June 2006
6686 Statistics S4
Mark Scheme

Question Number	Scheme	Marks
1.	$H_0: \mu = 1012$ $\quad H_1: \mu \neq 1012$ both	B1
	$\bar{x} = \frac{13700}{14}$ $(= 978.57\ldots)$	M1
	$S_x^2 = \frac{13\,448\,750 - 14\bar{x}^2}{13}$ $(= 3255.49\ldots)$ $(S_x = 57.056\ldots)$ S^2 or S	M1
	$t_{13} = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{978.6 - 1012}{57.06/\sqrt{14}} = -2.19\ldots$ AWRT -2.19	M1 A1
	$t_{13}(5\%)$ 2tail c.v. $= -2.160$ $(\pm)$	B1
	Significant result – there is evidence of a change in mean weight of squirrels (condone decrease) must mention weight	A1$\sqrt{}$ (7)
2.	(a) $\left(\bar{x} = \frac{466}{4} = 116.5\right)$ $\quad S_x^2 = \frac{54386 - 4\bar{x}^2}{3}$, $= 32.\dot{3}$ or $\frac{97}{3}$ or AWRT 32.3	M1, A1
	$0.216 < \frac{3S_x^2}{\sigma^2} < 9.348$	B1 M1 B1
	$10.376\ldots < \sigma^2 < 449.07\ldots$ AWRT $10.4, 449$	A1, A1 (7)
	(b) $H_0: \sigma_M^2 = \sigma_S^2$ ($\sigma_m = \sigma_s$) $\quad H_1: \sigma_M^2 > \sigma_S^2$ ($\sigma_m > \sigma_s$) (are OK) both	B1
	$\frac{S_M^2}{S_S^2} = \frac{318.8}{32.\dot{3}} = 9.859\ldots$ AWRT 9.86	M1 A1
	$F_{6,3}$ (1% c.v.) $= 27.91$	B1
	$9.85 < 27.91$, Insufficient evidence of an increase in variance / to say $\sigma_M^2 > \sigma_S^2$ is OK. / variance can be assumed to be the same is OK	A1$\sqrt{}$ (5) (12)
	(b) [NB. $\frac{32.\dot{3}}{318.8} = 0.101\ldots$ only gets M1A1 if appropriate F value attempted]	

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3.	(D = Without Solar heating − with Solar heating) $\mu_1 = \mu_2$ etc is B0 $\begin{bmatrix} \bar{D}=0 & \bar{D}>0 \\ D=0 & D>0 \end{bmatrix}$	B0 B0 B0
(a)	$H_0: \mu_D = 0$ $\quad H_1: \mu_D > 0$	B1
	d: 6, −3, 7, −2, −8, 6, 5, 11, 5 (Attempt d's)	M1
	$\bar{d} = 3$, $s_d = 6$ $\left(= \sqrt{\frac{369 - 9\times 3^2}{8}}\right)$ or $s_d^2 = 36$ $\left(\frac{\Sigma d}{9}\ \text{M1}\right)$	M1, M1
	$t_8 = \frac{3-0}{6/\sqrt{9}} = 1.5$ (±)	M1 A1 cao
	t_8 (5% 1tail c.v.) = 1.860	B1
	Not significant − insufficient evidence (that solar heating has) decreased weekly fuel consumption.	A1 ∫ (8)
(b)	Difference in weekly fuel consumption is normally distributed.	B1 (1) 9
4. (a)	($H_0: \sigma_A^2 = \sigma_B^2$ $\quad H_1: \sigma_A^2 \neq \sigma_B^2$)	
	$\frac{s_A^2}{s_B^2} = \frac{0.721^2}{0.572^2} = 1.588\ldots$ Awrt 1.59	M1 A1
	$F_{8,9}$ (5%) c.v. [= 10% 2tail] = 3.23	B1
	Not significant, can assume variances are equal. (accept $\sigma_A^2 = \sigma_B^2$)	B1 cao (4)
(b)	$s_p^2 = \frac{8\times 0.721^2 + 9\times 0.572^2}{8+9} = 0.41784\ldots$ 0.417… or Awrt 0.418	M1 A1
	t_{17} (2.5%) c.v. = 2.110	B1
	95% CI = $\bar{x}_B - \bar{x}_A \pm 2.110 \times s_p \times \sqrt{\frac{1}{9} + \frac{1}{10}}$	M1
	$= 0.02 \pm 2.110 \times \sqrt{0.417\ldots} \times \sqrt{\frac{1}{9} + \frac{1}{10}}$	A1 ∫
	= (−0.6066…, 0.6466…) Awrt (−0.607, 0.647)	A1, A1 (7)
(c)	± 0.7 is outside interval	B1 ∫
	∴ manager need not be concerned (dep) (allow ∫ if 0.7 inside)	B1 ∫ (2) 13

