RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE UNIT **4736/01**

MATHEMATICS

Decision Mathematics 1

TUESDAY 23 JANUARY 2007 Afternoon

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- There is an **insert** for use in Questions **4** and **5**.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **7** printed pages, **1** blank page and an insert.

1 An airline allows each passenger to carry a maximum of 25 kg in luggage. The four members of the Adams family have bags of the following weights (to the nearest kg):

Mr Adams:	10	4	2			
Mrs Adams:	13	3	7	5	2	4
Sarah Adams:	5	8	2	5		
Tim Adams:	10	5	3	5	3	

The bags need to be grouped into bundles of 25 kg maximum so that each member of the family can carry a bundle of bags.

(i) Use the first-fit method to group the bags into bundles of 25 kg maximum. Start with the bags belonging to Mr Adams, then those of Mrs Adams, followed by Sarah and finally Tim. [3]

(ii) Use the first-fit decreasing method to group the same bags into bundles of 25 kg maximum. [3]

(iii) Suggest a reason why the grouping of the bags in part **(i)** might be easier for the family to carry. [1]

2 A baker can make apple cakes, banana cakes and cherry cakes.

The baker has exactly enough flour to make either 30 apple cakes or 20 banana cakes or 40 cherry cakes.

The baker has exactly enough sugar to make either 30 apple cakes or 40 banana cakes or 30 cherry cakes.

The baker has enough apples for 20 apple cakes, enough bananas for 25 banana cakes and enough cherries for 10 cherry cakes.

The baker has an order for 30 cakes.

The profit on each apple cake is 4p, on each banana cake is 3p and on each cherry cake is 2p. The baker wants to maximise the profit on the order.

(i) The availability of flour leads to the constraint $4a + 6b + 3c \leqslant 120$. Give the meaning of each of the variables a, b and c in this constraint. [2]

(ii) Use the availability of sugar to give a second constraint of the form $Xa + Yb + Zc \leqslant 120$, where X, Y and Z are numbers to be found. [2]

(iii) Write down a constraint from the total order size. Write down constraints from the availability of apples, bananas and cherries. [3]

(iv) Write down the objective function to be maximised. [1]

[You are **not** required to solve the resulting LP problem.]

3 A *simple* graph is one in which any two vertices are directly connected by at most one arc and no vertex is directly connected to itself.

A *connected* graph is one in which every vertex is connected, directly or indirectly, to every other vertex.

A *simply connected* graph is one that is both simple and connected.

(i) A simply connected graph is drawn with 6 vertices and 9 arcs.

(a) What is the sum of the orders of the vertices? [1]

(b) Explain why if the graph has two vertices of order 5 it cannot have any vertices of order 1. [2]

(c) Explain why the graph cannot have three vertices of order 5. [2]

(ii) Draw an example of a simply connected graph with 6 vertices and 9 arcs in which one of the vertices has order 5 and all the orders of the vertices are odd numbers. [2]

(iii) Draw an example of a simply connected graph with 6 vertices and 9 arcs that is also Eulerian. [2]

4 **Answer this question on the insert provided.**

The table shows the distances, in units of 100 m, between seven houses, A to G.

	A	B	C	D	E	F	G
A	0	4	5	3	2	5	6
B	4	0	1	2	4	7	6
C	5	1	0	3	4	6	7
D	3	2	3	0	2	6	4
E	2	4	4	2	0	6	6
F	5	7	6	6	6	0	10
G	6	6	7	4	6	10	0

(i) Use Prim's algorithm on the table in the insert to find a minimum spanning tree. Start by crossing out row A. Show which entries in the table are chosen and indicate the order in which the rows are deleted. Draw your minimum spanning tree and state its total weight. [6]

Harry is an estate agent. He must visit each of the houses A to G to photograph them. The distances, in units of 100 m, from Harry's office (H) to each of the houses are listed below.

House	A	B	C	D	E	F	G
Distance from H	12	14	15	15	13	16	16

Harry wants to find the shortest route that starts at his office and visits each of the houses before returning to his office.

(ii) Which standard network problem does Harry need to solve? [1]

(iii) Use your answer from part **(i)** to calculate a lower bound for the length of Harry's route, showing all your working. [3]

