Pearson Edexcel

Mark Scheme (Results)

Summer 2019

Pearson Edexcel GCE In Statistics 4

Paper 6686/01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- $\sqsubset$ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer

Question Number	Scheme	Marks
1. (a)	$H_0: \sigma_x^2 = \sigma_y^2$ $\quad H_1: \sigma_x^2 \neq \sigma_y^2$	B1
	$s_x^2 = \frac{1}{8}\left(4202 - \frac{182^2}{9}\right) = 65.19\dot{4}$ $\left(= \frac{2347}{36}\right)$ $s_y^2 = \frac{1}{11}\left(8396 - \frac{310^2}{12}\right) = 35.\dot{2}\dot{4}$ $\left(= \frac{1163}{33}\right)$	M1
	$F_{8,11} = \frac{65.19\dot{4}}{35.\dot{2}\dot{4}} = 1.85$ (calc 1.8498…)	M1 A1
	$F_{8,11} = 2.95$	B1
	There is no evidence that the variances are different	A1
		(6)
(b)	$H_0: \mu_x = \mu_y$ $\quad H_1: \mu_x \neq \mu_y$	B1
	$s_p^2 = \frac{8\times 65.19\dot{4} + 11\times 35.\dot{2}\dot{4}}{19} = 47.85...$ or $s_p = 6.917...$	M1 A1
	$(t_{19} =)(\pm)\frac{20.\dot{2} - 25.8\dot{3}}{s_p\sqrt{\frac{1}{9}+\frac{1}{12}}} = (\pm)1.839...$	M1 A1
	Critical value $t_{19}(5\%) = \pm 2.093$	B1
	There is no evidence to reject H_0 There is no difference in the mean times taken on the two routes	A1
		(7)
(c)	Test in part (b) requires the variances to be equal. The test in part (a) showed that the variances could be assumed to be equal	B1
		(1)
		Total 14

Notes	
(a)	**B1** both hypotheses **M1** for $s_x^2 = \frac{1}{8}\left(4202 - \frac{182^2}{9}\right)$ or $s_y^2 = \frac{1}{11}\left(8396 - \frac{310^2}{12}\right)$ (correct expressions) **M1** use of $\frac{s_x^2}{s_y^2}$ **A1** awrt 1.85 **B1** for $F_{8,11} = 2.95$ **A1** for a correct contextual statement
(b)	**B1** both hypotheses **M1** correct use of formula for s_p^2 (ft their s_x^2 or their s_y^2) **A1** awrt 47.9 or awrt 6.92 **M1** correct use of formula with their s_p **A1** awrt 1.84 $(\pm)$ **B1** for $t_{19}(5\%) = \pm 2.093$ (compatible signs) **A1** for a correct contextual statement
(c)	**B1** for test in part (b) requires the variances to be equal. The test in part (a) showed that the variances could be assumed to be equal. Dependent on a not significant result in (a)

Question Number	Scheme	Marks
2. (a)	d: 5 1 3 0 2 4 0 4 3 2	M1
	$\bar{d} = \frac{24}{10} = 2.4$	M1
	$s_d^2 = \frac{84 - 10 \times 2.4^2}{9} = 2.9333...$	M1
	$H_0 : \mu_d = 0 \qquad H_1 : \mu_d > 0$	B1
	$t = \pm \frac{2.4 - 0}{\sqrt{\frac{2.92}{10}}} = 4.43$	M1 A1
	$t_9(1\%) = 2.821$	B1
	There is evidence to reject H_0. There is sufficient evidence to support the athlete's claim	A1 ft
		(8)
(b)	The **differences** in times are **normally** distributed	B1
		(1)
		Total 9

Notes

(a)	**M1** for attempting to find differences **M1** for $\bar{d}$ **M1** for s_d or s_d^2 **B1** both hypotheses correct in terms of μ_d or μ_d^2 $\mu_x - \mu_y$ etc is B0 unless a correct test statistics is achieved **M1** for use of $\frac{\bar{d}}{\left(\frac{s_d}{\sqrt{10}}\right)}$ **A1** awrt 4.43 **B1** awrt 2.82 (sign should match H_1) **A1** ft for a correct comment in context
(b)	**B1** for a comment that mentions "differences" and "normal" distribution

Question Number	Scheme	Marks
3.(a)	$E(X) = np \quad E(Y) = 2np$	B1
	$E(\hat{T}_1) = \frac{3E(Y) - 2E(X)}{4n} = \frac{6np - 2np}{4n} = p$ *	M1
	$E(\hat{T}_2) = \frac{5E(X) + 3E(Y)}{11n} = \frac{5np + 6np}{11n} = p$ *	M1 A1cso
		(4)
(b)	$\text{Var}(X) = np(1-p) \quad \text{Var}(Y) = 2np(1-p)$	B1
	$\text{Var}(\hat{T}_1) = \frac{9 \times 2np(1-p) + 4 \times np(1-p)}{16n^2} = \frac{11p(1-p)}{8n}$	M1
	$\text{Var}(\hat{T}_2) = \frac{25 \times np(1-p) + 9 \times 2np(1-p)}{121n^2} = \frac{43p(1-p)}{121n}$	M1 A1
		(4)
(c)	$\hat{T}_2$ is the better estimator. It has a smaller variance.	B1 ft
		(1)
		Total 9

