



1. A teacher wishes to test whether playing background music enables students to complete a task more quickly. The same task was completed by 15 students, divided at random into two groups. The first group had background music playing during the task and the second group had no background music playing.  
The times taken, in minutes, to complete the task are summarised below.

	Sample size $n$	Standard deviation $s$	Mean $\bar{x}$
With background music	8	4.1	15.9
Without background music	7	5.2	17.9

You may assume that the times taken to complete the task by the students are two independent random samples from normal distributions.

- (a) Stating your hypotheses clearly, test, at the 10% level of significance, whether or not the variances of the times taken to complete the task with and without background music are equal. (5)
- (b) Find a 99% confidence interval for the difference in the mean times taken to complete the task with and without background music. (7)

Experiments like this are often performed using the same people in each group.

- (c) Explain why this would not be appropriate in this case. (1)

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2. As part of an investigation, a random sample of 10 people had their heart rate, in beats per minute, measured whilst standing up and whilst lying down. The results are summarised below.

Person	1	2	3	4	5	6	7	8	9	10
Heart rate lying down	66	70	59	65	72	66	62	69	56	68
Heart rate standing up	75	76	63	67	80	75	65	74	63	75

(a) State one assumption that needs to be made in order to carry out a paired  $t$ -test. **(1)**

(b) Test, at the 5% level of significance, whether or not there is any evidence that standing up increases people’s mean heart rate by more than 5 beats per minute. State your hypotheses clearly. **(8)**

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3. A manager in a sweet factory believes that the machines are working incorrectly and the proportion  $p$  of underweight bags of sweets is more than 5%. He decides to test this by randomly selecting a sample of 5 bags and recording the number  $X$  that are underweight. The manager sets up the hypotheses  $H_0: p = 0.05$  and  $H_1: p > 0.05$  and rejects the null hypothesis if  $x > 1$ .

(a) Find the size of the test. (2)

(b) Show that the power function of the test is

$$1 - (1 - p)^4(1 + 4p) \quad (3)$$

The manager goes on holiday and his deputy checks the production by randomly selecting a sample of 10 bags of sweets. He rejects the hypothesis that  $p = 0.05$  if more than 2 underweight bags are found in the sample.

(c) Find the probability of a Type I error using the deputy's test. (2)

**Question 3 continues on page 12**

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**Question 3 continued**

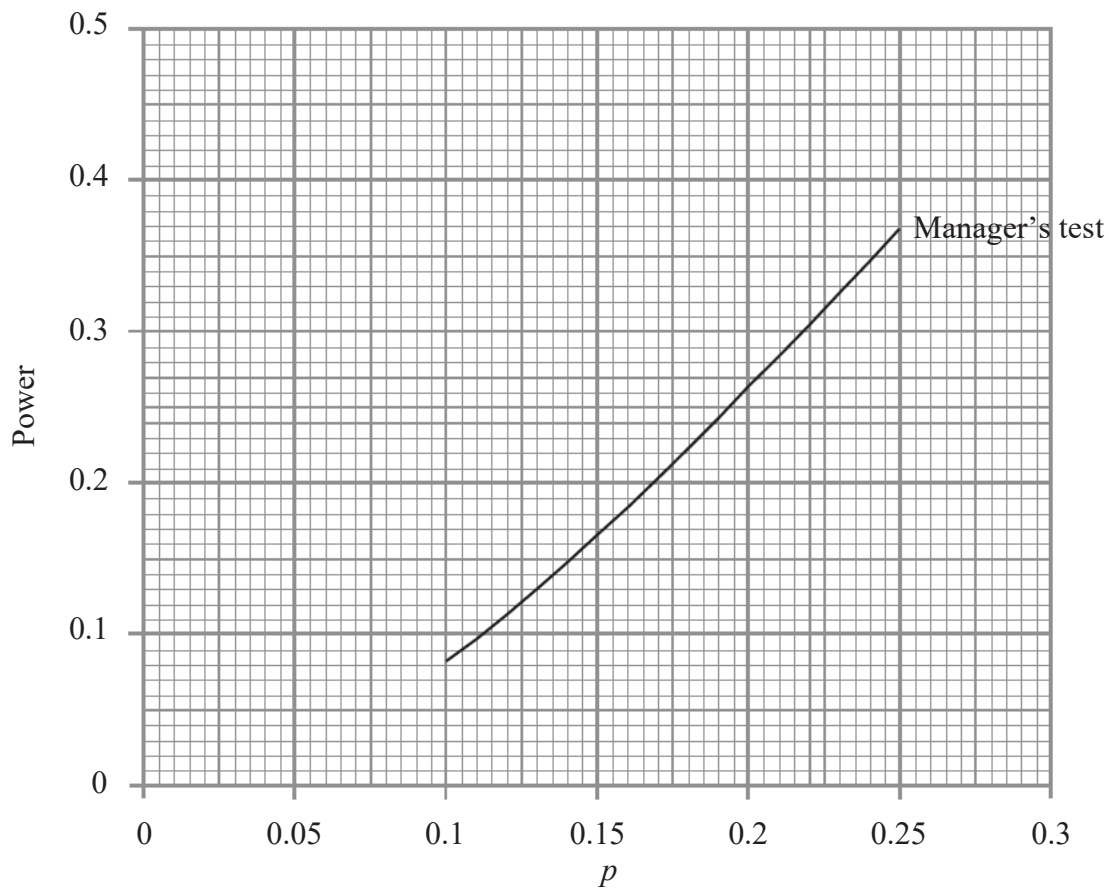
The table below gives some values, to 2 decimal places, of the power function for the deputy's test.

