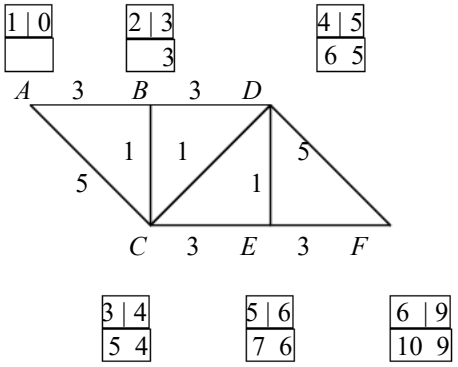
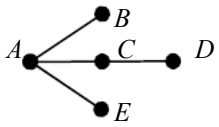
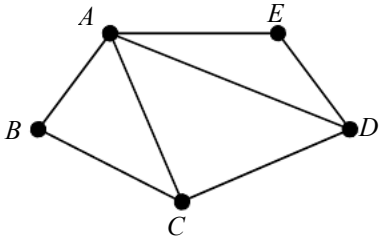
4736 Decision Mathematics 1

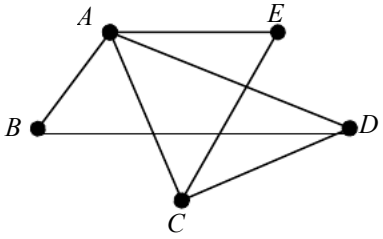
		TO BE ANSWERED ON INSERT			
1	**(i)**	 Path: $A-B-C-D-E-F$ Weight: 9	M1 A1 B1 B1 B1	Evidence of updating at C, D, E or F All temporary labels correct, with no extras All permanent labels correct cao cao	**[5]**
	(ii)	Total weight of all arcs = 25 Only odd nodes are B and E. Least weight path joining B to E is $B-C-E=3$. Weight: 28 Route: (example) $A–B–D–F–E–C–B–C–D–E-D-C-A$	B1 M1 A1 B1	Total weight = 25 (may be implied from weight) B to $E=3$ 28 (cao) A valid closed route that uses BC, CD and DE twice and all other arcs once	**[4]**
	(iii)	$A-B-E-F$ Graph is now Eulerian, so no need to repeat arcs	B1 B1	cao Eulerian (or equivalent)	**[2]**
				Total =	**11**

2	**(i)**	A graph cannot have an odd number of odd vertices (nodes)	B1	Or equivalent (eg 3×5 = 15 ⇒ 7½ arcs) Not from a diagram of a specific case	**[1]**
	(ii)	It has exactly two odd nodes eg *C A B C D E A D*	B1 B1	2 odd nodes A valid semi-Eulerian trail	**[2]**
	(iii)	*AE* = 2 *AC* = 3 *AB* = 5 *CD* = 7 Weight = 17 	B1 B1 B1	Correct tree (vertices must be labelled) Order of choosing arcs in a valid application of Prim, starting at *A* (working shown on a network or matrix) 17	**[3]**
	(iv)	Lower bound = 29 *A – E – D – F – C – B – A* = 34 *F – C – A – E – D* and *F –D – C – A - E* Vertex *B* is missed out	B1 M1 A1 B1	29 or 12 + their tree weight from **(iii)** *A – E – D – F – C –* 34, from correct working seen Correctly explaining why method fails, need to have explicitly considered both cases	**[4]**
					Total = 10

For reference

(ii)

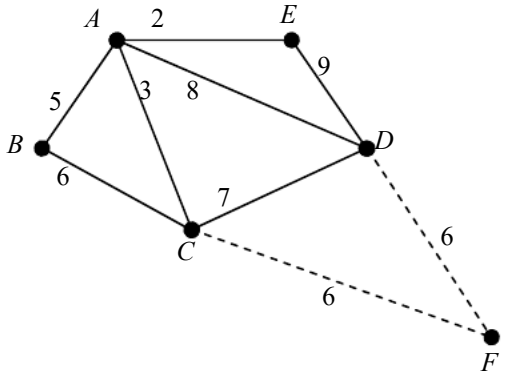


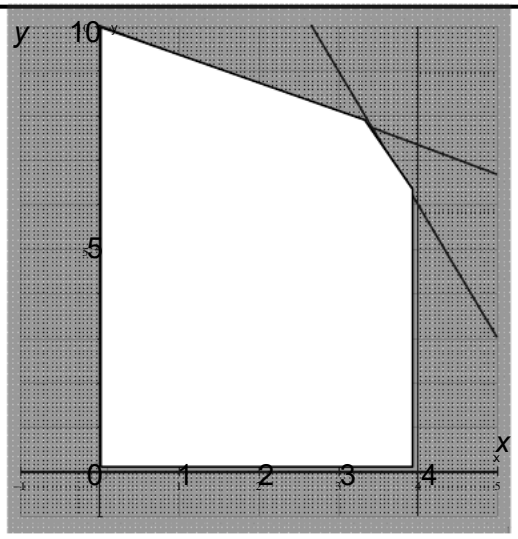


(iii) (iv)

