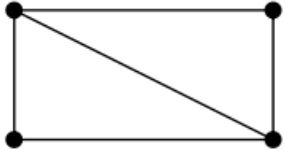
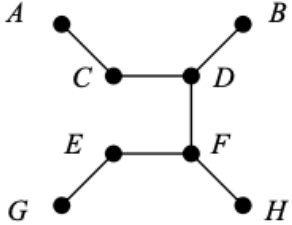
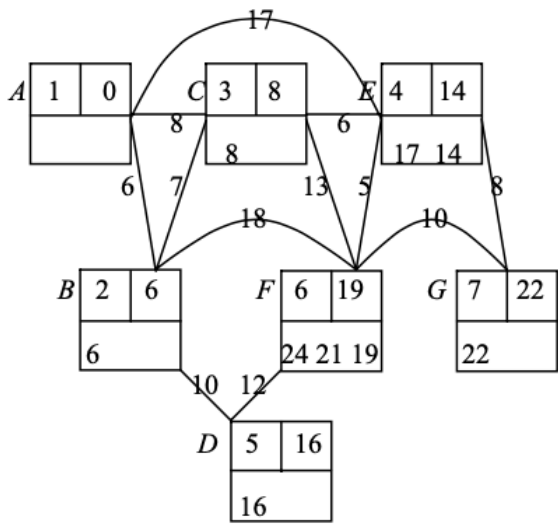


<p>1 (a) (i) 8 7 5 4 3 3 3 3 2 2</p> <p>First bag 8 2          Second bag 7 3          Third bag 5 4          Fourth bag 3 3 3          Fifth bag 2</p>	<p>M1          M1          A1</p>	<p>For sorting the list into decreasing order</p> <p>For trying to apply first-fit to their list</p> <p>For a completely correct solution</p>
<p>(ii) A packing that uses fewer bags could be</p> <p>First bag 8 2          Second bag 7 3          Third bag 5 3 2          Fourth bag 4 3 3</p>	<p>B1</p>	<p>For any valid packing into four bags (may be as an incorrect answer to using algorithm, need not be packed in this order)</p>
<p>(b) <math>\left(\frac{500}{100}\right)^3 \times 4</math> or <math>125000000 \times 0.000004</math>  <math>= 500</math></p>	<p>M1          A1          6</p>	<p>For scaling 4 seconds by <math>5^3</math> or for an equivalent valid and complete method. Condone minor errors with the number of zeros.          For 500 or 500 seconds or 500 s.          Accept 8 minutes 20 seconds or 8.3 minutes</p>
<p>2 (i) eg</p> 	<p>B1</p>	<p>For any <u>simple</u> graph with 4 vertices and 5 arcs</p> <p>Vertices need not be labelled          Need not be planar</p>
<p>(ii) The sum of the orders of the vertices is twice the number of arcs, and hence is even.</p> <p>Hence the sum of the odd orders must be even and so there must be an even number of odd vertices.</p>	<p>M1          A1</p>	<p>Or start from a null graph and successively add in arcs.          Each time an arc is added the number of odd vertices is either unchanged or it increases or decreases by 2.          So the number of odd nodes is always even</p>
<p>(iii) 5 arcs <math>\Rightarrow</math> sum of orders of vertices = 10          Simple graph connecting vertices so each vertex has order 1, 2 or 3  <math>1 + 3 + 3 + 3 = 10</math> or <math>2 + 2 + 3 + 3 = 10</math></p> <p>But <math>1 + 3 + 3 + 3</math> is not possible since if three vertices have order 3 they are all connected to the fourth vertex so it also has order 3.</p> <p>With <math>2 + 2 + 3 + 3</math> the two vertices of order 2 cannot be adjacent, since otherwise two arcs connect the other two vertices so not simple.</p> <p>Hence only one possible graph.</p>	<p>M1          A1          A1          6</p>	<p>Explaining why <math>1 + 3 + 3 + 3</math> is not possible.</p> <p>Explaining why there is only one graph with nodes of orders 2, 2, 3, 3.</p>
<p>3 (i)</p>  <p>Kruskal: <math>DF, CD, BD</math> and <math>EF, FH, AC, EG</math></p> <p>40</p> <p>(ii) <math>A C D F E G H B A</math></p>	<p>M1          M1          A1          B1          M1</p>	<p>For a correct tree (labels not required)</p> <p>For a valid order (using Prim or Kruskal)</p> <p>For length = 40</p> <p>At getting at least as far as <math>A C D F E</math> (or shown on a diagram)</p>

