







3. Manuel is planning to buy a new machine to squeeze oranges in his cafe and he has two models, at the same price, on trial. The manufacturers of machine *B* claim that their machine produces more juice from an orange than machine *A*. To test this claim Manuel takes a random sample of 8 oranges, cuts them in half and puts one half in machine *A* and the other half in machine *B*. The amount of juice, in ml, produced by each machine is given in the table below.

Orange	1	2	3	4	5	6	7	8
Machine <i>A</i>	60	58	55	53	52	51	54	56
Machine <i>B</i>	61	60	58	52	55	50	52	58

Stating your hypotheses clearly, test, at the 10% level of significance, whether or not the mean amount of juice produced by machine *B* is more than the mean amount produced by machine *A*.

(8)

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4. A proportion  $p$  of letters sent by a company are incorrectly addressed and if  $p$  is thought to be greater than 0.05 then action is taken.

Using  $H_0: p = 0.05$  and  $H_1: p > 0.05$ , a manager from the company takes a random sample of 40 letters and rejects  $H_0$  if the number of incorrectly addressed letters is more than 3.

(a) Find the size of this test. (2)

(b) Find the probability of a Type II error in the case where  $p$  is in fact 0.10 (2)

Table 1 below gives some values, to 2 decimal places, of the power function of this test.

$p$	0.075	0.100	0.125	0.150	0.175	0.200	0.225
Power	0.35	$s$	0.75	0.87	0.94	0.97	0.99

**Table 1**

(c) Write down the value of  $s$ . (1)

A visiting consultant uses an alternative system to test the same hypotheses. A sample of 15 letters is taken. If these are all correctly addressed then  $H_0$  is accepted. If 2 or more are found to have been incorrectly addressed then  $H_0$  is rejected. If only one is found to be incorrectly addressed then a further random sample of 15 is taken and  $H_0$  is rejected if 2 or more are found to have been incorrectly addressed in this second sample, otherwise  $H_0$  is accepted.

(d) Find the size of the test used by the consultant. (3)

**Question 4 continues on page 8**

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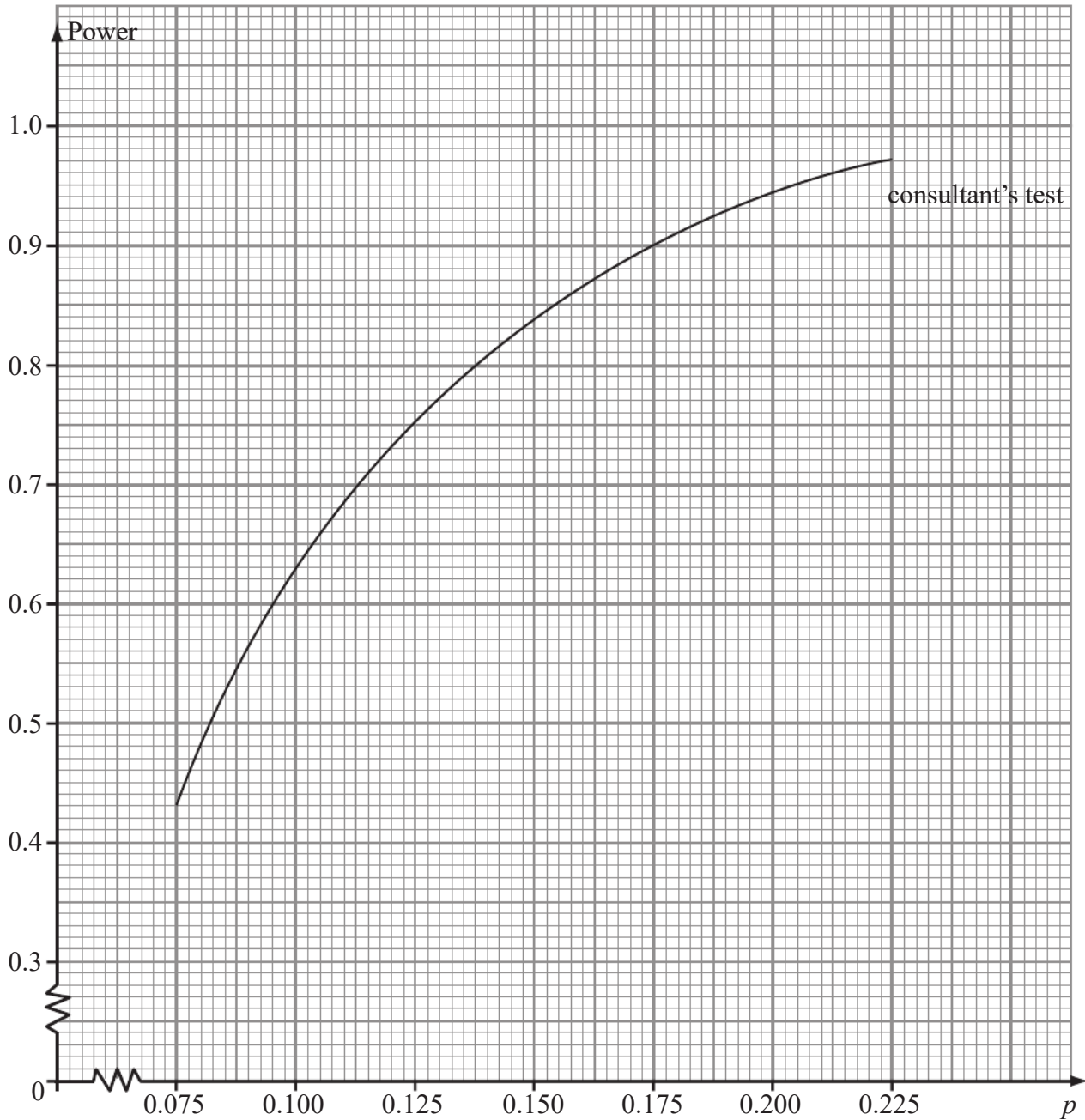