	A	*B*	*C*	*D*	*E*
A	-	5	3	8	2
B	5	-	6	-	-
C	3	6	-	7	-
D	8	-	7	-	9
E	2	-	-	9	-

CF = 6
DF = 6



3 (i)	x = number of clients who use program X y = number of clients who use program Y	B1	Number of clients on X and Y, respectively	**[1]**
(ii)	Spin cycle: $30x + 10y \leq 180$ $\Rightarrow 3x + y \leq 18$ Rower: $10x \leq 40$ $\Rightarrow x \leq 4$ Free weights: $20x + 30y \leq 300$ $\Rightarrow 2x + 3y \leq 30$	B1 B1 B1	$3x + y \leq 18$, or equivalent, simplified $x \leq 4$, or equivalent, simplified $2x + 3y \leq 30$, or equivalent, simplified Allow use of slack variables instead of inequalities	**[3]**
(iii)	Both must take non-negative integer values	B1	Non-negative and integer Accept $x + y \leq 12$ as an alternative answer	**[1]**
(iv)	 	B1 M1 A1	Axes scaled and labelled appropriately (on graph paper) Boundaries of their three constraints shown correctly (non-negativity may be missed) Correct graph with correct shading or feasible region correct and clearly identified (non-negativity may be missed) (cao)	**[3]**
	Checking vertices or using a profit line $(4, 6) \rightarrow 72$ $(3\frac{3}{7}, 7\frac{5}{7}) \rightarrow 77\frac{1}{7}$ or $(24/7, 54/7) \rightarrow 77\frac{1}{7}$ $(0, 10) \rightarrow 60$ $(4, 0) \rightarrow 36$	M1	Follow through their graph if possible $x = 3.4$, $y = 7.7$ may be implied from (3, 8)	
	Checking other feasible integer points near (non-integer) optimum for continuous problem $(3, 8) \rightarrow 75$	M1	Could be implied from identifying point (3, 8) in any form	
	Put 3 clients on program X, 8 on program Y and 1 on program Z	A1	cao, in context and including program Z	**[3]**
			Total =	**11**

4	(i)	*A A A A A A A A A A* *A A A A A D D D D C* *C C B B B B B B B B* Box 1: *A A A A A* Box 2: *A A A A A* Box 3: *A A A A A* Box 4: *D D D D C C C B* *B B B B B B B* Cannot fit all the items into box 4 There is only room for one *B* in a box	B1 M1 M1 A1 B1	15 *A*'s, 4 *D*'s, 3 *C*'s, 8*B*'s (but not just *A D C B*) Three boxes each containing *A A A A A* (or shown using weights) A box containing all the rest Completely correct, including order of packing into boxes Any identification of a (specific) volume conflict	**[5]**
	(ii)	*B B B B B B B B C C* *C D D D D A A A A A* *A A A A A A A A A A* Box 1: *B D A A* Box 2: *B D A A* Box 3: *B D A A* Box 4: *B D A A* Box 5: *B A A A A A A* Box 6: *B A* Box 7: *B* Box 8: *B* Box 9: *C C C* Box 5 is over the weight limit More than five A's is too heavy for one box	B1 M1 M1 A1 B1	8 *B*'s, 3 *C*'s, 4 *D*'s, 15 *A*'s (but not just *B C D A*) Four boxes each containing *B D A A* (in any order) Using exactly 9 boxes, the first eight of which each contain a *B* (with or without other items) and the ninth contains three *C*'s. Completely correct, including order of packing into boxes Any identification of a (specific) weight conflict	**[5]**
	(iii)	Items may be the wrong shape for the boxes eg too tall	B1	Reference to shape, height, etc. but not practical issues connected with the food	**[1]**
					Total = 11

For reference

Item type	*A*	*B*	*C*	*D*
Number to be packed	15	8	3	4
Length (cm)	10	40	20	10
Width (cm)	10	30	50	40
Height (cm)	10	20	10	10
Volume (cm^3)	1 000	24 000	10 000	4 000
Weight (g)	1 000	250	300	400

5 (i)

Minimise $2a - 3b + c + 18$
$\Rightarrow$ minimise $2(20\text{-}x) - 3(10\text{-}y) + (8\text{-}z) + 18$
$\Rightarrow$ minimise $-2x + 3y - z$
$\Rightarrow$ maximise $2x - 3y + z$ (given)

$a + b - c \geq 14$
$\Rightarrow (20\text{-}x) + (10\text{-}y) - (8\text{-}z) \geq 14$
$\Rightarrow x + y - z \leq 8$ (given)

$-2a + 3c \leq 50$
$\Rightarrow -2(20\text{-}x) + 3(8\text{-}z) \leq 50$
$\Rightarrow 2x - 3z \leq 66$ (given)

$10 + 4a \geq 5b$
$\Rightarrow 10 + 4(20\text{-}x) \geq 5(10\text{-}y)$
$\Rightarrow 4x - 5y \leq 40$ (given)