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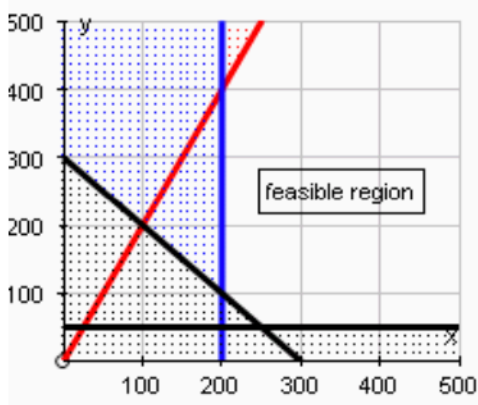
Mark Scheme

June 2005

<p>(iii) (A) <i>ACEG</i> and <i>ABGH</i> (B) 5 (C) <i>ABCD</i></p>	<p>A1 B1 B1 B1</p>	<p>For a correct cycle, ending back at <i>A</i> (if shown on a diagram, needs direction shown) For both, vertices in any order For 5 For <i>ABCD</i>, vertices in any order</p>
<p>4 (i)</p>  <p>Vertex <i>B C D E F G</i> Length 6 8 16 14 19 22</p> <p><i>A - C - E - G</i></p>	<p>M1 A1 B1 B1</p>	<p>Answer should be on insert sheet For using Dijkstra's algorithm – updating at <i>E</i> and <i>F</i> (even if incomplete) For all permanent labels correct For valid order of assigning permanent labels For copying their permanent labels, or correct values Correct answer only</p>
<p>(ii) The only odd nodes are <i>A</i> and <i>F</i> Shortest path from <i>A</i> to <i>F</i> has length 19 km 120 + 19 = 139 km</p>	<p>M1 M1 A1</p>	<p>For identifying <i>A</i> and <i>F</i> or value 19 or their 19 For 120 + their 19 For 139 (cao)</p>
<p>(iii) Need <i>A</i> and <i>G</i> odd and all other nodes even so need to connect <i>F</i> to <i>G</i> = 10 km 120 + 10 = 130 km</p>	<p>M1 A1</p>	<p>For identifying <i>F</i> and <i>G</i> or value 10 as only extra For 130 (cao)</p>

