

Mark Scheme (Final) Summer 2007

GCE

GCE Mathematics (6686/01)

June 2007
6686 Statistics S4
Mark Scheme

Question Number	Scheme	Marks
1. a	<p>d: 14 2 18 25 0 -8 4 4 12 20</p> $\bar{d} = \pm 9.1 \quad sd = \sqrt{106.7} = 10.332..$ $(\sum d = 91, \quad \sum x^2 = 1789)$ <p>$H_0 : \mu_d = 0 \quad H_1 : \mu_d \neq 0$</p> $t = \pm \frac{9.1\sqrt{10}}{10.332} = \pm 2.785 \quad \text{awrt } \pm 2.78 \text{ or } 2.79$ <p>Critical value $t_9 = \pm 1.833$</p> <p>Significant. There is a difference between <u>blood pressure</u> measured by arm cuff and finger monitor.</p>	M1 A1 A1 B1 M1 A1 B1 A1 (8)
b.	<p>The <u>difference in measurements</u> of blood pressure is <u>normally</u> distributed</p> <p>Notes.</p> <p>(a) One tail test Loses the first B1 . CV is 1.383 in this case. Can get 7/8</p> <p>(b) looking for the difference in measurements. Not just it is normally distributed.</p>	B1 (1)

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2. a)	$E(\bar{X}) = \mu$	B1
	$\text{Var}(\bar{X}) = \text{Var}\left(\frac{X_1 + X_2 + X_3 + \dots + X_n}{n}\right)$ $= \frac{\sigma^2}{n}$	B1 (2)
b)	$E(U) = \frac{1}{n+m}(nE(\bar{X}) + mE(\bar{Y}))$ $= \frac{1}{n+m}(n\mu + m\mu)$ $= \mu \Rightarrow U \text{ is unbiased}$	M1 A1 state unbiased A1 (3)
c)	$\text{Var}(\bar{Y}) = \frac{\sigma^2}{m}$ $\text{Var}(U) = \frac{n^2\text{Var}(\bar{X}) + m^2\text{Var}(\bar{Y})}{(n+m)^2}$ $= \frac{n^2 \frac{\sigma^2}{n} + m^2 \frac{\sigma^2}{m}}{(n+m)^2}$ $= \frac{n\sigma^2 + m\sigma^2}{(n+m)^2}$ $= \frac{\sigma^2}{n+m} \quad *$	B1 M1 A1 A1 cs0 A1 (4)
d)	$\frac{n\bar{X} + m\bar{Y}}{n+m}$ is a better estimate since variance is smaller.	B1 B1 (2)

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3. a	<p>$H_0 : \sigma_F = \sigma_M \quad H_1 : \sigma_F \neq \sigma_M$</p> <p>$s^2_F = \frac{1}{6}(17956.5 - 7 \times 50.6^2) = \frac{33.98}{6} = 5.66333\dots$</p> <p>$s^2_M = \frac{1}{9}(28335.1 - 10 \times 53.2^2) = \frac{32.7}{9} = 3.63333\dots$</p> <p>$\frac{s^2_F}{s^2_M} = 1.5587\dots$ (Reciprocal 0.6415)</p> <p>$F_{6,9} = 3.37$ (or 0.24)</p> <p>Not in critical region. <u>Variances</u> of the two distributions <u>are the same</u></p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>M1 A1</p> <p>B1</p> <p>A1</p> <p>(7)</p>
b.	<p>$H_0 : \mu_F = \mu_M \quad H_1 : \mu_F < \mu_M$</p> <p>Pooled estimate $s^2 = \frac{6 \times 5.66333\dots + 9 \times 3.63333}{15}$</p> <p>$= 4.44533$</p> <p>$s = 2.11$</p> <p>$t = \frac{50.6 - 53.2}{2.11 \sqrt{\frac{1}{7} + \frac{1}{10}}} = \pm 2.50$</p> <p>C.V. $t_{15}(5\%) = \pm 1.753$</p> <p>Significant. The mean length of the <u>females forewing</u> is less than the length of the males forewing</p> <p>Notes</p> <p>(a) need to have <u>variance</u> and <u>the same</u> o.e</p> <p>(b) need female and forewing(wing)</p>	<p>B1</p> <p>M1</p> <p>M1 A1</p> <p>B1</p> <p>A1</p> <p>(6)</p>

