

2. (a) Define

(i) a Type I error,

(ii) a Type II error.

(2)

Rolls of material, manufactured by a machine, contain defects at a mean rate of 6 per roll.

The machine is modified. A single roll is selected at random and a test is carried out to see whether or not the mean number of defects per roll has decreased. The significance level is chosen to be as close as possible to 5%.

(b) Calculate the probability of a Type I error for this test.

(3)

(c) Given that the true mean number of defects per roll of material made by the machine is now 4, calculate the probability of a Type II error.

(2)



3. A large number of chicks were fed a special diet for 10 days. A random sample of 9 of these chicks is taken and the weight gained, x grams, by each chick is recorded. The results are summarised below.

$$\sum x = 181 \quad \sum x^2 = 3913$$

You may assume that the weights gained by the chicks are normally distributed.

Calculate a 95% confidence interval for

- (a) (i) the mean of the weights gained by the chicks,
(ii) the variance of the weights gained by the chicks.

(10)

A chick which gains less than 16 g has to be given extra feed.

- (b) Using appropriate confidence limits from part (a), find the lowest estimate of the proportion of chicks that need extra feed.

(4)

Question 3 continued

Lined writing area for the answer to Question 3.



4. A random sample of 8 people were given a new drug designed to help people sleep.

In a two-week period the drug was given for one week and a placebo (a tablet that contained no drug) was given for one week.

In the first week 4 people, selected at random, were given the drug and the other 4 people were given the placebo. Those who were given the drug in the first week were given the placebo in the second week. Those who were given the placebo in the first week were given the drug in the second week.

The mean numbers of hours of sleep per night for each of the people are shown in the table.

Person	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>
Hours of sleep with drug	10.8	7.2	8.7	6.8	9.4	10.9	11.1	7.6
Hours of sleep with placebo	10.0	6.5	9.0	5.6	8.7	8.0	9.8	6.8

(a) State one assumption that needs to be made in order to carry out a paired *t*-test. (1)

(b) Stating your hypotheses clearly, test, at the 1% level of significance, whether or not the drug increases the mean number of hours of sleep per night by more than 10 minutes. State the critical value for this test. (8)

5. A statistician believes a coin is biased and the probability, p , of getting a head when the coin is tossed is less than 0.5

The statistician decides to test this by tossing the coin 10 times and recording the number, X , of heads. He sets up the hypotheses $H_0 : p = 0.5$ and $H_1 : p < 0.5$ and rejects the null hypothesis if $x < 3$

- (a) Find the size of the test. (1)

- (b) Show that the power function of this test is

$$(1 - p)^8 (36p^2 + 8p + 1) \quad (3)$$

Table 1 gives values, to 2 decimal places, of the power function for the statistician's test.

p	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45
Power	0.93	0.82	r	0.53	0.38	0.26	s	0.10

Table 1

- (c) Calculate the value of r and the value of s . (2)