Notes

(a)	B1 for use of $E(X) = np$ and $E(Y) = 2np$ M1 for use of $\frac{3E(Y) - 2E(X)}{4n}$ M1 for use of $\frac{5E(X) + 3E(Y)}{11n}$ A1 for both solutions correct with no incorrect working seen
(b)	B1 for use of $\text{Var}(X) = np(1-p)$ and $\text{Var}(Y) = 2np(1-p)$ M1 for use of $\frac{9\text{Var}(Y) + 4\text{Var}(X)}{16n^2}$ M1 for use of $\frac{25\text{Var}(X) + 9\text{Var}(Y)}{121n^2}$ A1 for $\frac{11p(1-p)}{8n}$ and $\frac{43p(1-p)}{121n}$
(c)	B1 ft for identifying the estimator with the smallest variance and stating that it has the smallest variance

Question Number	Scheme	Marks
4(a)	χ^2 value = 19.023 or 2.700	B1
	$\frac{9s^2}{19.023} = 12.31$ or $\frac{9s^2}{2.7} = 86.7$	M1
	$s^2 = 26.01$ <u>or</u> 26.0192…	M1
	$s = 5.1$*	A1 cso
		(4)
(b)	t-value = 3.250	B1
	$85 \pm 3.250 \times \frac{5.1}{\sqrt{10}}$	M1
	(79.8, 90.2)	A1 A1
		(4)
		Total 8

Notes

(a)	**B1** for 19.023 or awrt 2.7 **M1** for $\frac{9s^2}{\chi^2 \text{ value}} = 12.31$ or $\frac{9s^2}{\chi^2 \text{value}} = 86.7$ **M1** for solving either equation leading to $s^2 = \ldots$ **A1** for a correct solution with no incorrect working seen
(b)	**B1** for 3.250 **M1** use of $85 \pm t\text{-value} \times \frac{5.1}{\sqrt{10}}$ (Allow 8.5 for 85 to achieve M1) **A1** awrt 79.8 **A1** awrt 90.2

Question Number	Scheme	Marks
5.(a)	$\Sigma w = 852 \quad \Sigma w^2 = 60578$	B1
	$\bar{w} = 71 \quad s_w^2 = \frac{86}{11}$	M1 A1
	$H_0: \mu = 72 \quad H_1: \mu < 72$	B1
	$t_{11}(5\%) = -1.796$	B1
	$t = \frac{71-72}{\sqrt{\frac{\left(\frac{86}{11}\right)}{12}}} = -1.24$	M1 A1
	Not significant. There is not sufficient evidence that the mean weight of Great Dane dogs is less than 72 kg	A1
		(8)
(b)	$H_0: \sigma^2 = 36 \quad H_1: \sigma^2 \neq 36 \quad [H_0: \sigma = 6 \quad H_1: \sigma \neq 6]$	B1
	$\chi_{11}^2(0.975) = 3.816$	B1
	Critical region $\frac{(n-1)s^2}{\sigma^2} \sim \chi_{11}^2$ test statistic = 2.39	M1 A1
	2.39 is in the critical region. There is evidence that the standard deviation of the weight of Great Dane dogs is not 6 kg	A1
		(5)
		Total 13

Notes

(a)	**B1** for both $\Sigma w = 852$ and $\Sigma w^2 = 60578$
	M1 for either $\bar{w} = 71$ (allow $\frac{852}{12}$) or $s_w^2 = \frac{86}{11}$ or $s_w = 2.796...$
	A1 for both correct
	B1 both hypotheses correct
	B1 for $t_{11}(5\%) = -1.796$ (accept awrt -1.80)
	M1 for attempting the correct test statistic
	A1 for awrt -1.24
	A1 for a correct contextual statement
(b)	**B1** both hypotheses correct
	B1 for $\chi_{11}^2(0.975) = 3.816$ and $[\chi_{11}^2(0.025) = 21.920]$
	M1 for a correct test statistic
	A1 awrt 2.39
	A1 for a correct contextual statement

Question Number	Scheme	Marks
6.(a)	$H_0 : \mu = 38 \quad H_1 : \mu < 38$	B1
		(1)
(b)	5% critical value for z is –1.6449	B1
	Reject H_0 if $\dfrac{\bar{X}-38}{\frac{2.5}{\sqrt{20}}} \le -1.6449$	M1
	$\bar{X} \le \dfrac{2.5}{\sqrt{20}} \times -1.6449 + 38$	dM1
	Critical region is $\bar{X} \le 37.080$	A1
		(4)
(c)	$P\left(Z > \dfrac{37.08-37.5}{\sqrt{\frac{2.5^2}{20}}}\right) = P(Z > -0.75...)$	M1 M1
	$P(\bar{X} > 37.08) = 0.7734$ (calc 0.77351…)	A1
		(3)
(d)	Reject H_0 if $P\left(\bar{X}_n < d \mid \mu = 38\right) < 0.05$	
	$\dfrac{d-38}{\frac{2.5}{\sqrt{n}}} = -1.6449$	M1
	$d = 38 - 1.6449 \times \dfrac{2.5}{\sqrt{n}}$	A1
	Require $P\left(\bar{X}_n > d \mid \mu = 37.5\right) < 0.2$	
	$P\left(Z > \dfrac{d-37.5}{\frac{2.5}{\sqrt{n}}}\right) < 0.2$	dM1
	$\dfrac{d-37.5}{\frac{2.5}{\sqrt{n}}} > 0.8416$	B1
	$\dfrac{38 - 1.6449 \times \frac{2.5}{\sqrt{n}} - 37.5}{\frac{2.5}{\sqrt{n}}} > 0.8416$	A1
	$\sqrt{n} > 12.4325$	M1
	n = 155	A1
		(7)
		Total 15