For your convenience Table 1 is repeated here

$p$	0.075	0.100	0.125	0.150	0.175	0.200	0.225
Power	0.35	$s$	0.75	0.87	0.94	0.97	0.99

**Table 1**

Figure 1 shows the graph of the power function of the test used by the consultant.



**Figure 1**

(e) On Figure 1 draw the graph of the power function of the manager's test.

(2)

(f) State, giving your reasons, which test you would recommend.

(2)





5. The weights of the contents of breakfast cereal boxes are normally distributed. A manufacturer changes the style of the boxes but claims that the weight of the contents remains the same.

A random sample of 6 old style boxes had contents with the following weights (in grams).

$$512 \quad 503 \quad 514 \quad 506 \quad 509 \quad 515$$

The weights,  $y$  grams, of the contents of an independent random sample of 5 new style boxes gave

$$\bar{y} = 504.8 \text{ and } s_y = 3.420$$

- (a) Use a two-tail test to show, at the 10% level of significance, that the variances of the weights of the contents of the old and new style boxes can be assumed to be equal. State your hypotheses clearly. **(5)**
- (b) Showing your working clearly, find a 90% confidence interval for  $\mu_x - \mu_y$ , where  $\mu_x$  and  $\mu_y$  are the mean weights of the contents of old and new style boxes respectively. **(7)**
- (c) With reference to your confidence interval comment on the manufacturer's claim. **(2)**

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**Question 5 continued**

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6. A random sample  $X_1, X_2, \dots, X_n$  is taken from a population where each of the  $X_i$  have a continuous uniform distribution over the interval  $[0, \beta]$ .  
The random variable  $Y = \max\{X_1, X_2, \dots, X_n\}$ .  
The probability density function of  $Y$  is given by

$$f(y) = \begin{cases} \frac{n}{\beta^n} y^{n-1} & 0 \leq y \leq \beta \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that  $E(Y^m) = \frac{n}{n+m} \beta^m$ . (3)

(b) Write down  $E(Y)$ . (1)

(c) Using your answers to parts (a) and (b), or otherwise, show that

$$\text{Var}(Y) = \frac{n}{(n+1)^2(n+2)} \beta^2$$
 (3)

(d) State, giving your reasons, whether or not  $Y$  is a consistent estimator of  $\beta$ . (3)

The random variables  $M = 2\bar{X}$ , where  $\bar{X} = \frac{1}{n}(X_1 + X_2 + \dots + X_n)$ , and  $S = kY$ , where  $k$  is a constant, are both unbiased estimators of  $\beta$ .

(e) Find the value of  $k$  in terms of  $n$ . (1)

(f) State, giving your reasons, which of  $M$  and  $S$  is the better estimator of  $\beta$  in this case. (3)

Five observations of  $X$  are: 8.5 6.3 5.4 9.1 7.6

(g) Calculate the better estimate of  $\beta$ . (2)

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Question 6 continued

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7. A machine produces components whose lengths are normally distributed with mean 102.3 mm and standard deviation 2.8 mm. After the machine had been serviced, a random sample of 20 components were tested to see if the mean and standard deviation had changed. The lengths,  $x$  mm, of each of these 20 components are summarised as

$$\sum x = 2072 \quad \sum x^2 = 214\,856$$

(a) Stating your hypotheses clearly, test, at the 5% level of significance, whether or not there is evidence of a change in standard deviation.

(7)

(b) Stating your hypotheses clearly, test, at the 5% level of significance, whether or not the mean length of the components has changed from the original value of 102.3 mm using

- (i) a normal distribution,
- (ii) a  $t$  distribution.

(9)

(c) Comment on the mean length of components produced after the service in the light of the tests from part (a) and part (b). Give a reason for your answer.

(2)

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