Question 5 parts (d) and (e) continue on page 16

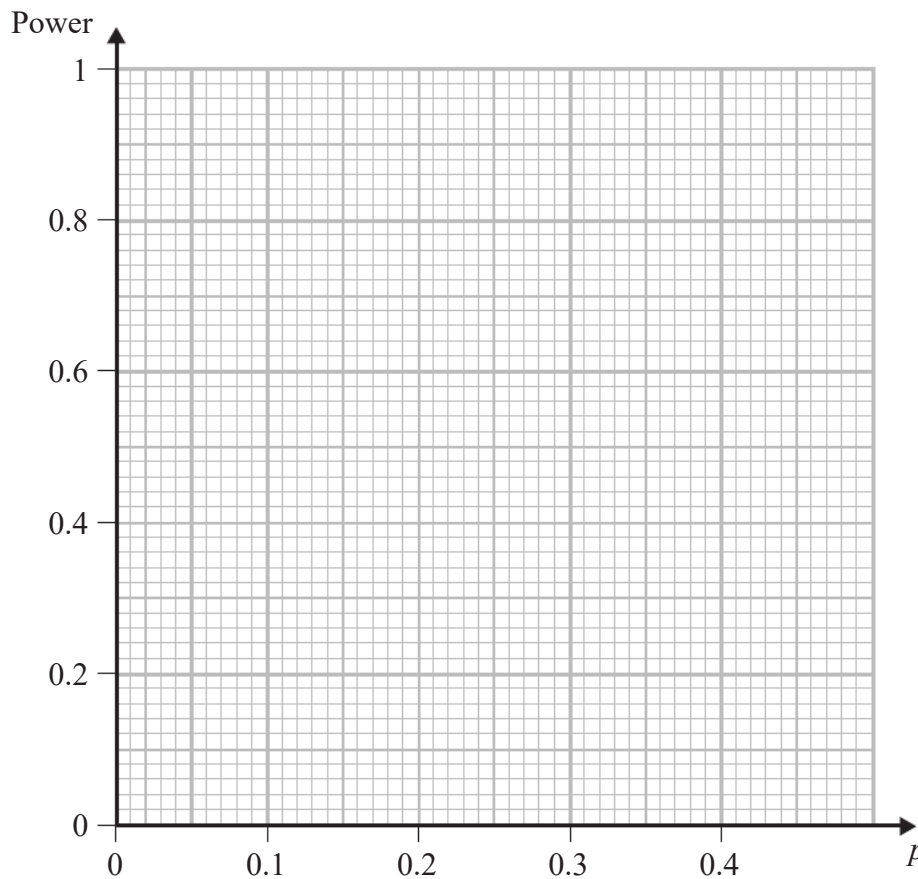
Question 5 continued

For your convenience Table 1 is repeated here.

p	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45
Power	0.93	0.82	r	0.53	0.38	0.26	s	0.10

Table 1

- (d) On the axes below draw the graph of the power function for the statistician's test. (2)
- (e) Find the range of values of p for which the probability of accepting the coin as unbiased, when in fact it is biased, is less than or equal to 0.4 (3)



Question 5 continued

Lined writing area for the answer.

(Total 11 marks)

Q5

6. (a) Explain what is meant by the sampling distribution of an estimator T of the population parameter θ . (1)

(b) Explain what you understand by the statement that T is a biased estimator of θ . (1)

A population has mean μ and variance σ^2

A random sample X_1, X_2, \dots, X_{10} is taken from this population.

(c) Calculate the bias of each of the following estimators of μ .

$$\hat{\mu}_1 = \frac{X_3 + X_5 + X_7}{3}$$

$$\hat{\mu}_2 = \frac{5X_1 + 2X_2 + X_9}{6}$$

$$\hat{\mu}_3 = \frac{3X_{10} - X_1}{3} \quad (4)$$

(d) Find the variance of each of these three estimators. (6)

(e) State, giving a reason, which of these three estimators for μ is

(i) the best estimator,

(ii) the worst estimator. (3)

Question 6 continued

Lined area for writing the answer to Question 6 continued.

7. Two groups of students take the same examination.

A random sample of students is taken from each of the groups.

The marks of the 9 students from Group 1 are as follows

$$30 \quad 29 \quad 35 \quad 27 \quad 23 \quad 33 \quad 33 \quad 35 \quad 28$$

The marks, x , of the 7 students from Group 2 gave the following statistics

$$\bar{x} = 31.29 \qquad s^2 = 12.9$$

A test is to be carried out to see whether or not there is a difference between the mean marks of the two groups of students.

You may assume that the samples are taken from normally distributed populations and that they are independent.

(a) State **one** other assumption that must be made in order to apply this test and show that this assumption is reasonable by testing it at a 10% level of significance. State your hypotheses clearly. (7)

(b) Stating your hypotheses clearly, test, using a significance level of 5%, whether or not there is a difference between the mean marks of the two groups of students. (7)