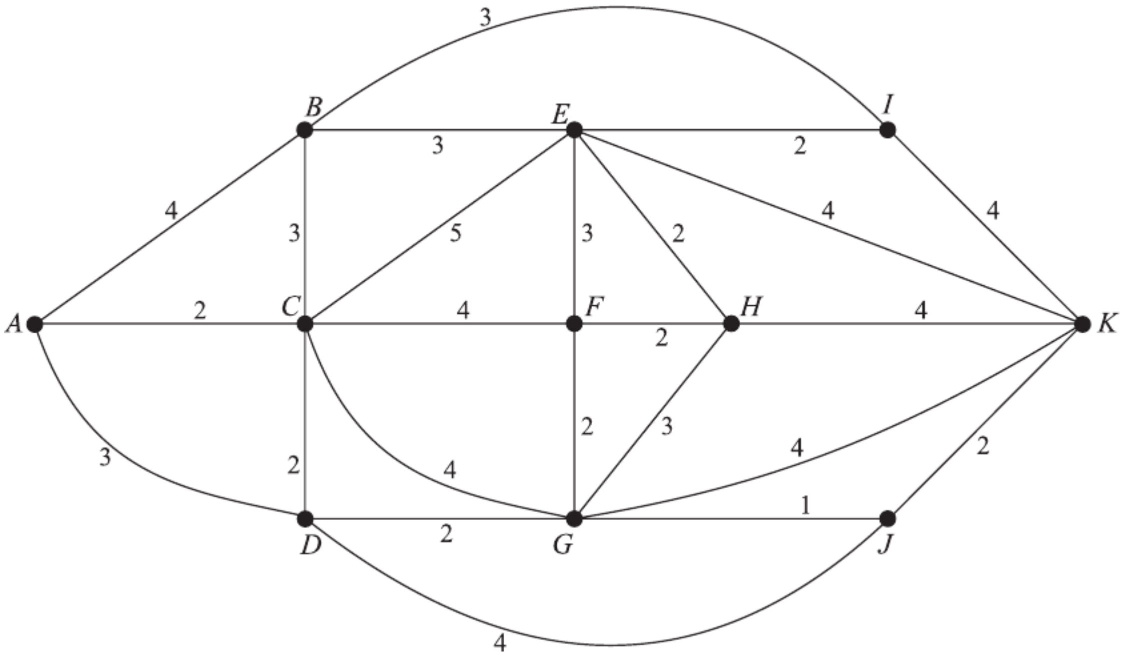
(iv) Use the nearest neighbour method, starting from Harry's office, to find a tour that visits each of the houses. Hence find an upper bound for the length of Harry's route. [4]

5 **Answer part (i) of this question on the insert provided.**

Rhoda Raygh enjoys driving but gets extremely irritated by speed cameras.

The network represents a simplified map on which the arcs represent roads and the weights on the arcs represent the numbers of speed cameras on the roads.

The sum of the weights on the arcs is 72.



(i) Rhoda lives at Ayton (A) and works at Kayton (K). Use Dijkstra's algorithm on the diagram in the insert to find the route from A to K that involves the least number of speed cameras and state the number of speed cameras on this route. [7]

(ii) In her job Rhoda has to drive along each of the roads represented on the network to check for overhanging trees. This requires finding a route that covers every arc at least once, starting and ending at Kayton (K). Showing all your working, find a suitable route for Rhoda that involves the least number of speed cameras and state the number of speed cameras on this route. [6]

(iii) If Rhoda checks the roads for overhanging trees on her way home, she will instead need a route that covers every arc at least once, starting at Kayton and ending at Ayton. Calculate the least number of speed cameras on such a route, explaining your reasoning. [3]

6 Consider the linear programming problem:

$$\begin{aligned} &\text{maximise} && P = 3x - 5y + 4z, \\ &\text{subject to} && x + 2y - 3z \leqslant 12, \\ & && 2x + 5y - 8z \leqslant 40, \\ &\text{and} && x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0. \end{aligned}$$

(i) Represent the problem as an initial Simplex tableau. [3]

(ii) Explain why it is not possible to pivot on the z column of this tableau. Identify the entry on which to pivot for the first iteration of the Simplex algorithm. Explain how you made your choice of column and row. [3]

(iii) Perform **one** iteration of the Simplex algorithm. Write down the values of x, y and z after this iteration. [3]

(iv) Explain why P has no finite maximum. [1]

The coefficient of z in the objective is changed from +4 to −40.

(v) Describe the changes that this will cause to the initial Simplex tableau and the tableau that results after one iteration. What is the maximum value of P in this case? [4]

Now consider this linear programming problem:

$$\begin{aligned} &\text{maximise} && Q = 3x - 5y + 7z, \\ &\text{subject to} && x + 2y - 3z \leqslant 12, \\ & && 2x - 7y + 10z \leqslant 40, \\ &\text{and} && x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0. \end{aligned}$$

Do **not** use the Simplex algorithm for these parts.

(vi) By adding the two constraints, show that Q has a finite maximum. [1]

(vii) There is an optimal point with $y = 0$. By substituting $y = 0$ in the two constraints, calculate the values of x and z that maximise Q when $y = 0$. [3]

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MATHEMATICS

Decision Mathematics 1 INSERT for Questions 4 and 5

TUESDAY 23 JANUARY 2007 Afternoon

Time: 1 hour 30 minutes

Candidate Name	

Centre Number						Candidate Number					

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the boxes above.
- This insert should be used to answer Questions **4** and **5 (i)**.
- Write your answers to Questions **4** and **5 (i)** in the spaces provided in this insert, and attach it to your answer booklet.

