edexcel
advancing learning, changing lives

Mark Scheme (Results)
Summer 2009

GCE

GCE Mathematics (6686/01)

A PEARSON COMPANY

June 2009
6686 Statistics S4
Mark Scheme

Question Number	Scheme		Marks
Q1	H_0: $\mu = 5$; H_1: $\mu < 5$	both	B1
	CR: $t_9(0.01) > 2.821$		B1
	$\bar{x} = 4.91$		B1
	$s^2 = \frac{1}{9}\left(241.2 - \frac{49.1^2}{10}\right) = 0.0132222$	s= awrt 0.115	M1 A1
	$t = \frac{\lvert 4.91 - 5 \rvert}{\frac{\sqrt{0.013222}}{\sqrt{10}}} = \pm 2.475$	2.47 – 2.48	M1 A1
	Since 2.475 is not in the critical region there is insufficient evidence to reject H_0 and conclude that the mean diameter of the bolts is not less than (not equal to) 5mm.		A1ft **[8]**

Question Number	Scheme	Marks
Q2 (a)	The differences are normally distributed	B1 (1)
(b)	The data is collected in pairs or small sample size and variance unknown or samples not independent	B1 (1)
(c)	d: 2.5, 1.6, 1.6, –1.9, –0.6, 4.5 at least 2 correct	M1
	$(\Sigma d = 7.7, \Sigma d^2 = 35.59)$ $\bar{d} = \pm 1.2833$, sd = 2.2675. (Var = 5.141)	A1, A1
	H_0: $\mu_d = 0$, H_1: $\mu_d > 0$ (H_1 : $\mu_d < 0$ if d - 2.5, -1.6, -1.6 etc) both depend on their d's	B1
	$t = \dfrac{\pm 1.2833\sqrt{6}}{2.2675} = \pm 1.386......$ formula and substitution, 1.38 – 1.39	M1, A1
	Critical value $t_5(5\%) = 2.015$ (1 tail)	B1
	Not significant. Insufficient evidence to support that the device reduces CO_2 emissions.	A1 ft (8)
(d)	The idea that the device reduces $C0_2$ emissions has been rejected when in fact it does reduce emissions. OR Concluding that the device does not reduce emissions when in fact it does (if not in context can get B1 only)	B1 B1 (2) [12]
	(b) Allow because the same car has been used (c) awrt ± 1.28 , 2.27	

Question Number	Scheme	Marks
3 (a)	Size is the probability of H_0 being rejected when it is in fact true. or P(reject H_0/ H_0 is true) oe	B1 (1)
(b)	The power of the test is the probability of rejecting H_0 when H_1 is true. or P(rejecting H_0/H_1 is true) / P(rejecting H_0/H_0 is false) oe	B1 (1)
(c)	$X \sim B(12,0.5)$	B1
	$P(X \le 2) = 0.0193$	M1
	$P(X \ge 10) = 0.0193$	
	$\therefore$ critical region is $\{X \le 2 \cup X \ge 10\}$	A1A1 (4)
(d)(i)	P(Type II error) = $P(3 \le X \le 9 \mid p = 0.4)$	M1
	$= P(X \le 9) - P(X \le 2)$	M1dep
	$= 0.9972 - 0.0834$	
	$= 0.9138$	A1
(ii)	Power = $1 - 0.9138$ $= 0.0862$	B1 ft (4)
(e)	Increase the sample size	B1
	Increase the significance level/larger critical region	B1 (2)
		[12]
Notes	(d) (i) first M1 for either correct area or follow through from their critical region 2nd M1 dependent on them having the first M1. for finding their area correctly A1 cao (ii) B1 follow through from their (i)	