$p$	0.10	0.15	0.20	0.25
Power	0.07	$s$	0.32	0.47

(d) Find the value of  $s$ .

(1)

The graph of the power function for the manager's test is shown in Figure 1.



**Figure 1**

(e) On the same axes, draw the graph of the power function for the deputy's test.

(1)

(f) (i) State the value of  $p$  where these graphs intersect.

(ii) Compare the effectiveness of the two tests if  $p$  is greater than this value.

(2)

The deputy suggests that they should use his sampling method rather than the manager's.

(g) Give a reason why the manager might not agree to this change.

(1)



### Question 3 continued

Lined writing area for the answer to Question 3.

**(Total 12 marks)**

**Q3**







5. A car manufacturer claims that, on a motorway, the mean number of miles per gallon for the Panther car is more than 70. To test this claim a car magazine measures the number of miles per gallon,  $x$ , of each of a random sample of 20 Panther cars and obtained the following statistics.

$$\bar{x} = 71.2 \quad s = 3.4$$

The number of miles per gallon may be assumed to be normally distributed.

- (a) Stating your hypotheses clearly and using a 5% level of significance, test the manufacturer's claim. (5)

The standard deviation of the number of miles per gallon for the Tiger car is 4.

- (b) Stating your hypotheses clearly, test, at the 5% level of significance, whether or not there is evidence that the variance of the number of miles per gallon for the Panther car is different from that of the Tiger car. (6)

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6. Faults occur in a roll of material at a rate of  $\lambda$  per  $\text{m}^2$ . To estimate  $\lambda$ , three pieces of material of sizes  $3 \text{ m}^2$ ,  $7 \text{ m}^2$  and  $10 \text{ m}^2$  are selected and the number of faults  $X_1$ ,  $X_2$  and  $X_3$  respectively are recorded.

The estimator  $\hat{\lambda}$ , where

$$\hat{\lambda} = k(X_1 + X_2 + X_3)$$

is an unbiased estimator of  $\lambda$ .

- (a) Write down the distributions of  $X_1$ ,  $X_2$  and  $X_3$  and find the value of  $k$ . (4)

- (b) Find  $\text{Var}(\hat{\lambda})$ . (3)

A random sample of  $n$  pieces of this material, each of size  $4 \text{ m}^2$ , was taken. The number of faults on each piece,  $Y$ , was recorded.

- (c) Show that  $\frac{1}{4}\bar{Y}$  is an unbiased estimator of  $\lambda$ . (2)

- (d) Find  $\text{Var}(\frac{1}{4}\bar{Y})$ . (3)

- (e) Find the minimum value of  $n$  for which  $\frac{1}{4}\bar{Y}$  becomes a better estimator of  $\lambda$  than  $\hat{\lambda}$ . (2)

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