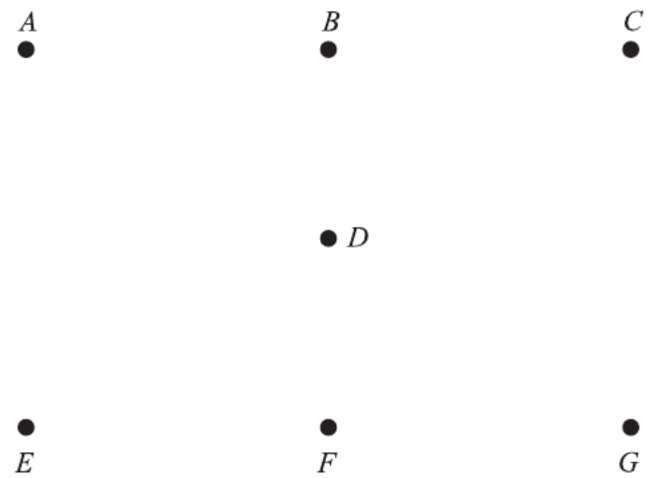
This document consists of **3** printed pages and **1** blank page.

4 **(i)**

	↓ *A*	*B*	*C*	*D*	*E*	*F*	*G*
A	0	4	5	3	2	5	6
B	4	0	1	2	4	7	6
C	5	1	0	3	4	6	7
D	3	2	3	0	2	6	4
E	2	4	4	2	0	6	6
F	5	7	6	6	6	0	10
G	6	6	7	4	6	10	0

Order in which rows were deleted: ....*A*..

Minimum spanning tree:



Total weight: ....

(ii) ....

(iii) ....

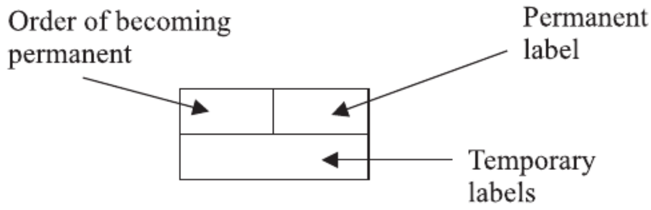
Lower bound: ....

(iv) Tour: ....

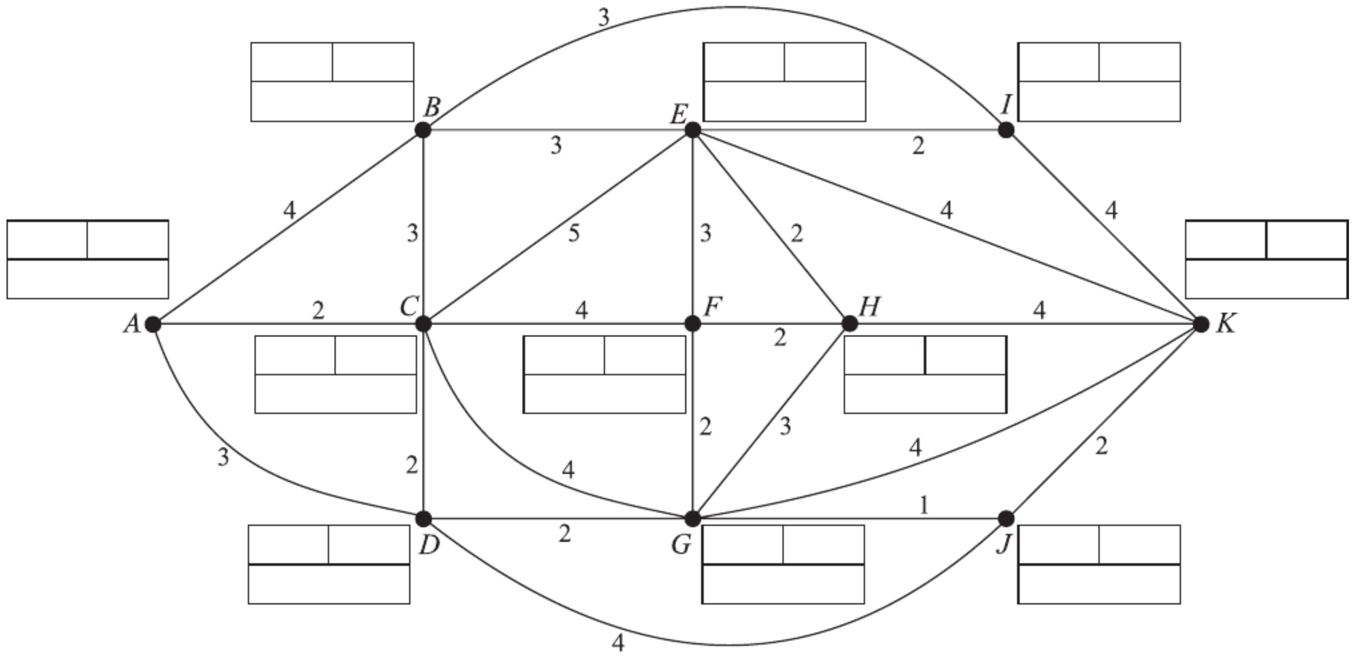
Upper bound: ....

5 (i)

Key:



Do not cross out your working values (temporary labels)



Route: ....

Number of speed cameras on route: ....

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