Question Number	Scheme	Marks
Q4 (a)	$H_0 : \sigma_A^2 = \sigma_B^2,\ H_1 : \sigma_A^2 \neq \sigma_B^2$	B1
	critical values $F_{12,8} = 3.28$ and $\frac{1}{F_{8,12}} = 0.35$	B1
	$\frac{s_B^2}{s_A^2} = 2.40 \quad \left(\frac{s_A^2}{s_B^2} = 0.416\right)$	M1A1
	Since 2.40 (0.416) is not in the critical region we accept H_0 and conclude there is no evidence that the two variances are different.	A1ft
		(5)
(b)	$S_p^2 = \frac{8 \times 1.02 + 12 \times 2.45}{20}$	M1
	$= 1.878$	A1
	$(27.94 - 25.54) \pm 2.086 \times \sqrt{1.878} \times \sqrt{\frac{1}{9} + \frac{1}{13}}$	B1M1 A1ft
	$(1.16,\ 3.64)$	A1 A1
		(7)
(c)	To calculate the confidence interval the variances need to be equal.	B1
	In part (a) the test showed they are equal.	B1
		(2)
		[14]

Question Number	Scheme	Marks
Q5 (a)	95% confidence interval for μ is 2.145	B1
	$560 \pm t_{14}(2.5\%)\sqrt{\frac{25.2}{15}} = 560 \pm 2.145\sqrt{\frac{25.2}{15}} = (557.2,\ 562.8)$	M1 A1 A1
		(4)
(b)	95% confidence interval for σ^2 is	
	$5.629 < \frac{14\times 25.2}{\sigma^2} < 26.119$	B1, M1, B1
	$\sigma^2 < 62.675\ \ \sigma^2 > 13.507$	
	$13.507 < \sigma^2 < 62.675$ awrt 13.5, 62.7	A1, A1
		(5)
(c)	Require $P(X > 565) = P\left(Z > \frac{565-\mu}{\sigma}\right)$ to be as large as possible OR	
	$\frac{565-\mu}{\sigma}$ to be as small as possible; both imply highest σ and μ.	M1
	$\frac{565-562.8}{\sqrt{62.675}} = 0.28$	M1A1
	$P(Z > 0.28) = 1 - 0.6103 = 0.3897$ awrt 0.39 – 0.40	M1 A1
		(5)
	(c) M1 for using their largest σ and μ	**[14]**
	M1 for using $\frac{x-\mu}{\sigma}$	
	M1 1 – their prob	

Question Number	Scheme	Marks
Q6 (a)	$E(\frac{2}{3}X_1 + \frac{1}{2}X_2 + \frac{5}{6}X_3) = \frac{2}{3} \times \frac{k}{2} + \frac{1}{2} \times \frac{k}{2} + \frac{5}{6} \times \frac{k}{2} = k$	M1 A1
	$E(X_1 + X_2 + X_3) = k \Rightarrow$ unbiased	B1
		(3)
(b)	$E(aX_1 + bX_2) = a\frac{k}{2} + b\frac{k}{2} = k$	M1
	$a + b = 2$	A1
	$\text{Var}(aX_1 + bX_2) = a^2\frac{k^2}{12} + b^2\frac{k^2}{12}$	M1A1
	$= a^2\frac{k^2}{12} + (2-a)^2\frac{k^2}{12}$	M1
	$= (2a^2 - 4a + 4)\frac{k^2}{12}$	
	$= (a^2 - 2a + 2)\frac{k^2}{6}$ (*) since answer given	A1 cso
(c)		(6)
	Min value when $(2a-2)\frac{k^2}{6} = 0$ $\frac{d}{da}(\text{Var}) = 0$, all correct, condone missing $\frac{k^2}{6}$	M1A1
	$\Rightarrow 2a - 2 = 0$ $a = 1, b = 1.$	A1A1
	$\frac{d^2(\text{Var})}{da^2} = \frac{2k^2}{6} > 0$ since $k^2 > 0$ therefore it is a minimum	M1
	min variance $= (1 - 2 + 2)\frac{k^2}{6}$ $= \frac{k^2}{6}$	B1
		(6)
	Alternative $\frac{k^2}{6}(a-1)^2 - \frac{k^2}{6} + \frac{2k^2}{6}$	M1 A1
	$\frac{k^2}{6}(a-1)^2 + \frac{k^2}{6}$	M1
	Min when $\frac{k^2}{6}(a-1)^2 = 0$	A1A1
	$a = 1$ $b = 1$	B1
	min var $= k^2/6$	