$a \leq 20 \Rightarrow 20\text{-}x \leq 20 \Rightarrow x \geq 0$
$b \leq 10 \Rightarrow 10\text{-}y \leq 10 \Rightarrow y \geq 0$
$c \leq 8 \Rightarrow 8\text{-}z \leq 8 \Rightarrow z \geq 0$

B1 (Constant has no effect on slope of objective) Replacing a, b and c in objective to get $-2x + 3y - z$ (Condone omission of conversion to maximisation here)

M1 Replacing a, b and c in the first three constraints to get given expressions

A1 Showing how $a \leq 20$, $b \leq 10$, $c \leq 8$ give $x \geq 0$, $y \geq 0$, $z \geq 0$ **[3]**

(ii)

P	x	y	z	s	t	u	RHS
1	-2	3	-1	0	0	0	0
0	1	1	-1	1	0	0	8
0	2	0	-3	0	1	0	66
0	4	-5	0	0	0	1	40

M1 Constraint rows correct, with three slack variable columns
A1 Objective row correct **[2]**

x and z columns have negative entries in objective row, but z column has no positive entries in constraint rows, so pivot on x col
$8 \div 1 = 8$; $66 \div 2 = 33$; $40 \div 4 = 10$
Least ratio is $8 \div 1$, so pivot on 1 from x col

M1 Choosing to pivot on x column (may be implied from pivot choice)

A1 Calculations seen or referred to and correct pivot choice made (cao) **[2]**

New row 2 = row 2
New row 1 = row 1 + 2(new row 2)
New row 3 = row 3 – 2(new row 2)
New row 4 = row 4 – 4(new row 2)

M1 Pivot row unchanged (may be implied) or follow through for their +ve pivot

A1 Calculations for other rows shown (cao) **[2]**

P	x	y	z	s	t	u	RHS
1	0	5	-3	2	0	0	16
0	1	1	-1	1	0	0	8
0	0	-2	-1	-2	1	0	50
0	0	-9	4	-4	0	1	8

M1 An augmented tableau with three basis columns, non-negative values in final column and value of objective having not decreased

A1 Correct tableau after one iteration (cao) **[2]**

$x = 8, y = 0, z = 0$

B1 Non-negative values for x, y and z from their tableau

$x = 8 \Rightarrow a = 20 - 8 = 12$
$y = 0 \Rightarrow b = 10 - 0 = 10$
$z = 0 \Rightarrow c = 8 - 0 = 8$

M1 Putting their values for x, y and z into $a = 20 - x$, $b = 10 - y$ and $c = 8 - z$

A1 Correct values for a, b and c, from their non-negative x, y and z **[3]**

(iii) $x \leq 20$, $y \leq 10$ and $z \leq 8$

M1 20, 10, 8
A1 Correct inequalities for x, y and z **[2]**

Total = 16

		TO BE ANSWERED ON INSERT			
6	**(i)**	10 ½$n(n-1)$	B1 B1	10 1+2+…+(n-1) seen, or equivalent Check that sum stops at n-1 not n	**[2]**
	(ii)(a)	9 1 2 3 45	B1 M1 A1	Their 10 minus 1 1, 2 and 3 45 following from method mark earned cao	**[3]**
	(b)	1+2+3+…+(N-1) = ½$N(N-1)$, where N = ½$n(n-1)$ = ¼$n(n-1)$(½$n(n-1) - 1$) (given)	M1 A1	1+2+3+…+(N-1) or ½$N(N-1)$, where N = ½$n(n-1)$ Convincingly achieving the given result	**[2]**
	(iii)	M1 Vertices in tree / M2 Arcs in tree / M3 Vertices not in tree / M4 Sorted list *ABCDE* *D E* | *D* | 2 | *E* | *A B C* | ~~*D* | 2 | *E*~~ *D E A* | *D* | 2 | *E*, *A* | 3 | *E* | *B C* | ~~*A* | 3 | *E*~~, ~~*A* | 4 | *C*~~ *D E A C* | *D* | 2 | *E*, *A* | 3 | *E*, *A* | 4 | *C* | *B* | ~~*C* | 5 | *D*~~, ~~*B* | 6 | *E*~~, ~~*B* | 7 | *C*~~ *DEACB* | *D* | 2 | *E*, *A* | 3 | *E*, *A* | 4 | *C*, *B* | 6 | *E* | ~~*A* | 8 | *B*~~, ~~*C* | 9 | *E*~~	 M1 M1 M1 A1	(Order of entries in *M1*, *M2* and *M3* does not matter) Arc *A* | 3 | *E* is added to *M2*, *A* is added to *M1* and deleted from *M3* Arc *A* | 4 | *C* is added to *M2*, *C* is added to *M1* and deleted from *M3* Arc *C* | 5 | *D* is not added to *M2* and arc *B* | 6 | *E* is added to *M2* cao (lists *M*1, *M*2 and *M*3 totally correct, ignore what is done in list *M*4).	**[4]**
	(iv)	$30\times\left(\frac{500}{100}\right)^4$ = 18750 seconds	M1 A1	Or equivalent cao, with units (312 min 30 sec or 5 hours 12 min 30 sec)	**[2]**
				Total =	**13**