

6. Emily is monitoring the level of pollution in a river. Over a period of time she has found that the amount of pollution, X , in a 100 ml sample of river water has a continuous distribution with probability density function $f(x)$ given by

$$f(x) = \begin{cases} \frac{2x}{a^2} & 0 \leq x \leq a \\ 0 & \text{otherwise} \end{cases}$$

where a is a constant.

Emily takes a random sample $X_1, X_2, X_3, \dots, X_n$ to try to estimate the value of a .

- (a) Show that $E(\bar{X}) = \frac{2a}{3}$ and $\text{Var}(\bar{X}) = \frac{a^2}{18n}$ (4)

The random variable $S = p\bar{X}$, where p is a constant, is an unbiased estimator of a .

- (b) Write down the value of p and find $\text{Var}(S)$. (2)

Felix suggests using the statistic $M = \max\{X_1, X_2, X_3, \dots, X_n\}$ as an estimator of a .

He calculates $E(M) = \frac{2n}{2n+1}a$ and $\text{Var}(M) = \frac{n}{(n+1)(2n+1)^2}a^2$

- (c) State, giving your reasons, whether or not M is a consistent estimator of a . (3)

The random variable $T = qM$, where q is a constant, is an unbiased estimator of a .

- (d) Write down, in terms of n , the value of q and find $\text{Var}(T)$. (3)

- (e) State, giving your reasons, which of S or T you would recommend Emily use as an estimator of a . (3)

Emily took a sample of 5 values of X and obtained the following:

5.3 4.3 5.7 7.8 6.9

- (f) Calculate the estimate of a using your recommended estimator from part (e). (2)

- (g) Find the standard error of your estimate, giving your answer to 2 decimal places. (2)