	Notes
(a)	**B1** for both hypotheses
(b)	**B1** for -1.6449 (allow $\pm$) **M1** for use of $\dfrac{\overline{X}-\mu}{\frac{\sigma}{\sqrt{n}}} \le -1.6449$ or $-z$ where $-2.5 < z < -1.5$ (Must have correct sign and $<$ or $\le$) **M1** for rearranging to the form $\overline{X} \le \ldots$ **A1** allow any letter for $\bar{X}$ but do not allow probability statements
(c)	**M1** for standardising using their 37.08 and $\mu = 37.5$ and $\frac{2.5}{\sqrt{20}}$ **dM1** $\dfrac{37.08-37.5}{\sqrt{\frac{2.5^2}{20}}}$ (may be implied by $-0.75\ \ldots$) <u>and</u> attempt a prob > 0.5 **A1** awrt 0.773 or 0.774
(d)	**M1** for use of $\dfrac{d-\mu}{\frac{\sigma}{\sqrt{n}}} = -1.6449$ **A1** for $d = 38 - 1.6449 \times \dfrac{2.5}{\sqrt{n}}$ **dM1** for $P\left(Z > \dfrac{d-37.5}{\frac{2.5}{\sqrt{n}}}\right) < 0.2$ or $\dfrac{d-37.5}{\frac{2.5}{\sqrt{n}}} > 0.8416$ or z where $0.8 < \lvert z \rvert < 0.9$ Allow equality rather than inequality here but d must be a function of n **B1** for 0.8416 used correctly. Allow their d to be a value but should be + 0.8416 and $\frac{2.5}{\sqrt{n}}$ **A1** for $\dfrac{38-1.6449\times\frac{2.5}{\sqrt{n}}-37.5}{\frac{2.5}{\sqrt{n}}} > 0.8416$ (Must have correct inequality from here onwards) **M1** for solving as far as $\sqrt{n} = \ldots$ or $n = \ldots$ **A1** for n = 155

Question Number	Scheme		Marks
7.(a)	$\frac{1}{2}\times 10\times 10k = 1$ so $k=\frac{1}{50}$	$\int_0^{10} kx\,dx = 1$ so $k = \frac{1}{50}$	B1
	$\frac{1}{2}k(10+9.5)\times 0.5 = 0.0975$	$\int_{9.5}^{10}\frac{1}{50}x\,dx = \left[\frac{x^2}{100}\right]_{9.5}^{10} = 0.0975$	M1 A1
			(3)
(b)	$\frac{1}{2}(2k+12k)\times 10 = 1$	$\int_2^{12} kx\,dx = 1$ so $\left[\frac{1}{2}kx^2\right]_2^{12} = 1$	M1
	$k=\frac{1}{70}$	$k=\frac{1}{70}$	A1
	$\frac{1}{2}\left(\frac{12}{70}+\frac{9.5}{70}\right)\times 2.5 = 0.3839\ ...$	$\int_{9.5}^{12}\frac{1}{70}x\,dx = \left[\frac{x^2}{140}\right]_{9.5}^{12} = 0.3839\ ...$	dM1 A1
			(4)
			Total 7

Notes

(a)	B1 for $k=\frac{1}{50}$ M1 for $\frac{1}{2}k(10+9.5)\times 0.5$ or $\int_{9.5}^{10} kx\,dx = \left[\frac{x^2}{2}k\right]_{9.5}^{10}$ or $1-\int_0^{9.5} kx\,dx = 1-\left[\frac{x^2}{2}k\right]_0^{9.5}$ or $1-45.125k$ or $4.875k$ A1 for 0.0975
(b)	M1 for $\frac{1}{2}(2k+12k)\times 10 = 1$ or $\int_2^{12} kx\,dx = 1$ so $\left[\frac{1}{2}kx^2\right]_2^{12} = 1$ A1 for $k=\frac{1}{70}$ dM1 for $\frac{1}{2}\left(\frac{12}{70}+\frac{9.5}{70}\right)\times 2.5$ or $\int_{9.5}^{12}\frac{1}{70}x\,dx = \left[\frac{x^2}{140}\right]_{9.5}^{12}$ or $1-\frac{86.25}{2}\times$"k" Must have attempted a new k and have a value for it A1 awrt 0.384

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