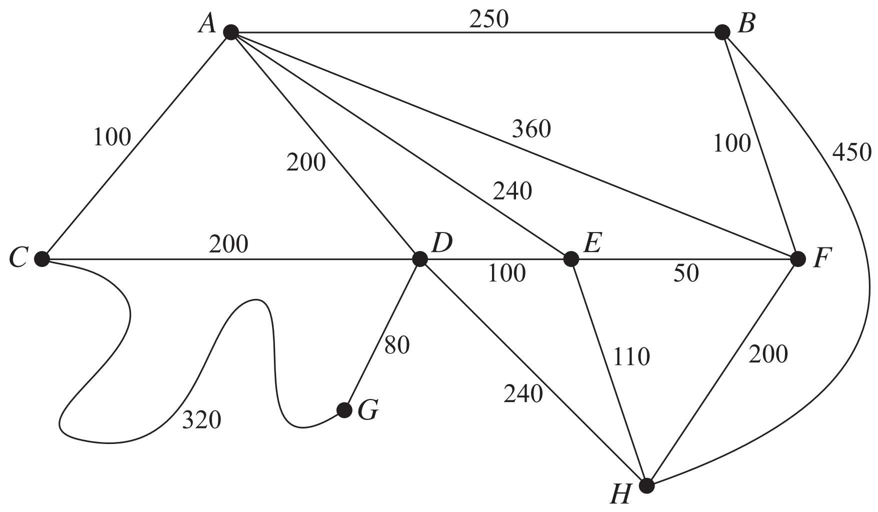
Laura's problem gives the linear programming problem:

$$\begin{aligned}
&\text{Maximise} && P = x + 2y,\\
&\text{subject to} && -x + 2y \leqslant 3,\\
& && x + y \leqslant 3,\\
&\text{and} && x \geqslant 0,\ y \geqslant 0, \quad \text{with } x \text{ and } y \text{ both integers.}
\end{aligned}$$

(iv) Represent this problem as an initial Simplex tableau. [2]

(v) Use the Simplex algorithm, pivoting first on a value chosen from the y column, to find the values of x and y at the optimum point. [6]

6 The network below represents a simplified map of a forest. The nodes represent locations in the forest and the arcs represent footpaths. The weights on the arcs represent distances, in metres.



(a) Woody the forest ranger wants to start from rangers' hut (H) and walk along every footpath at least once using the shortest possible total distance.

(i) Which standard network problem does Woody need to solve to find the shortest route that covers every arc? [1]

The total length of all the footpaths shown is 3000 metres.

(ii) Use an appropriate algorithm to find the length of the shortest route that Woody can use. Show all your working. (You may find the lengths of shortest paths between nodes by inspection.) [4]

Suppose that, instead, Woody wants to start from the car park (C) and walk along every footpath at least once using the shortest possible total distance.

(iii) What is the length of the shortest route that Woody can use if he starts from the car park? At which node does this route end? [3]

(b) There is a nesting box at each node of the network.

Cyril the squirrel lives in the forest. He wants to start from his drey (D) and check each nesting box, to see whether he has stored any nuts there, before returning to his drey. Cyril is a vain squirrel, so he wants to use the footpaths so that people can see him. However he is also a very lazy squirrel, so he would like to check the boxes in the shortest distance possible.

(i) Apply the nearest neighbour method starting at node D to find a tour through all the nodes that starts and ends at D. Calculate the total weight of this tour. Explain why the nearest neighbour method fails if you start at node A. [3]

(ii) Construct a minimum spanning tree by using Prim's algorithm on the reduced network formed by deleting node A and all the arcs that are directly joined to node A. **Start building your tree at node *B*.** (You do *not* need to represent the network as a matrix.) Give the order in which nodes are added to your tree and draw a diagram to show the arcs in your tree. Calculate the total weight of your tree. [5]

(iii) From your previous answers, what can you say about the shortest possible distance that Cyril must travel to visit each nesting box and return home to his drey? [2]

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