10

<p><b>5</b> (i)</p> <table border="1"> <thead> <tr> <th><math>X</math></th> <th><math>N</math></th> <th><math>T</math></th> <th><math>S</math></th> </tr> </thead> <tbody> <tr> <td></td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <td>3</td> <td>1</td> <td>3</td> <td>9</td> </tr> <tr> <td>6</td> <td>2</td> <td>9</td> <td>45</td> </tr> <tr> <td>5</td> <td>3</td> <td>14</td> <td>70</td> </tr> <tr> <td>7</td> <td>4</td> <td>21</td> <td>119</td> </tr> <tr> <td>3</td> <td>5</td> <td>24</td> <td>128</td> </tr> </tbody> </table> <p><math>M = 4.8</math> <math>D = 1.6</math></p> <p>(ii) 15 additions and 5 multiplications <math>20 + 5 = 25</math></p> <p>(iii) <math>3n + n + 5</math> <math>= 4n + 5</math></p> <p>(iv) <math>(5000 \div 1000) \times 2 = 10</math> seconds</p>	$X$	$N$	$T$	$S$		0	0	0	3	1	3	9	6	2	9	45	5	3	14	70	7	4	21	119	3	5	24	128	<p>M1 M1  A1  B1 B1  B1 B1  M1  A1 B1</p>	<p>For initial pass through step 3 correct For updating each of <math>N</math>, <math>T</math> and <math>S</math> correctly</p> <p>For final values of <math>N</math>, <math>T</math> and <math>S</math> correct</p> <p>For 4.8 (ft their <math>T \div N</math>) For 1.6 (ft their <math>\sqrt{\{(S \div N) - (M^2)\}}</math>)</p> <p>For 'their 20' + 5</p> <p>For any function of <math>n</math> that gives their answer from (ii) when <math>n = 5</math> For any expression that simplifies to <math>4n + 5</math> Or <math>2 \div 4005 \times 20005 = 9.99 \approx 10</math> seconds</p>																																																																												
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$P$	$x$	$y$	$z$	$s$	$t$	$u$																																																																																																				
1	-15	4	4	0	0	0	0																																																																																																			
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0	10	6	9	0	1	0	72																																																																																																			
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1	0	-2	16	1.5	0	0	60																																																																																																			
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<p>7 (i) Minimise <math>70x + 80y + 50z</math></p> <p>'No more than twice as many packs of type Y as packs of type X'</p> <p>Other constraints  <math>x \geq 200, 0 \leq z \leq 50</math>  <math>y \geq z</math>  <math>x + z \geq 220</math>  <math>x + y \geq 300</math></p>	<p>B1 B1 B1 B1 B1 B1</p>	<p>For 'minimise' a (non-zero) multiple of <math>7x+8y+5z</math></p> <p>For identifying this constraint from the list, or equivalent</p> <p>Ignore extra 'constraints' unless contradictions</p> <p>For boundary constraints on <math>x</math> and <math>z</math></p> <p>For this, or an equivalent correct answer</p> <p>For this, or an equivalent correct answer</p> <p>For this, or an equivalent correct answer</p> <p>Use of strict inequalities – penalise first time only</p>
<p>(ii) (a) Minimise <math>70x + 80y (+ 2500)</math> (or scaled through)          Subject to  <math>y \leq 2x</math>  <math>x \geq 200</math>  <math>y \geq 50</math>  <math>x + y \geq 300</math></p> 	<p>M1 A1 M1 M1 A1</p>	<p>For replacing <math>z</math> by <math>50</math></p> <p>For their <math>y \geq 50</math></p> <p>For at least two appropriate lines drawn on a graph with plausibly scaled axes.</p> <p>For boundary lines drawn correctly (follow through their equations provided there are at least two horizontal or vertical lines and at least two lines that 'slope')</p> <p>Feasible region correctly identified (correct answer only, not follow through)</p>
<p>(b) <math>(200, 400), (200, 100), (250, 50)</math></p> <p><math>(200, 100)</math> gives <math>70x + 80y = 22000</math> (£245)  <math>(250, 50)</math> gives <math>70x + 80y = 21500</math> (£240)</p> <p>Cost is minimised when <math>x = 250, y = 50</math>          Cost = £240</p>	<p>M1 A1 M1 A1 B1</p>	<p>For reading off or calculating at least one of their vertices</p> <p>For getting these three vertices correct with no extras</p> <p>For calculating their cost at one of their vertices or using an appropriate line of constant cost</p> <p>For identifying vertex <math>(250, 50)</math></p> <p>For £240 or 24000 p (with units)</p>
<p>(iii) eg <math>x = 300, y = 0, z = 0</math>          only costs £210</p>	<p>M1 A1</p>	<p>For finding a feasible point with <math>z &lt; 50</math></p> <p>Or a written explanation</p> <p>For finding such a feasible point with a lower cost than that in (ii)(b) and showing that cost is lower.</p>