Question Number	Scheme	Mark
4.a)	$H_0: \sigma^2 = 0.9 \quad H_1: \sigma^2 \neq 0.9$ $\nu = 19$ <p>CR (Lower tail 10.117) Upper tail 30.144</p> $\text{Test statistic} = \frac{19 \times 1.5}{0.9} = 31.6666, \quad \text{significant}$ <p>There is sufficient evidence that the <u>variance</u> of the length of spring is <u>different to 0.9</u></p>	<p>B1</p> <p>B1 B1</p> <p>M1 A1 A1</p> <p>(6)</p>
b)	$H_0: \mu = 100 \quad H_1: \mu > 100$ $t_{19} = 1.328$ $t = \frac{100.6 - 100}{\sqrt{\frac{1.5}{20}}} = 2.19$ <p>Significant. The mean <u>length of spring is greater than 100</u></p> <p>Notes (a) only need to see 30.144 need variance in conclusion (b) conclusion must be in context. Length of spring needed</p>	<p>B1</p> <p>B1</p> <p>M1 A1 A1</p> <p>B1</p> <p>(6)</p>

Question Number	Scheme	Mark
5.a)	Power = P (X ≤ 3 / λ) $= e^{-\lambda} + e^{-\lambda}\lambda + \frac{e^{-\lambda}\lambda^2}{2} + \frac{e^{-\lambda}\lambda^3}{6}$ $= \frac{e^{-\lambda}}{6} (6 + 6\lambda + 3\lambda^2 + \lambda^3)$	M1 A1 A1 (3)
b)	CR is X ≤ 3 Size = P[X ≤ 3 / λ = 7] = 0.0818	M1 A1 (2)
c)	P(Type II error) = 1 – power $= 1 - \frac{e^{-4}}{6} (6 + 6 \times 4 + 3 \times 4^2 + 4^3)$ = 0.5665..	M1 A1 (2)
6.a)	$\frac{\bar{X} - 250}{\frac{4}{\sqrt{15}}} > 2.3263 \quad \text{or} \quad \frac{\bar{X} - 250}{\frac{4}{\sqrt{15}}} < -2.3263$ 2.3262	± B1 M1
	$\bar{X} > 252.40... \quad \text{or} \quad \bar{X} < 247.6...$ 252 and 248	awrt A1 (3)
b)	$P(\bar{X} < 252.4 / \mu = 254) - P(\bar{X} < 247.6 / \mu = 254)$ $= P\left(Z < \frac{252.4 - 254}{\frac{4}{\sqrt{15}}}\right) - P\left(Z < \frac{247.6 - 254}{\frac{4}{\sqrt{15}}}\right)$ $= P(Z < -1.5492) - P(Z < -6.20)$ $= (1 - 0.9394) - (1 - 1)$ $= 0.0606$	using their '252.4' and '247.6' M1 stand using 4/√15, 254 their '252.4' or '247.6' M1 -1.5492 and -6.20 o.e. A1 M1 A1 (5)
	Notes (a) only needs to try and find one side for M1 (b) only need to see one of the standardisation for second M1 if consider only 252.4 and get 0.0606 they get M0 M1 A0 M1 A1 ie they can get 3/5	

Question Number	Scheme	Marks
7.	$\bar{x} = 4.01$ $s = 0.7992\dots$	B1 M1 A1
(a)	$4.01 \pm 2.5\% \frac{0.7992\dots}{\sqrt{10}} = 4.01 \pm 2.262 \frac{0.7992\dots}{\sqrt{10}}$ $= 4.5816\dots \text{ and } 3.4383\dots$	2.262 B1 M1 A1√ A1 (7)
(b)	$2.700 < \frac{9 \times 0.7992\dots^2}{s^2} < 19.023$ $\sigma^2 < 2.13, \quad \sigma^2 > 0.302$	2.7, 19.023 9 × s²/σ² B1 B1 M1 both awrt 2.13, 0.302 A1 (4)
(c)	$P(X > 7) = P\left(Z > \frac{7 - \mu}{\sigma}\right)$ <p>needs to be as high as possible</p> <p>Therefore μ and σ must be as big as possible</p> $= P\left(Z > \frac{7 - 4.581}{\sqrt{2.13}}\right)$ $= 1 - 0.9515$ $= 0.0485$ $= 4.85\%$	M1 M1 A1√ 4.8 to 4.9 A1 (4)
<p>Notes</p> <p>(a) $s^2 = 0.63877\dots$</p> <p>(c) M1 may be implied by them using their highest μ and σ.</p>		