5 (a)	X_1 = no. of defects in 15 m. $X_1 \sim Po(4.5)$ Use of $Po(4.5)$	M1
	Size = $P(X_1 \geq 9) = 1 - P(X \leq 8) = 1 - 0.9597 = \underline{0.0403}$ (AWRT)	A1 (2)
(b)	$r = P(X_2 \geq 9 \mid X_2 \sim Po(9))$, $= 1 - P(X_2 \leq 8) = 1 - 0.4557 = \underline{0.54}(43)$ (AWRT)	M1, A1 (2)
(c)	Y_1 = no. of defects in 10 m $Y_1 \sim Po(3)$ Use of $Po(3)$ to find $P(Y \geq c)$	M1
	Require smallest c so that $P(Y_1 \geq c) < 0.10$. Tables $\underline{Y_1 \geq 6}$	A1 (2)
(d)	Size = $P(Y_1 \geq 6) = 1 - P(Y_1 \leq 5) = 1 - 0.9161 = \underline{0.0839}$	B1 (1)
(e)	$s = 1 - P(Y_2 \leq 5 \mid Y_2 \sim Po(8))$, $= 1 - 0.1912 = 0.8088$ (AWRT $\underline{0.81}$)	M1, A1 (2)
(f)	See graph	4 (4)
(g)	(i) $0.62 \sim 0.67$ (ii) Test 1 is more powerful	B1 B1 (2)
(h)	Test 2 has higher P(Type I error) but cost of this is low	B1 Test 2
	Test 2 is more powerful for $\lambda < 0.7$ and $\lambda > 0.7$ is rare	B1 Reason (2)
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6. (a)	$E(X^n) = \int_0^t x^n \frac{1}{t}\,dx = \left[\frac{x^{n+1}}{t(n+1)}\right]_0^t = \left(\frac{t^{n+1}}{t(n+1)} - 0\right) = \underline{\underline{\frac{t^n}{n+1}}}$ $\int x^n \frac{1}{t}$ $\int_0^t dx$	M1 M1 A1 cso (3)
(b)	$\left(E(X) = \frac{t}{2}\right)$ $\underline{E(S) = kE(X)E(Y)}$, $= k \cdot \frac{t^2}{4}$	M1, A1
	$E(S) = t^2 \Rightarrow \underline{k = 4}$	A1 (3)
(c)	$Var(XY) = E(X^2)E(Y^2) - [E(XY)]^2$	M1
	$= \frac{t^2}{3} \times \frac{t^2}{3} - \left(\frac{t^2}{4}\right)^2 = \left\{\frac{7t^4}{144}\right\}$	M1
	$Var(S) = k^2 Var(XY) = 16 \times \frac{7t^4}{144} = \underline{\frac{7t^4}{9}}$	A1 cso (3)
(d)	$E(U) = t^2 \Rightarrow 2E(X^2)q = t^2, \Rightarrow 2\frac{t^2}{3}q = t^2, \Rightarrow \underline{q = \frac{3}{2}}$	M1, M1, A1 cso (3)
(e)	$Var(U) = q^2[Var(X^2) + Var(Y^2)] = 2q^2 Var(X^2)$	M1
	$Var(X^2) = E(X^4) - [E(X^2)]^2 = \frac{t^4}{5} - \left(\frac{t^2}{3}\right)^2 = \left(\frac{4}{45}t^4\right)$	M1
	$Var(U) = 2 \times \frac{9}{4} \times \frac{4}{45} t^4 = \underline{\underline{\frac{2}{5}t^4}}$	A1 (3)
(f)	$\frac{2}{5} < \frac{7}{9}$ $\therefore$ U is better $\because$ smaller variance	B1ft (1)
(g)	Using U estimate is: $\frac{3}{2}(2^2 + 3^2) = \frac{3}{2} \times 13 = \underline{\underline{\frac{39}{2}}}$ or $\underline{\underline{19.5}}$	B1ft (1)
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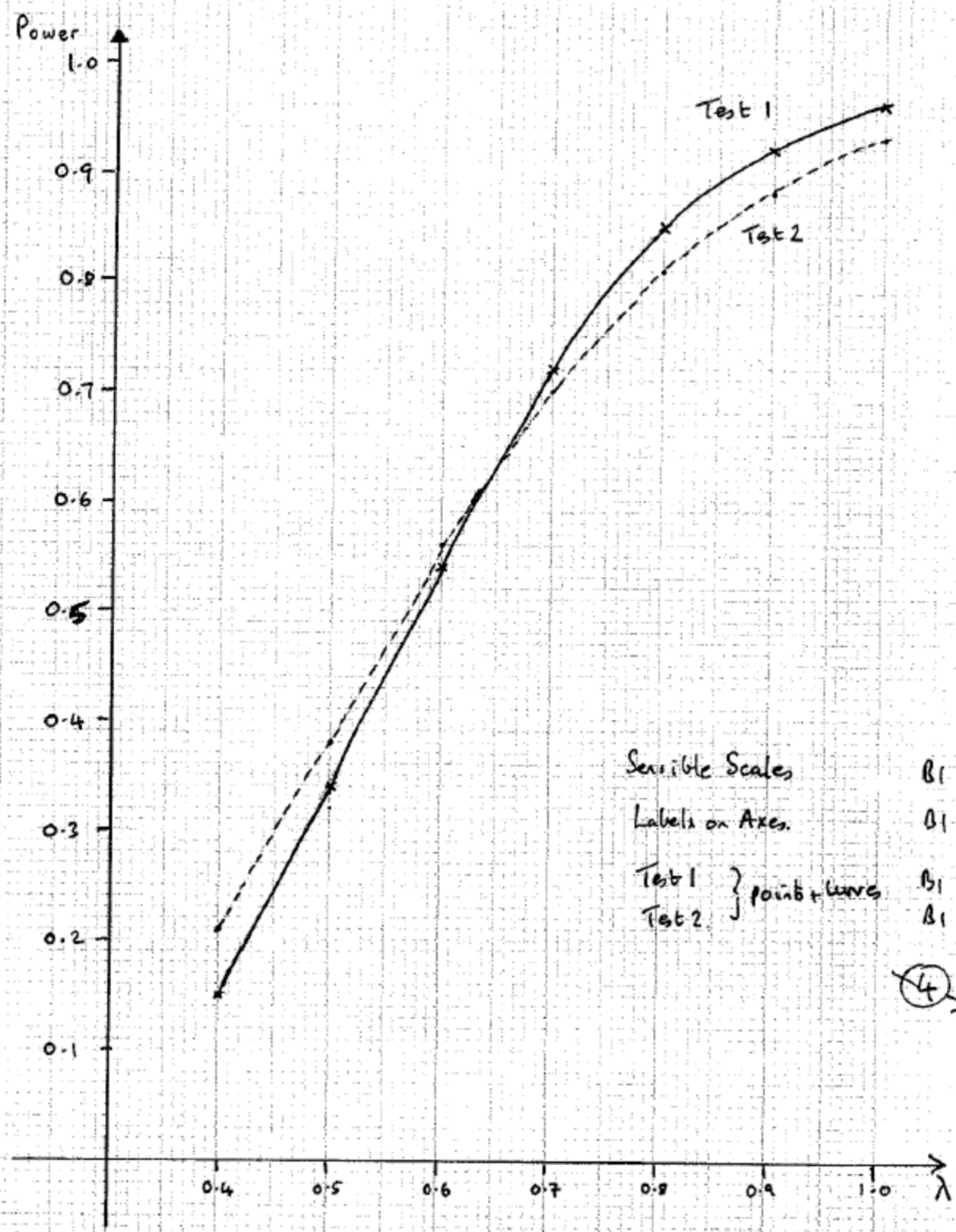
Power Functions



Sensible Scales B1

Labels on Axes. B1

Test 1 } point + curve B1

Test 2 } B1

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